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NAME:

MEMO



SAHETI SCHOOL

PRELIMINARY EXAMINATION 2021

MATHEMATICS GRADE 12

PAPER 1

Time: 3 hours

Total: 150

Examiner: P R Mhuka

Moderators: S McConkey
N Eleftheriades

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions on the question paper and hand it in at the end of examination.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

Page 1 of 25

SECTION A- 75 MARKS**QUESTION 1:**

a) A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

- 1) Show that the x -coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. (2)

$$x^2 - x + 3 = 3x + a \quad \checkmark$$

$$x^2 - 4x + 3 - a = 0 \quad \checkmark$$

- 2) For the case where the line intersects the curve at two points, it is given that the x -coordinate of one of the points of intersection is -1 . Find the x -coordinate of the other point of intersection. (3)

$$(-1)^2 - 4(-1) + 3 - a = 0$$

$$a = 8 \quad \checkmark$$

$$x^2 - 4x - 5 = 0 \quad \checkmark$$

$$x = 5 \quad \checkmark \quad x = -1$$

- 3) For the case where the line is a tangent to the curve at a point P , find the value of a . (4)

$$f'(x) = 2x - 1 \quad \checkmark \quad m = 3$$

$$\therefore 2x - 1 = 3 \quad \checkmark$$

$$2x = 4$$

$$x = 2 \quad \checkmark$$

$$\therefore y = (2)^2 - 2 + 3 = 5$$

$$\therefore 5 = 3(2) + a$$

$$\underline{-1 = a} \quad \checkmark$$

b) Given that $\frac{2^{3x+1}+8}{2^{3x}-7} = 5$

1) Find the value of 2^{3x}

(3)

$$2^{3x} \cdot 2^1 + 8 = 5(2^{3x} - 7)$$

$$2^{3x} \cdot 2 + 8 = 5 \cdot 2^{3x} - 35 \quad \checkmark$$

$$43 = 3 \cdot 2^{3x} \quad \checkmark$$

$$\frac{43}{3} = 2^{3x} \quad \checkmark$$

2) Hence, using logarithms, find the value of x .

(2)

$$3x = \log_2 \left(\frac{43}{3} \right) \quad \checkmark$$

$$x = 1,28 \quad \checkmark$$

c) Solve for x : $\log x + \log x^2 + \log x^3 + \dots + \log x^{10} = 110$

(4)

$$\log x^{55} = 110 \quad \checkmark$$

$$55 \log x = 110 \quad \checkmark$$

$$\log x = 2 \quad \checkmark$$

$$x = 10^2 = 100 \quad \checkmark$$

[18]

QUESTION 2:

Consider the quadratic sequence $-12; -8; 0; 12; \dots$

- a) Determine T_n , the general term of the pattern, in the form $T_n = an^2 + bn + c$.

(4)

$$\begin{aligned} 2a &= 4 \\ a &= 2 \end{aligned}$$

$$3(2) + b = 4$$

$$b = -2$$

$$2 + (-2) + c = -12$$

$$c = -12$$

$$\underline{T_n = 2n^2 - 2n - 12}$$

$$\begin{array}{cccc} -12 & -8 & 0 & 12 \\ \vee & \vee & \vee & \\ 4 & 8 & 12 & \\ & \vee & & \\ & 4 & & \end{array}$$

- b) Between which two consecutive terms does the first difference of 192 lie?

(3)

$$T_n = 4n - \text{first difference}$$

$$4n = 192$$

$$n = 48$$

Between 48th and 49th term

- c) A new pattern, P_n , is formed. It is given that $P_n = T_n - 168$, determine the number of negative terms in the new pattern. (4)

$$P_n = 2n^2 - 2n - 12 - 168 \quad \checkmark$$

$$= 2n^2 - 2n - 180$$

$$2n^2 - 2n - 180 < 0 \quad \checkmark$$

$$-9 < n < 10 \quad \checkmark$$

$$\therefore \text{number of negative terms} = \underline{9} \quad \checkmark$$

- d) Show that T_n is always an even number. (2)

$$T_n = 2(n^2 - n - 6) \quad \checkmark \quad \checkmark$$

[13]

QUESTION 3:a) Given $h = \{(3 - 2a; 2a); (a^2; a - 4)\}$ 1) For which value(s) of a is h not a function?

(3)

↓
one to many

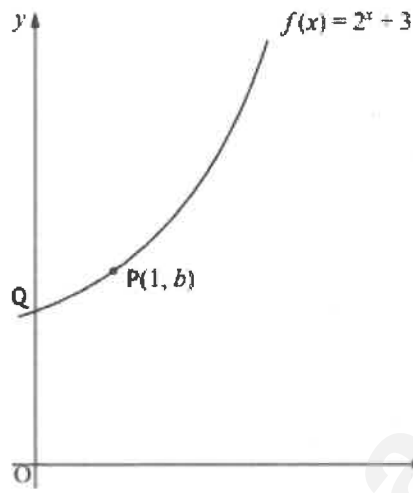
$$\begin{aligned} \therefore a^2 &= 3 - 2a \checkmark \\ a^2 + 2a - 3 &= 0 \checkmark \\ \underline{a = 1} \quad \text{or} \quad \underline{a = -3} \end{aligned}$$

2) For which value(s) of a is h a many to one function?

(2)

$$\begin{aligned} 2a &= a - 4 \checkmark \\ \underline{a = -4} \checkmark \end{aligned}$$

- b) The function $f(x) = 2^x + 3$ passes through the point $P(1; b)$ and cuts the y -axis at Q as shown in the diagram.



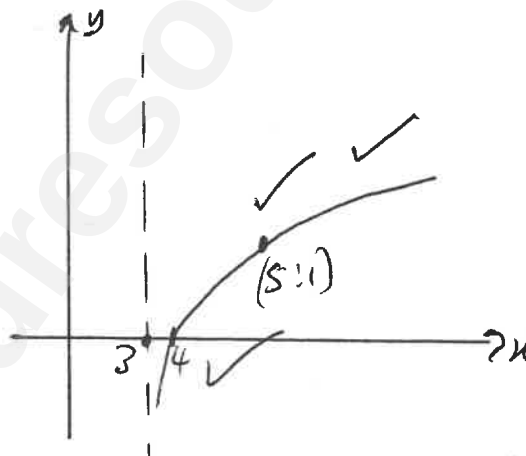
- 1) Write down the value of b .

(1)

$$\underline{b = 5} \quad \checkmark$$

- 2) Sketch the graph of $f^{-1}(x)$ and write the coordinates of the images of P and Q .

(3)



- 3) Write down values of x where $f^{-1}(x) \leq 1$.

(2)

$$3 \leq x \leq 5 \quad \checkmark \checkmark$$

- 4) $R(3; 11)$ also lies on the graph of $f(x)$. Find the coordinates of the image of R on the graph with equation $y = f(x + 1) + 4$. (2)

$$(2; 15) \checkmark \checkmark$$

[13]

QUESTION 4:

- a) A curve has equation $f(x) = x^2 - 12x + 5$.
The point A on the curve has x coordinate -4
The point B on the curve has x coordinate $-4 + h$

- 1) Show that the gradient of line AB is $h - 20$ ✓ (4)

$$M_{AB} = \frac{(-4+h)^2 - 12(-4+h) + 5 - (-4)^2 - 12(-4) + 5}{-4+h+4} \checkmark$$

$$= \frac{16 - 8h + h^2 + 48 - 12h + 5 - 69}{h} \checkmark$$

$$= \frac{h^2 - 20h}{h} \checkmark$$

$$= \underline{h - 20}$$

- 2) Hence, explain how the result of part (1) can be used to show that A is a non-stationary point on the curve. (2)

$$\lim_{h \rightarrow 0} (h - 20) = -20$$

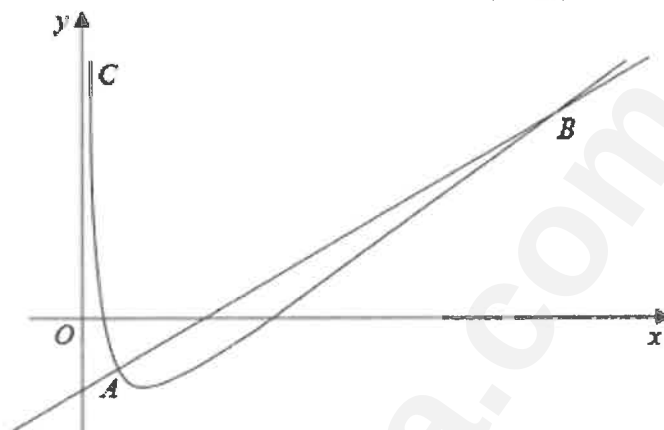
$\therefore$ non stationary

- b) Determine $\frac{dy}{dx}$ if $y = -3gx^2 + \frac{3}{g\sqrt{x}} + e$ (4)

$$y = -3gx^2 + \frac{3}{g}x^{-1/2} + e$$

$$\frac{dy}{dx} = -6gx - \frac{3}{2g}x^{-3/2} + 0$$

- c) The diagram shows a part of the curve C with equation $y = \frac{1}{2}x + \frac{27}{x} - 12$, for $x > 0$. The point A lies on C and has coordinates $(3; \frac{-3}{2})$



Show that the equation of the normal to C at A can be written as $10y = 4x - 27$

(5)

$$f'(x) = \frac{1}{2} - \frac{27}{x^2} \checkmark$$

$$f'(3) = \frac{1}{2} - \frac{27}{3^2} = -\frac{5}{2} \checkmark$$

$$\therefore m \text{ of normal} = \frac{2}{5} \checkmark$$

$$y = \frac{2}{5}x + c$$

$$-\frac{3}{2} = \frac{2}{5}(3) + c$$

$$-\frac{27}{10} = c \checkmark$$

$$y = \frac{2}{5}x - \frac{27}{10} \checkmark$$

$$\underline{10y = 4x - 27}$$

[15]

QUESTION 5:

- a) On 1 October 2019, Miron took a loan of R85 000 from the bank to pay for his studies. He will make his first payment of R3 380 on 1 June 2020 and continue to make payments of R3 380 on the first of each month thereafter until he settles the loan. The bank charges interest at 13,2% p.a. compounded monthly.

- 1) Calculate how much Miron will owe the bank on 1 May 2020. (3)

$$85000 \left(1 + \frac{13,2\%}{12}\right)^7 = 91764,99$$

- 2) How many installments of R3 380 must he pay? (4)

$$91764,99 = 3380 \left[\frac{1 - \left(1 + \frac{13,2\%}{12}\right)^{-n}}{\frac{13,2\%}{12}} \right]$$

$$n = 32,42$$

∴ 32 payments of R3 380.

- 3) Calculate the final payment Miron has to pay to settle the loan. (5)

$$B_0 = 91764,44 \left(1 + \frac{13,2\%}{12}\right)^{32} - 3380 \left[\frac{\left(1 + \frac{13,2\%}{12}\right)^{32} - 1}{\frac{13,2\%}{12}} \right]$$

$$= 1428,81 \checkmark$$

$$\therefore \text{Final Payment} = 1428,81 \left(1 + \frac{13,2\%}{12}\right)^1$$

$$= \underline{\underline{R1444,53 \checkmark}}$$

- b) On his 18th birthday, Michael decides to start saving to go on a long tour of Asia when he turns 30. He takes out an investment where he makes 144 payments of R1500, starting at the beginning of the month, up to his 30th birthday. His investment earns an interest rate of 12,6% p.a. compounded monthly.

What is the value of his investment on his 30th birthday?

(4)

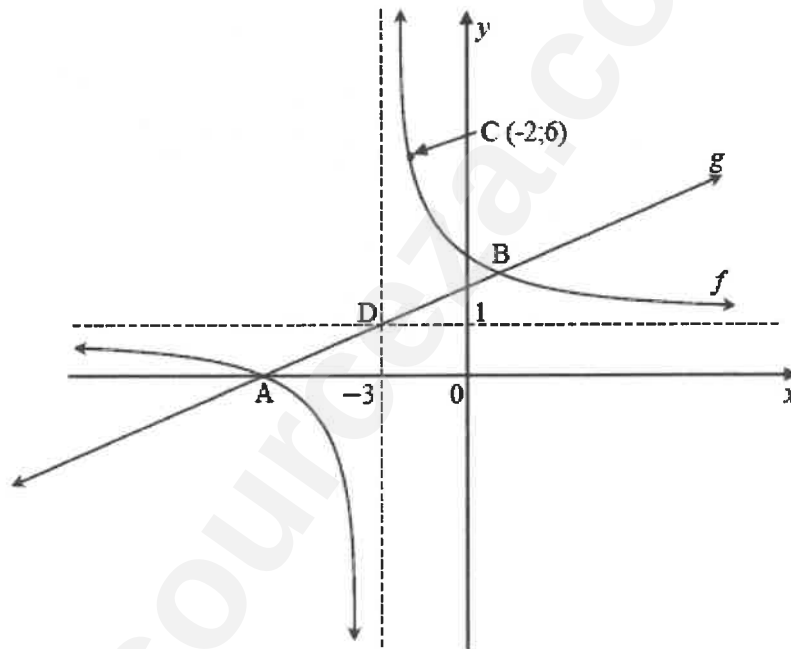
$$F = 1500 \left[\frac{\left(1 + \frac{12,6\%}{12}\right)^{144} - 1}{\frac{12,6\%}{12}} \right] \left(1 + \frac{12,6\%}{12}\right)$$

$$= \underline{\underline{R505275,81 \checkmark}}$$

[16]

SECTION B-75 MARKS**QUESTION 6:**

The diagram below shows the graphs of $f(x) = \frac{a}{x+p} + q$ and $g(x) = bx + c$. The asymptotes of $f(x)$ are at $x = -3$ and at $y = 1$. The graphs intersect at A, the x -intercept of $f(x)$, and B, while $C(-2;6)$ is a point that lies on $f(x)$. $g(x)$ passes through the point $D(-3;1)$, the point of intersection of the asymptotes of $f(x)$.



a) Determine the values of a , p and q .

(3)

$$p = 3 \checkmark \quad q = 1 \checkmark$$

$$6 = \frac{a}{-2+3} + 1$$

$$\underline{a = 5} \checkmark$$

- b) Determine the coordinates of the x -intercept of $f(x)$. (2)

$$0 = \frac{5}{x+3} + 1$$

$$\underline{-8 = x}$$

- c) Determine the values of b and c and write down the equation of $g(x)$. (3)

$$b = \frac{1-0}{-3+8} = \frac{1}{5} \checkmark$$

$$0 = \frac{1}{5}(-8) + c$$

$$\underline{c = 8/5} \checkmark$$

$$\underline{y = \frac{1}{5}x + 8/5} \checkmark$$

- d) Determine the axes of symmetry of $g(x)$ with positive y -intercept. (2)

$$y = x + 4 \checkmark$$

- e) For which values of x is $f'(x) \cdot g(x) < 0$ (2)

$$x > -8 \quad x \neq -3 \checkmark$$

[12]

QUESTION 7:

Find the number of different arrangements that can be made of all 9 letters in the word **CAMERAMAN** in each of the following cases.

- a) There are no restrictions.

(2)

$$\frac{9!}{3!2!} = 30240$$

- b) The As occupy the 1st, 5th and 9th positions.

(3)

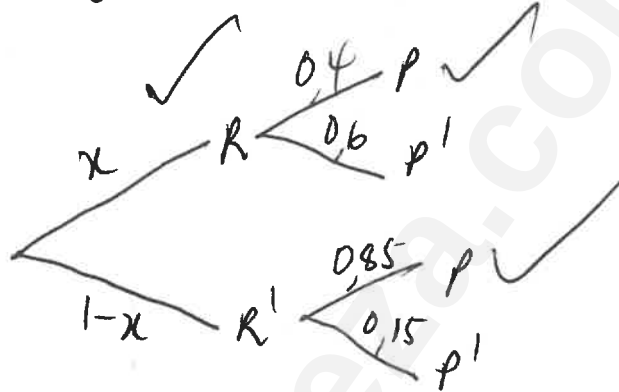
$$\frac{6! \times 3!}{3!} = 720$$

[5]

QUESTION 8:

- a) The probability that it will rain on Saturday is x . If it rains there is a 40% chance that the hockey team will play their match while the probability that they will play if it doesn't rain is 85%. The probability that the hockey team will play is 0,505.

- 1) Draw the tree diagram to illustrate the situation above. (3)



- 2) Calculate the value of x . (3)

$$x \times 0,4 + (1-x)0,85 = 0,505 \checkmark \checkmark$$

$$0,4x + 0,85 - 0,85x = 0,505$$

$$x = \frac{23}{30} \checkmark (0,7667)$$

- b) Events M and N are independent. $P(\text{not } M) = 0,2$ and $P(N) = 0,7$. Complete the contingency table below and show all calculations.

	M	Not M	Totals
N	39,2 ✓	9,8 ✓	49
Not N	16,8 ✓	4,2 ✓	21
Totals	56	14	70

(5)

$$P(M \cap N) = 0,7 \times 0,8$$

$$= \underline{0,56}$$

$$\therefore m \cap N = 0,56 \times 70 = \underline{39,2}$$

$$P(M \cap N') = 0,8 \times 0,3$$

$$= \underline{0,24}$$

$$\therefore m \cap N' = 0,24 \times 70$$

$$= \underline{16,8}$$

$$M' \cap N = 0,2 \times 0,7 \times 70$$

$$= \underline{9,8}$$

[11]

QUESTION 9:

- a) A geometric sequence has first term a and common ratio r and sum to infinity S . A second geometric sequence has first term a , common ratio R and sum to infinity $2S$.

1) Show that $r = 2R - 1$

(3)

$$S = \frac{a}{1-r}$$

$$2S = \frac{a}{1-R}$$

$$\frac{2a}{1-r} = \frac{a}{1-R}$$

$$2-2R = 1-r$$

$$\underline{r = 2R - 1}$$

- 2) It is now given that the 3rd term of the first sequence is equal to the 2nd term of the second sequence. Express S in terms of a .

(4)

$$3^{\text{rd}} \text{ term} = ar^2 \quad 2^{\text{nd}} \text{ term} = aR$$

$$\therefore ar^2 = aR$$

$$r^2 = R$$

$$\therefore r = 2r^2 - 1$$

$$2r^2 - r - 1 = 0$$

$$r \neq 1 \quad r = -\frac{1}{2}$$

$$\therefore S = \frac{a}{1 + \frac{1}{2}} = \frac{2a}{3}$$

- b) The sum of the first twenty terms of an arithmetic sequence is 320. The twenty-first term is 37. What is the sum of the first ten terms? (5)

$$\frac{20}{2}(2a + 19d) = 320$$

$$2a + 19d = 32 \checkmark$$

$$a + 20d = 37 \checkmark$$

$$2(37 - 20d) + 19d = 32$$

$$74 - 40d + 19d = 32$$

$$-21d = -42 \checkmark$$

$$d = 2$$

$$\therefore a = 37 - 20(2)$$

$$= -3 \checkmark$$

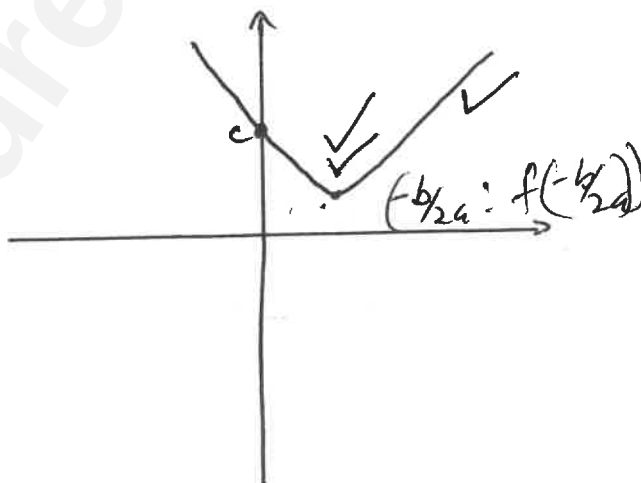
$$S_{10} = 5(2(-3) + 9(2))$$

$$= 60 \checkmark$$

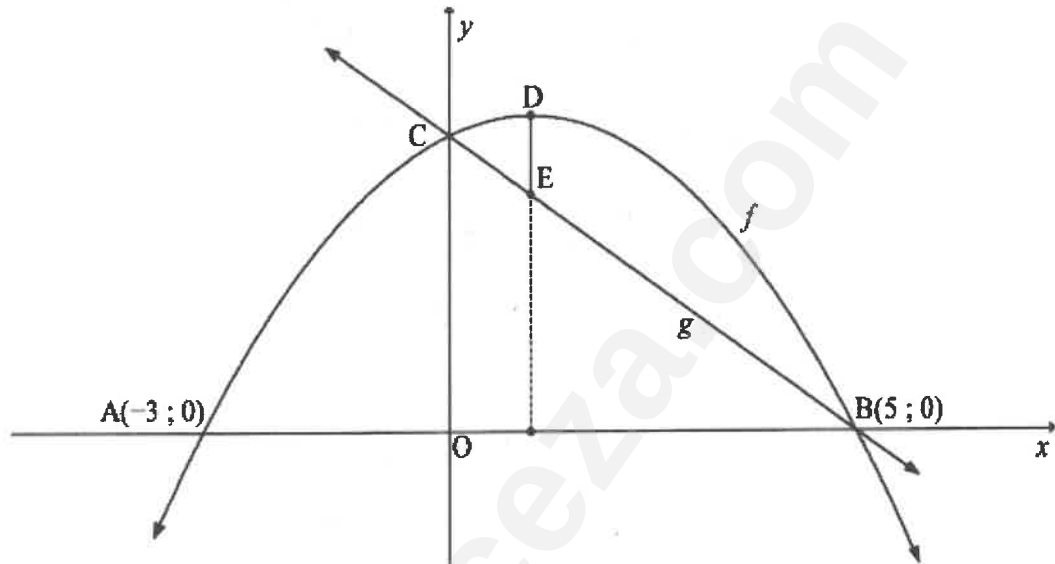
[12]

QUESTION 10:

- a) Sketch the graph of $y = ax^2 + bx + c$, where $a > 0$, $b < 0$ and $b^2 - 4ac < 0$ (3)



- b) The diagram shows the graphs of $f(x) = -x^2 + 2x + 15$ and $g(x) = -3x + k$. Graph f cuts the x -axis at $A(-3; 0)$ and $B(5; 0)$, the y -axis at C and has a turning point at D . Graph g cuts the x -axis at B and the y -axis at C . E is a point on g such DE is parallel to y -axis.



- 1) Determine the coordinates of D , the turning point of f . (2)

$$(1; 16) \checkmark \checkmark$$

- 2) Calculate the length of DE . (2)

$$g(x) = -3x + 15$$

$$\begin{aligned} \therefore \Delta E &= 16 - (12) \\ &= 4 \checkmark \end{aligned}$$

- 3) Find the equation of h if $h(x) = f(x - 1) - 2$, determine the equation of h in the form $h(x) = a(x + p)^2 + q$. (4)

$$\begin{aligned} h(x) &= -(x-1)^2 + 16 - 2 \checkmark \checkmark \\ &= -(x-1)^2 + 14 \checkmark \end{aligned}$$

- 4) Determine the values of k for which the equation $g(x + k) = f(x + k)$ has one positive and one negative root. (2)

$$\underline{0 < k < 5} \checkmark$$

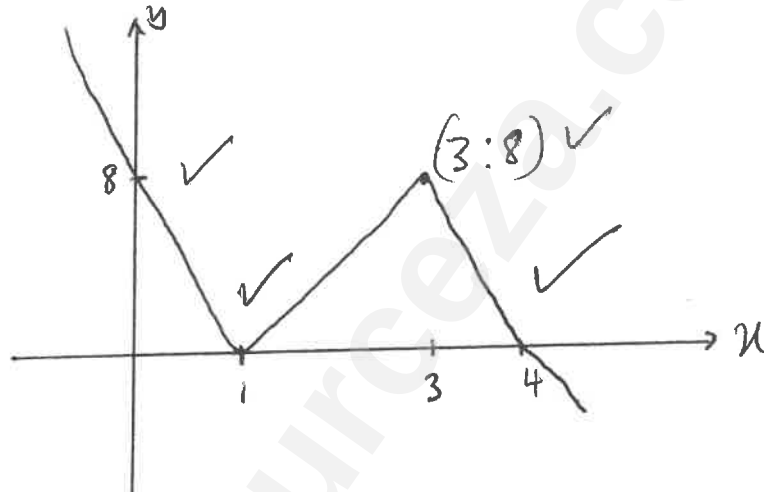
[13]

QUESTION 11:

A cubic function has the following essential properties:

- $f(0) = 8$
- $f(4) = f(1) = 0$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

- a) Sketch the graph of $f(x)$ indicating turning points and the points of intersection of the graph with the axes. (4)



- b) Determine the equation of the graph of $f(x)$ in the form $f(x) = ax^3 + bx^2 + cx + d$. (5)

$$f(x) = a(x-1)^2(x-4) \checkmark \checkmark$$

$$8 = a(-1)^2(-4)$$

$$\underline{a = -2} \checkmark$$

$$f(x) = -2(x^2 - 2x + 1)(x - 4)$$

$$= -2(x^3 - 6x^2 + 9x - 4)$$

$$= -2x^3 + 12x^2 - 18x + 8 \checkmark \checkmark$$

- c) Determine the values of x for which the graph of $f(x)$ is concave down. (2)

$$x > 2 \checkmark \checkmark$$

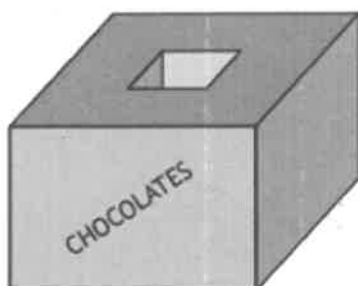
- d) Write down the values of x for which: $x \cdot f'(x) \geq 0$ (3)

$$x \leq 0 \checkmark \text{ or } 1 \leq x \leq 3 \checkmark \checkmark$$

[14]

QUESTION 12:

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



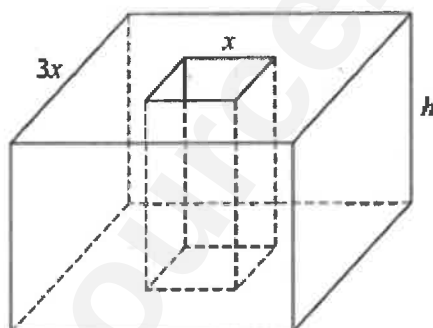
The box is a cuboid with a cuboid shaped tunnel through it.

The height of the box is h centimetres.

The top of the box is a square of side $3x$ centimetres.

The end of the tunnel is a square of side x centimetres.

The volume of the box is $2\,000\text{ cm}^3$



- a) Show that the total surface area, $A\text{ cm}^2$, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$

(5)

$$V = 3x \cdot 3x \cdot h - x \cdot x \cdot h$$

$$2000 = 8x^2h$$

$$h = \frac{250}{x^2}$$

$$SA = 3xh \times 4 + 4xh + (3x \times 3x - x \times x) \times 2$$

$$= 16xh + 16x^2$$

$$= 16x \cdot \frac{250}{x^2} + 16x^2$$

$$= \frac{4000}{x} + 16x^2$$

24

- b) To minimize the cost of production, the surface area, of the box should be as small as possible. Find the minimum value of A. (3)

$$A'(x) = -\frac{4000}{x^2} + 32x = 0 \quad \checkmark$$

$$x^3 = \frac{4000}{32}$$

$$\underline{x = 5}$$

$$\therefore \text{Minimum Area} = \underline{1200 \text{ cm}^2} \quad \checkmark$$

[8]

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