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NAME: MEMORANDUM



SAHETI SCHOOL

PRELIMINARY EXAMINATION 2022

MATHEMATICS GRADE 12

PAPER 1

Time: 3 hours

Total: 150

Examiner: P R Mhuka

Moderator: S McConkey
N Eleftheriades

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 23 pages and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Answers must be rounded off to the first decimal place, unless otherwise stated.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.

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1 21	2 14	3 18	4 16	5 5	6 11	7 16	8 11	9 12	10 19	11 7	Total 150
										1	

SECTION A**QUESTION 1:**a) Solve for x and show all your working:

1) $\log_3(2x - 1) = 2$

(3)

$$\begin{aligned}
 2x - 1 &= 3^2 \checkmark \\
 2x &= 10 \\
 x &= 5 \checkmark \\
 \hline
 \end{aligned}$$

2) $\sqrt{5-x} - 1 = x$

(4)

$$\begin{aligned}
 \sqrt{5-x} &= x+1 \\
 5-x &= x^2 + 2x+1 \checkmark \\
 0 &= x^2 + 3x - 4 \checkmark \\
 0 &= (x+4)(x-1) \checkmark \\
 \underline{x \neq -4} \quad \underline{x=1} \checkmark
 \end{aligned}$$

3) $4 \cdot 2^{2x} + 7 \cdot 2^x = 2$

(4)

Let $2^x = k$

$4 \cdot k^2 + 7k - 2 = 0$ ✓

$k = 1/4 \quad k = -2$ ✓

$2^x = 2^{-2} \quad 2^x \neq -2$ ✓

$x = -2$ ✓

- b) Solve for x and y simultaneously if:
 $x - y - 3 = 0$ and $x^2 + xy - 2y^2 = 0$.

(5)

$x = y + 3$ ✓

$(y+3)^2 + y(y+3) - 2y^2 = 0$ ✓

$y^2 + 6y + 9 + y^2 + 3y - 2y^2 = 0$ ✓

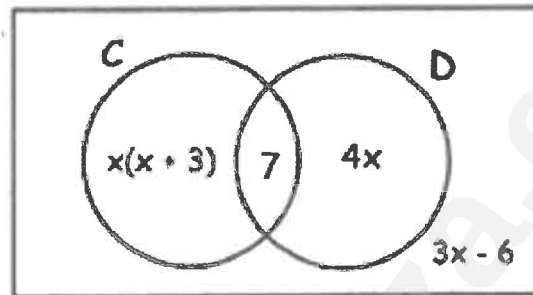
$9y = -9$

$y = -1$ ✓

$x = 2$ ✓

- c) The Venn diagram shows information about the pets owned by 40 students.

C = students who own a cat
 D = students who own a dog



A student is chosen at random. Work out the probability that they own a dog.

$$x^2 + 3x + 7 + 4x + 3x - 6 = 40 \quad (5) \quad \checkmark$$

$$x^2 + 10x - 39 = 0 \quad \checkmark$$

$$(x+13)(x-3) = 0$$

$$x = -13 \quad ; \quad x = 3 \quad \checkmark$$

$$P(D) = \frac{19}{40} \quad \checkmark$$

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QUESTION 2:

- a) Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 8$ (4)

$$\begin{aligned}
 f(x+h) &= 3(x+h)^2 - 8 \\
 &= 3x^2 + 6xh + 3h^2 - 8 \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 8 - (3x^2 - 8)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h} \\
 &= \underline{\underline{6x}}
 \end{aligned}$$

- b) Determine:

- 1) $f'(x)$, if it is given that $f(x) = 3x^{-3} + x$ (2)

$$\begin{aligned}
 f'(x) &= -9x^{-4} + 1 \\
 &= \frac{-9}{x^4} + 1
 \end{aligned}$$

$$\begin{aligned}
 2) \quad D_x \left[\frac{-3\sqrt[3]{x}-x}{x^3} \right] &= D_x \left[-3x^{-8/3} - x^{-2} \right] \\
 &= 8x^{-11/3} + 2x^{-3} \\
 &= \frac{8}{x^{11/3}} + \frac{2}{x^3}
 \end{aligned}
 \tag{4}$$

- c) A function is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$. Determine whether $f\left(\frac{x}{2}\right)$ is increasing or decreasing when $x = 2$. (4)

$$\begin{aligned}
 f\left(\frac{x}{2}\right) &= \left(\frac{x}{2}\right)^3 - 7\left(\frac{x}{2}\right) - 6 \\
 &= \frac{x^3}{8} - \frac{7x}{2} - 6
 \end{aligned}$$

$$f'\left(\frac{x}{2}\right) = \frac{3x^2}{8} - \frac{7}{2}$$

$$f'(2) = -2$$

$\therefore$ Decreasing at $x=2$.

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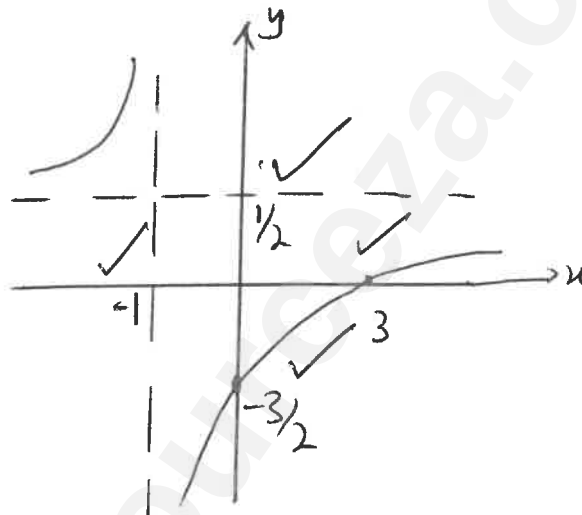
QUESTION 3:

a) Given $f(x) = \frac{-2}{x+1} + \frac{1}{2}$

- 1) Write down the equations of the asymptotes of $f(x)$. (2)

$y = \frac{1}{2}$ ✓ $x = -1$ ✓

- 2) Sketch the graph of $f(x)$, labelling the graph clearly. (4)



- 3) The line $y = x + k$ is the line of symmetry of $f(x)$. Write down the value of k . (2)

$y = x + \frac{3}{2}$

$k = \frac{3}{2}$ ✓✓

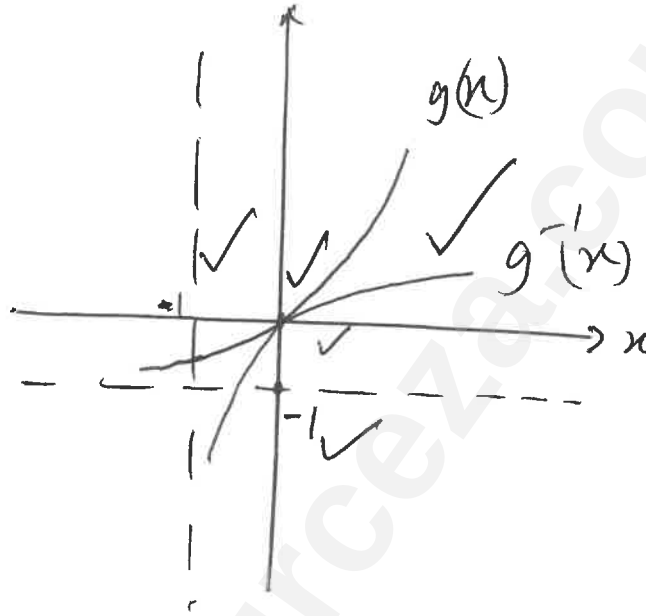
- 4) Write down the domain and range of $f(x)$. (2)

$x \in \mathbb{R} : x \neq -1$ ✓

$y \in \mathbb{R} : y \neq \frac{1}{2}$ ✓

b) Consider the function $g(x) = 3^x - 1$

- 1) Draw a neat sketch of $g(x)$ and its inverse on the same set of axes. Clearly show the asymptotes and intercepts with the axes. (5)



- 2) Write down the domain of the inverse. (1)

$$x > -1 \quad \checkmark$$

- 3) For which values of x would $g^{-1}(x) < g(x)$ (2)

$$x > -1 \quad \checkmark \quad x \neq 0 \quad \checkmark$$

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QUESTION 4:

- a) A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, $140 + 2d$ in week 3 and so on, until the company is producing 206 bicycles in week 12.

- 1) Find the value of d .

(2)

$$206 = 140 + 11d \quad \checkmark$$

$$\underline{d = 6} \quad \checkmark$$

- 2) After week 12 the company plans to continue making 206 bicycles each week. Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1. (5)

$$S_{12} = \frac{12}{2} (140 + 206) \quad \checkmark$$

$$= \underline{2076} \quad \checkmark$$

$$S_{40} = 40 \times 206 \quad \checkmark$$

$$= \underline{8240}$$

$$S_{52} = 8240 + 2076 \quad \checkmark$$

$$= \underline{10316}$$

$$\text{or } S_{11} = \frac{11}{2} (140 + 206 - 6) + (52 - 11) \times 206$$

$$= \underline{10316}$$

b) Given the quadratic sequence: 34; 42; 48; 52; ...

- 1) Determine an expression for the general term, T_n , of the quadratic sequence. (4)

$$\begin{array}{cccc}
 34 & 42 & 48 & 52 \\
 \swarrow & \searrow & \swarrow & \searrow \\
 & 8 & 6 & 4 \\
 & \swarrow & \searrow & \\
 & & -2 &
 \end{array}$$

$$\begin{aligned}
 2a &= -2 \\
 a &= -1 \\
 3(-1) + b &= 8 \\
 b &= 11 \\
 1 + 11 + c &= 34 \\
 c &= 24
 \end{aligned}$$

$$T_n = -n^2 + 11n + 24 \quad \checkmark$$

- 2) For a certain value of k , the k^{th} term of the quadratic sequence is equal to the k^{th} term in the first row of the differences. Determine the value of k . (5)

$$\begin{aligned}
 T_k &= -k^2 + 11k + 24 \quad \checkmark & T_k &= -2k + 10 \quad \checkmark \\
 -k^2 + 11k + 24 &= -2k + 10 \quad \checkmark \\
 0 &= k^2 - 13k - 14 \\
 k &= 14 \quad \checkmark & k &\neq -1 \quad \checkmark
 \end{aligned}$$

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QUESTION 5:

Freddie has 6 toy cars and 3 toy buses, all different.

Freddie arranges these 9 toys in a line.

- a) Find the number of possible arrangements of 9 toys. (2)

$$9! = 362\,880$$

- b) Find the number of possible arrangements if there is a car at each end of the line and no buses are next to each other. (3)

$$6 \times 4! \times 3! \times 5 \times 2!$$

$$= 8640$$

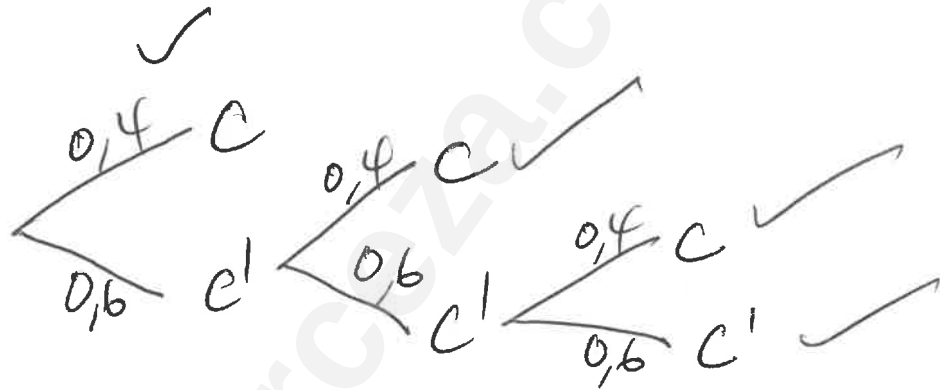
[5]

SECTION B

QUESTION 6:

- a) Eric is very forgetful. Every time he logs in to his online bank he only has a 40% chance of remembering his password correctly. He is allowed 3 unsuccessful attempts on any one day and then the bank will not let him try again until the next day.

- 1) Draw a fully labelled tree diagram to illustrate this situation. (4)

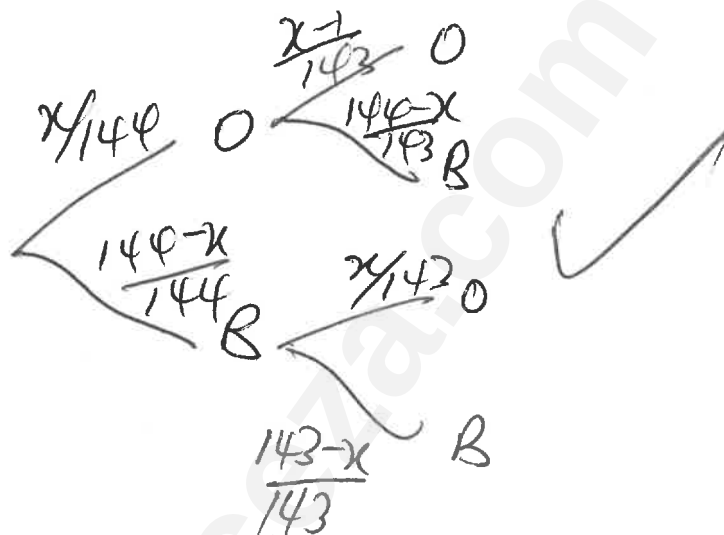


- 2) Calculate the probability of two unsuccessful attempts. (3)

$$0,6 \times 0,6 \times 0,4 \checkmark \checkmark$$

$$= 0,144 \checkmark$$

- b) A bag contains 144 ping-pong balls. More than half of the balls are painted orange, and the rest are painted blue. Two balls are drawn at random without replacement. The probability of drawing two balls of the same colour is the same as the probability of drawing two balls of different colours. How many orange balls are in the bag? (5)



$$P(OO) + P(BB) = P(OB) + P(BO)$$

$$\frac{x}{144} \cdot \frac{x-1}{143} + \frac{144-x}{144} \cdot \frac{143-x}{143} = \frac{x}{144} \cdot \frac{144-x}{143} + \frac{144-x}{144} \cdot \frac{x}{143} \quad [12]$$

$$x(x-1) + (144-x)(143-x) = x(144-x) + (144-x)x$$

$$x^2 - x + 20592 - 287x + x^2 = 144x - x^2 + 144x - x^2$$

$$4x^2 - 576x + 20592 = 0$$

$$x = 78 \quad x = 66$$

$\therefore$ Orange balls 78

QUESTION 7:

- a) Sihle obtained a loan at an annual interest rate of 5,68%, compounded monthly. She will pay back the loan with equal monthly instalments of R5 154,26, starting in three months' time and completing the repayment within three years of the date of the loan being granted.

- 1) Calculate the value of the loan, correct to the nearest rand. (4)

$$P = 5154,26 \left[\frac{1 - \left(1 + \frac{5,68\%}{12}\right)^{-36}}{\frac{5,68\%}{12}} \right] \left(1 + \frac{5,68\%}{12}\right)^2$$

$$= \underline{160\,000,15}$$

- 2) Calculate the balance outstanding on the loan, two years after it was approved. (3)

$$BO = 5154,26 \left[\frac{1 - \left(1 + \frac{5,68\%}{12}\right)^{-12}}{\frac{5,68\%}{12}} \right]$$

$$= \underline{59\,989,47}$$

b) A bus company owns vehicles to the value of R2 500 000.

- 1) The vehicles are depreciating at a rate of 17.9% per annum on a reducing balance. Calculate their market value after 6 years. (2)

$$A = 2\,500\,000 (1 - 17,9\%)^6 \checkmark$$

$$= \underline{765\,544,76} \checkmark$$

- 2) The average rate of inflation over the next 6 years is expected to be 5,5% per annum and management expects to sell the old vehicles for their market value in 6 years' time. Calculate the value of the sinking fund which will be necessary to buy brand new vehicles after 6 years. (3)

$$A = 2\,500\,000 (1 + 5,5\%)^6 \checkmark$$

$$= \underline{3\,447\,107,017} \checkmark$$

$$\therefore \text{Sinking fund value} = \underline{2\,681\,512,26} \checkmark$$

- 3) What quarterly payments will the management of the bus company need to make in order to achieve this target if they are offered an interest rate of 8,9% per annum, compounded quarterly, on their savings? (4)

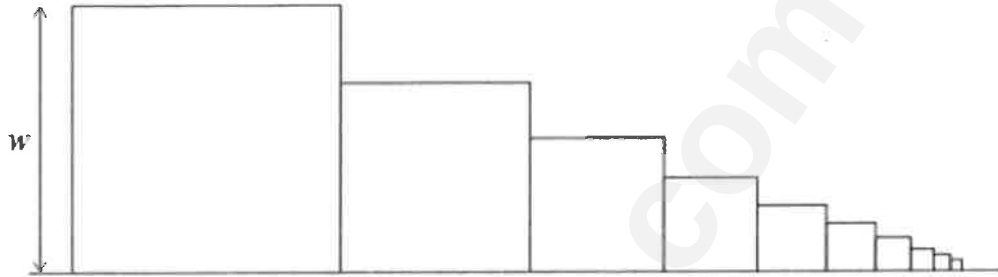
$$2\,681\,512,26 = x \left[\frac{(1 + \frac{8,9\%}{4})^{24} - 1}{\frac{8,9\%}{4}} \right] \checkmark$$

$$x = \underline{85\,750,09} \checkmark$$

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QUESTION 8:

- a) Mpho is creating a mosaic pattern by placing square tiles next to each other along a straight line indefinitely.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

- 1) Find, in terms of w , the length of the sides of the second largest tile. (3)

$$\text{length} = \frac{w}{\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}w}{2}$$

$$\text{Area} = w^2 \quad \text{Area for 2 tiles} = \frac{w^2}{2}$$

- 2) Assume the tiles are in contact with adjacent tiles, but do not overlap. Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$. (4)

$$a = w \quad r = \frac{\sqrt{2}}{2}$$

$$S_{\infty} = \frac{w}{1 - \frac{\sqrt{2}}{2}} = 3.41w < 3.5w$$

b) Calculate: $\sum_{t=4}^{21} 3 \cdot (2)^{-t+2}$

(4)

$$n = 18 \checkmark$$

$$a = 3(2)^{-4+2} = 3/4 \checkmark$$

$$ar = 3(2)^{-3} = 3/8$$

$$r = 1/2 \checkmark$$

$$S_{18} = \frac{3/4 ((1/2)^{18} - 1)}{1/2 - 1}$$

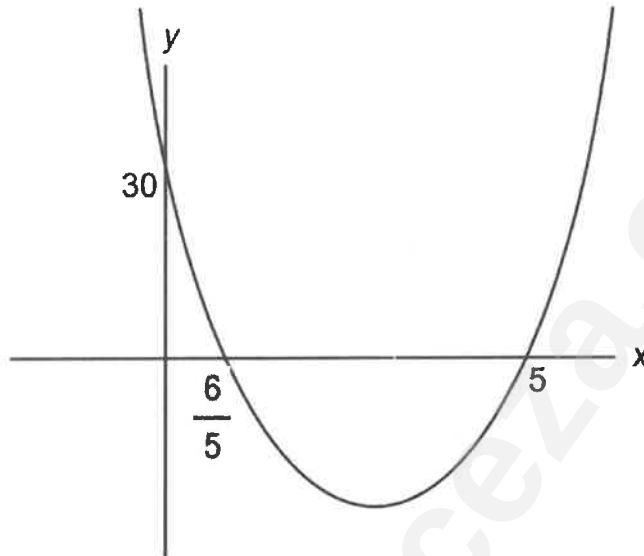
[11]

$$= \underline{\underline{1,5}}$$

QUESTION 9:

In the diagram below, the graph of $f(x) = ax^2 + bx + c$ is given.

$$f\left(\frac{6}{5}\right) = f(5) = 0 \text{ and } f(0) = 30$$



- a) Determine the values of a , b and c if they are integers.

(4)

$$y = a(x-5)(5x-6)$$

$$30 = a(-5)(-6)$$

$$a = 1$$

$$y = 5x^2 - 31x + 30$$

If it is given that $f(x) = 5x^2 - 31x + 30$.

- b) Determine the coordinates of the turning point $f(x)$. (2)

$$f'(x) = 10x - 31 = 0$$

$$x = \frac{31}{10}$$

$$y = \frac{-361}{20} \quad \left(\frac{31}{10}, -\frac{361}{20} \right)$$

- c) Write down one possible way to restrict the domain of $f(x)$ so that $f^{-1}(x)$ will be a function. (1)

$$x \leq \frac{31}{10} \quad \checkmark$$

$$\text{or } x \geq \frac{31}{10}$$

- d) Use the graph to find the values of t for which $f(x) + 2t = 0$ has two positive roots. (3)

$$f(x) = -2t$$

$$-\frac{361}{20} < -2t < 30 \quad \checkmark$$

$$\frac{361}{40} > t > -15$$

$$-15 < t < \frac{361}{40} \quad \checkmark$$

- e) Write down the maximum value of $-f\left(\frac{x}{2}\right) + 3$ (2)

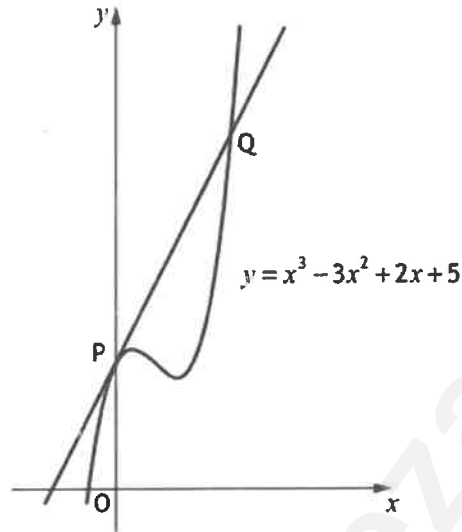
$$\frac{361}{20} + 3 \quad \checkmark$$

$$\frac{421}{20} \quad \checkmark$$

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QUESTION 10:

The sketch below represents the graph of $y = f(x) = x^3 - 3x^2 + 2x + 5$.



- a) Write down the coordinates of P, the point where the curve crosses y-axis. (2)

$P(0; 5)$ ✓✓

- b) Determine the equation of the tangent to the curve at P. (3)

$y = 2x + 5$ ✓✓✓

or $f'(x) = 3x^2 - 6x + 2$ ✓✓

$f'(0) = 2$ ✓

$y = 2x + 5$

- c) Find the coordinates of Q, the point where this tangent meets the curve again. (3)

$$x^3 - 3x^2 + 2x + 5 = 2x + 5 \quad \checkmark$$

$$x^2(x-3) = 0 \quad \checkmark$$

$$x = 0 \quad x = 3$$

$$y = 0 \quad y = 11$$

$$\underline{Q(3; 11)} \quad \checkmark$$

- d) Calculate the coordinates of the turning points of $f(x)$. (4)

$$f'(x) = 3x^2 - 6x + 2 = 0 \quad \checkmark$$

$$x = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{6}$$

$$(1,6; 4,6) \quad \checkmark = \frac{3 + \sqrt{3}}{3} (1,6) \text{ or } \frac{3 - \sqrt{3}}{3} (0,4)$$

$$(0,4; 5,4) \quad \checkmark y = \frac{45 - 2\sqrt{3}}{4} (4,6) \quad y = 5,4$$

- e) Write the values of x where the graph is concave upward. (2)

$$f''(x) = 6x - 6 > 0 \quad \checkmark$$

$$\underline{x > 1} \quad \checkmark$$

- f) Calculate the maximum vertical distance between the graphs of $f(x)$ and the tangent between P and Q . (4)

$$\Delta(x) = 2x + 5 - x^3 + 3x^2 - 2x - 5 \checkmark$$

$$= -x^3 + 3x^2 \checkmark$$

$$\Delta'(x) = -3x^2 + 6x = 0 \checkmark$$

$$x = 0 \quad x = 2$$

$$\therefore \Delta(2) = -8 + 12$$

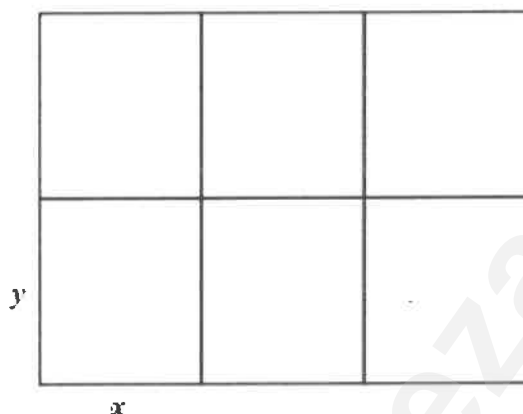
$$= \underline{\underline{4}} \checkmark$$

[18]

QUESTION 11:

A council is setting aside an area of land to create six fenced plots where local residence can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram below.



- a) The area of land being set aside is 108 m^2 .
Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x} \quad (3)$$

$$3x \times 2y = 108$$

$$y = \frac{18}{x} \quad \checkmark$$

$$L(x) = 9x + 8y \quad \checkmark$$

$$= 9x + 8\left(\frac{18}{x}\right) = 9x + \frac{144}{x} \quad \checkmark$$

- b) Find the value of x ^{and y} that minimizes the length of fencing required. (4)

$$L(x) = 9x + 144x^{-1}$$

$$L'(x) = 9 - \frac{144}{x^2} = 0 \quad \checkmark \checkmark$$

$$x^2 = 16$$

$$x = 4 \quad \checkmark \quad y = 9/2 \quad \checkmark$$

[7]

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