



**[eduresourceza.com](http://eduresourceza.com)**

NAME: \_\_\_\_\_



**PRELIMINARY EXAMINATION 2023**

**MATHEMATICS GRADE 12**

**PAPER 1**

Time: 3 hours

Total: 150

Examiner: P R Mhuka

Moderator: S McConkey  
N Eleftheriades

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 22 pages and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Answers must be rounded off to the first decimal place, unless otherwise stated.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.

Page 1 of 22

|         |         |         |         |        |         |         |         |        |          |          |              |
|---------|---------|---------|---------|--------|---------|---------|---------|--------|----------|----------|--------------|
| 1<br>23 | 2<br>17 | 3<br>15 | 4<br>14 | 5<br>7 | 6<br>10 | 7<br>13 | 8<br>15 | 9<br>8 | 10<br>18 | 11<br>10 | Total<br>150 |
|         |         |         |         |        |         |         |         |        |          |          |              |

**SECTION A:**

**QUESTION 1:**

a) Simplify  $\frac{12^{x-2} \times 2^{x+2}}{8^x \times 3^{x-4}}$  (4)

b) Solve for  $x$  and show all your working:

1)  $3x^2 - 4x - 2 = 0$ . (3)

$$2) \sqrt{6x^2 - 15} = x + 1 \quad (4)$$

$$3) \log_3(x^2 - 4x + 4) = 1 + \log_3(x - 2) \quad (5)$$

- c)  $f(x) = 2x^2 - 3x + k$  for constant  $k$ . For which value of  $k$  will the  $x$ -axis be a tangent to  $f(x)$ ? (3)

- d) Let  $f(x) = px^3 - qx$ . At  $x = 0$ , the gradient of the curve of  $f(x)$  is 2. Given that  $f^{-1}(12) = 2$ , find the value of  $p$  and  $q$ . (4)

[23]

**QUESTION 2:**

- a) Consider the function  $f(x) = -2ax^2 + 2x$ .

Determine  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ . (5)

- b) The line  $y = 4x + n$  is tangent to the curve  $f(x) = -2x^3 + mx^2 - 1$  at  $x = 1$ . Find the values of  $m$  and  $n$ . (4)

c) A curve has the equation  $y = \sqrt{x} + \frac{9}{\sqrt{x}}$ .

1) Find the expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  (i. e.  $f''(x)$ ). (4)

2) Show that the curve has a stationary value when  $x = 9$ . (2)

3) Find the nature of this stationary value. (2)

[17]

**QUESTION 3:**

a) Consider the function  $p(x) = \log_2(2 - x)$

1) Is  $p(x)$  an increasing or decreasing function? (1)

2) Draw a neat sketch of  $p(x)$ . Clearly show the asymptote and intercepts with the axes. (3)

3) Write down the range of the  $p^{-1}(x) - 5$ . (1)

b) Given  $f(x) = \frac{4}{x-1} + 2$

1) Write down the domain and range of  $f(x)$ . (2)

2) Sketch the graph of  $f(x)$ , label all asymptotes and indicate the intercepts with the axes. (4)

3) Use your graph to determine the values of  $x$  for which  $\frac{4}{x-1} \geq -2$ . (2)

4) Determine the equation of the axis of symmetry of  $f(x-2)$ , that has a negative gradient. (2)

[15]

**QUESTION 4:**

a) 1, 3, 5, 7, . . .

The **first** odd number can be expressed as  $1 = 1^2 - 0^2$

The **second** odd number can be expressed as  $3 = 2^2 - 1^2$

The **third** odd number can be expressed as  $5 = 3^2 - 2^2$

1) Express the number 19 in this form. (1)

2) Write down a formula for the  $n^{th}$  odd number and simplify this expression. (2)

3) Prove that the product of two consecutive odd numbers is always odd. (3)

b) The first four terms of a quadratic sequence are:

$$6; a^2 + 2; 6a + 2; 11a$$

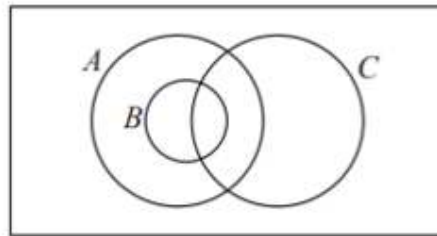
1) Determine the value(s) of  $a$ . (4)

2) Hence, determine an expression for the general term,  $T_n$ , of the quadratic sequence if  $a > 0$ . (4)

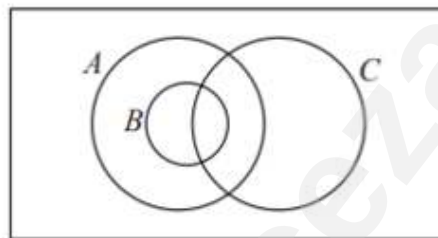
[14]

**QUESTION 5:**

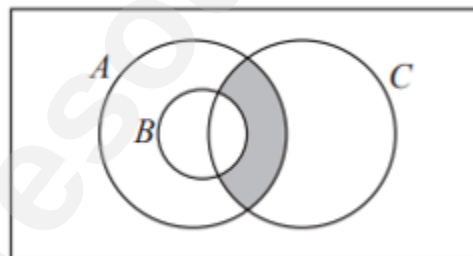
- a) The diagram below shows a universal set and three sets  $A$ ,  $B$  and  $C$ .



- 1) Shade on the diagram below the region that represents  $A \cap B \cap C$ . (2)

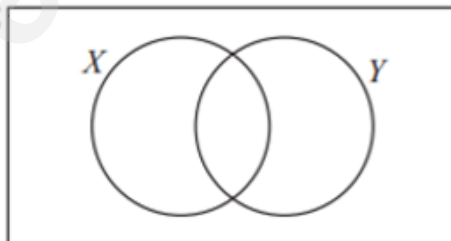


- 2) Express, in set notation, the set represented by the shaded region in the diagram below. (2)

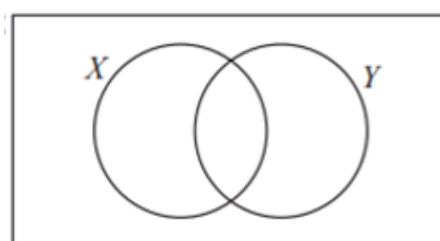


- b) The diagram below shows sets  $X$  and  $Y$ . Show, by means of two diagrams, that the set  $(X \cup Y)'$  is not the same as the set  $(X' \cup Y')$  (3)

$(X \cup Y)'$



$(X' \cup Y')$



[7]

**SECTION B:**

**QUESTION 6:**

- a) Abigail, Akhona, Ronewa, Maria, Griseld, Mira and Zane are waiting in line to buy matric dance concert tickets.
- 1) How many ways can they line up if Zane insists on being in the middle? (2)
  - 2) How many ways can they line up if Zane, Akhona and Griseld insists on being together? (3)
- b) Two players,  $A$  and  $B$ , alternately roll a fair six-sided dice, with  $A$  starting, until one of them obtains a six. Find the probability that  $B$  obtains the first six. (5)

[10]

**QUESTION 7:**

- a) Katerina takes out a bank loan of R100 000 to buy a premium electric car, at an annual rate of 5,49 %. The interest is calculated at the end of each year and added to the amount outstanding.
- 1) Find the amount of money Katerina would owe the bank after 10 years. (2)
  - 2) To pay off the loan, Katerina makes quarterly deposits of  $Rx$  at the end of every quarter. She makes her first deposit at the end of the first quarter after taking out the loan. Given that Katerina's aim is to own the electric car after 10 years, find the value of  $Rx$ . (6)

- b) Ronan opened a savings account with a single deposit of R1 200 on 1 June 2023. He then makes monthly deposit of R650 at the end of every month. His first payment is made on 30 June 2023 and his last payment on 30 November 2024. The account earns interest at 13% per annum compounded monthly. Determine the amount that should be in his savings account immediately after his last deposit is made that is on 30 November 2024. (5)

[13]

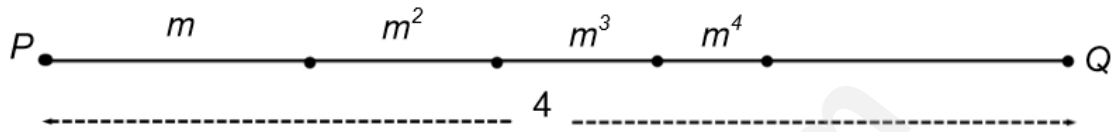
**QUESTION 8:**

- a) The first term of an arithmetic series is  $a$  and the common difference is  $d$ .  
The 18<sup>th</sup> term of the series is 25 and the 21<sup>st</sup> term of the series is  $32\frac{1}{2}$ . The  
sum of the first  $n$  terms is 2 750. Calculate the value of  $n$ . (6)

eduresourceza.com

- b) A tool for putting fence posts into the ground is called a post-rammer. The distance in millimeters that the post sinks into the ground on each impact of the post-rammer follow a geometric sequence. The first three impacts cause the post to sink into the ground by  $50\text{ mm}$ ,  $40\text{ mm}$  and  $32\text{ mm}$  respectively.
- 1) Calculate the first impact in which the post sinks less than  $10\text{ mm}$  into the ground. (3)
  - 2) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. (3)

- c) The following diagram shows  $PQ$ , with length  $4\text{ cm}$ . The line is divided into an infinite number of line segments. The diagram shows the first four segments.

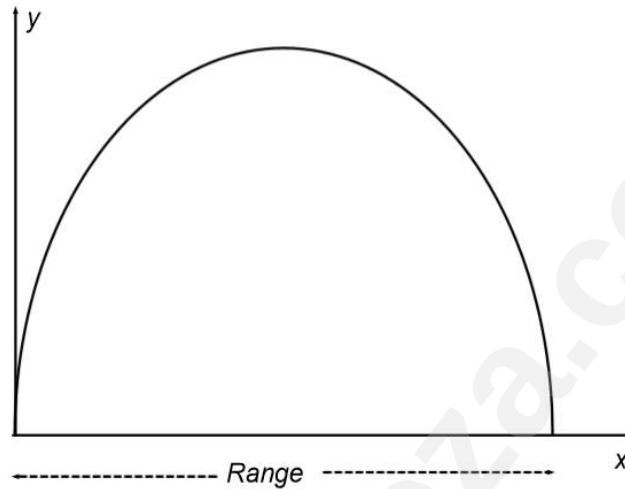


The length of the line segments are  $m\text{ cm}$ ;  $m^2\text{ cm}$ ;  $m^3\text{ cm}$ ;  $m^4\text{ cm}$ ; ...  
 where  $0 < m < 1$ . Show that  $m = \frac{4}{5}$ . (3)

[15]

**QUESTION 9:**

The path of a projectile is given by the equation  $y = \frac{-49x^2}{9000} + \sqrt{3}x$ , where  $x$  and  $y$  are measured in metres.



a) Find the range of this projectile. (2)

b) Find the maximum height reached by this projectile. (3)

c) Use the graph to find the values of  $t$  for which  $y - \frac{t}{2} = 0$  does not intersect the projectile. (3)

[8]

**QUESTION 10:**

A cubic function  $f(x) = ax^3 + bx^2 + cx + d$  has a maximum at  $(0 ; 5)$  and a point of inflection at  $(1 ; 1)$ .

- a) Find an expression of the cubic function in terms of  $x$ . (6)

- b) If  $f(x) = 2x^3 - 6x^2 + 5$ , find the  $x$ -coordinate of the minimum point and justify that it is a minimum. (3)

c) Sketch the graph of  $f(x)$ . (4)

d) Write the values of  $x$  where the graph is concave downward. (2)

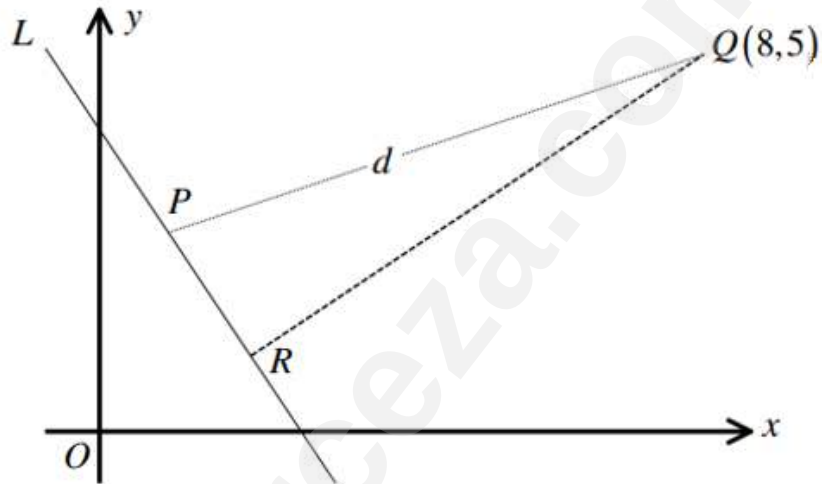
e) Determine the equation of the tangent to the curve at  $x = -1$ . (3)

[18]

**QUESTION 11:**

The straight line  $L$  has equation  $3x + 2y = 8$ .

The point  $P(x; y)$  lies on  $L$  and the point  $Q(8; 5)$  lies outside  $L$ . The point  $R$  lies on  $L$  so that  $QR$  is perpendicular to  $L$ . The length  $PQ$  is denoted by  $d$ .



- a) Show clearly that:  $d^2 = 65 - 13x + \frac{13x^2}{4}$ . (5)

Let  $f(x) = 65 - 13x + \frac{13}{4}x^2$ .

b) Use differentiation to find the stationary value of  $f(x)$ . (2)

c) State the coordinates of  $R$  and find, as an exact surd, the shortest distance of the point  $Q$  from  $L$ . (3)

[10]