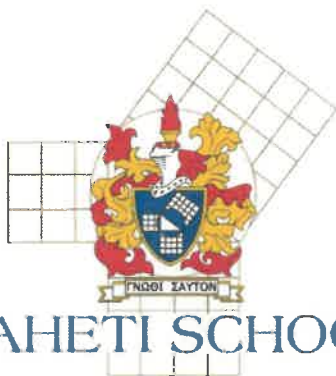




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NAME: MARKING GUIDELINE



**SAHETI SCHOOL**

**PRELIMINARY EXAMINATION 2020**

**MATHEMATICS GRADE 12**

**PAPER 1**

**Time: 3 hours**

**Total: 150**

**Examiner: P R Mhuka**

**Moderators: E Zachariou  
M Torrani  
N Eleftheriades**

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 20 pages and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

Page 1 of 20

SECTION A- 76 MARKSQUESTION 1:

a) Given  $x^2 - 3x + 5m = 0$

- 1) Show that the roots of the equation will equal to
- $\frac{3 \pm \sqrt{9-20m}}{2}$
- (2)

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5m)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-20m}}{2}$$

- 2) For which values of
- $m$
- will the equation have equal roots. (2)

$$9 - 20m = 0$$

$$m = \frac{9}{20}$$

b) Solve for  $x$ :

1)  $5^{x+3} - 5^{x+1} = 24$  (4)

$$5^x \cdot 5^3 - 5^x \cdot 5^1 = 24$$

$$5^x(5^3 - 5^1) = 24$$

$$5^x = \frac{24}{120}$$

$$5^x = 5^{-1}$$

$$x = -1$$

2)  $\sqrt{5-x} + 1 + x = 0$

(5)

$$(\sqrt{5-x})^2 = (-x-1)^2 \checkmark$$

$$5-x = x^2 + 2x + 1 \checkmark$$

$$x^2 + 3x - 4 = 0 \checkmark$$

$$(x+4)(x-1) = 0$$

$$x = -4 \checkmark \quad x \neq 1 \checkmark$$

c) State the positive value of  $n$  that satisfies  $\sum_{r=1}^n (r-2) = 0$

(2)

$$n = 3 \checkmark \checkmark$$

$$(1-2) + (2-2) + (3-2) = 0$$

[15]

**QUESTION 2:**a) A geometric sequence  $x; \sqrt{x}; 1 \dots$ 1) For which values of  $x$  is this convergent?

(3)

$$r = \frac{\sqrt{x}}{x} = x^{-1/2} \checkmark$$

$$-1 < \frac{1}{x^{1/2}} < 1 \checkmark$$

$$\therefore \underline{x > 1} \checkmark$$

2) Determine the sum to infinity of the series if  $x = 2$ 

(2)

$$S_{\infty} = \frac{a}{1-r} = \frac{2}{1-\frac{1}{\sqrt{2}}} \checkmark$$

$$= 4 + 2\sqrt{2} \quad (6.8) \checkmark$$

b) Consider the quadratic sequence  $69; 0; -63; \dots$ 1) Determine the  $n^{\text{th}}$  term of the sequence.

(4)

$$69 : 0 : -63$$

$$\begin{array}{ccc} \swarrow & & \swarrow \\ -69 & & -63 \\ \searrow & & \searrow \\ & 6 & \end{array}$$

$$2a = 6 \checkmark$$

$$a = 3 \checkmark$$

$$9 + b = -69 \checkmark$$

$$b = -78 \checkmark$$

$$3 - 78 + c = 69 \checkmark$$

$$\underline{c = 144} \checkmark$$

$$T_n = 3n^2 - 78n + 144 \checkmark$$

- 2) Determine the value of the smallest term in the sequence (2)

$$\begin{aligned} 6n - 78 &= 0 \\ n &= 13 \checkmark \\ \therefore T_{13} &= -363 \checkmark \end{aligned}$$

[11]

**QUESTION 3:**

- a) The function  $f(x) = \left(\frac{2}{3}\right)^x + 1$ .

- 1) Write down range of  $f(x)$ . (1)

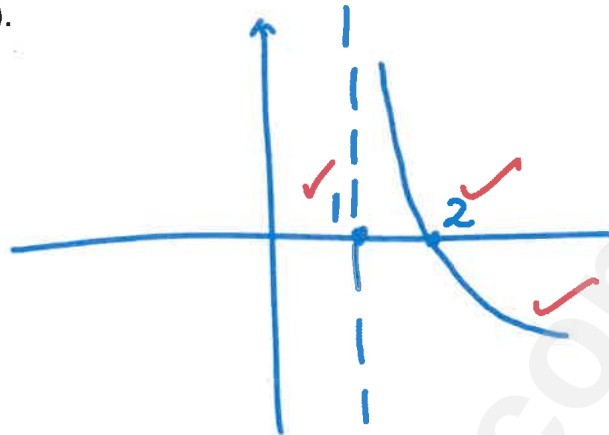
$$y > 1 \checkmark$$

- 2) Write down  $f^{-1}(x)$ . (3)

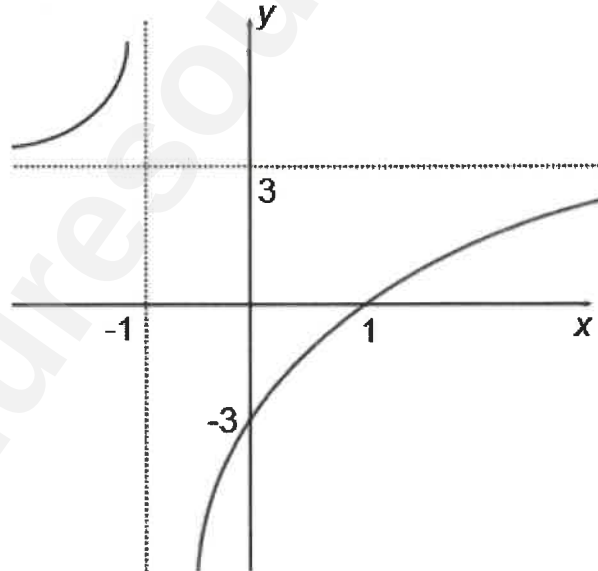
$$\begin{aligned} x-1 &= \left(\frac{2}{3}\right)^y \checkmark \\ \log_{\frac{2}{3}}(x-1) &= y = f^{-1}(x) \checkmark \end{aligned}$$

3) Sketch  $f^{-1}(x)$ .

(3)



b) The diagram shows the curve with equation  $g(x) = \frac{-6}{x+a} + b$



1) Determine  $a$  and  $b$

(2)

$$a = 1 \quad \checkmark$$

$$b = 3 \quad \checkmark$$

- 2) Determine the axes of symmetry of  $g(x)$  with negative gradient. (2)

$$y = -(x+1) + 3 \\ = -x + 2 \quad \checkmark \checkmark$$

- 3) Write down the domain and range of  $g(x)$  (2)

$$x \in \mathbb{R}: x \neq -1 \quad \checkmark$$

$$y \in \mathbb{R}: y \neq 3 \quad \checkmark$$

- 4) For which values of  $x$  is  $g'(x) > 0$  (2)

$$x \in \mathbb{R}: x \neq -1 \quad \checkmark \checkmark$$

[15]

**QUESTION 4:**

- a) Determine  $f'(x)$  from first principles if  $f(x) = -3x^2 + 2x$ . (5)

$$f(x+h) = -3(x^2 + 2xh + h^2) + 2(x+h) \\ = -3x^2 - 6xh - 3h^2 + 2x + 2h \quad \checkmark$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-3x^2 - 6xh - 3h^2 + 2x + 2h - (-3x^2 + 2x)}{h} \quad \checkmark \checkmark \\ = \lim_{h \rightarrow 0} \frac{-6x - 3h + 2}{1} \quad \checkmark = -6x + 2 \quad \checkmark$$

- b) Determine  $\frac{dy}{dx}$  if  $y = 5x + \frac{3}{\sqrt[3]{x}}$  (3)

$$y = 5x + 3x^{-1/3} \checkmark$$

$$\frac{dy}{dx} = 5 - x^{-4/3} \checkmark$$

$$= 5 - \frac{1}{x^{4/3}} \checkmark$$

- c) Given  $g(x) = \frac{2x^3 + x^2 - 3x}{x-1}$ ,

- 1) Determine  $g'(2)$  if  $x \neq 1$  (4)

$$g(x) = \frac{x(2x+3)(x-1)}{x-1} \checkmark$$

$$g(x) = 2x^2 + 3x \checkmark$$

$$g'(x) = 4x + 3 \checkmark$$

$$g'(2) = 11 \checkmark$$

- 2) Hence, find the equation of the normal at  $x = 2$  (3)

$$m_{\perp} = -\frac{1}{11} \checkmark \quad @ \quad x=2 \checkmark$$

$$y=14 \checkmark$$

$$14 = -\frac{1}{11}(2) + c$$

$$y = -\frac{1}{11}x + \frac{156}{11} \checkmark$$

[15]

**QUESTION 5:**

- a) A mining company buys mining equipment at a cost of R1,65 million. The depreciation on the mining equipment is calculated at 12.8% p.a. on a reducing balance.

- 1) Calculate the expected cost of new mining equipment in 8 years time if the rate of inflation is estimated to be 4,9% p.a. (2)

$$A = 1650\,000(1 + 4,9\%)^8 \checkmark$$

$$= \underline{2419\,289,55} \checkmark$$

- 2) Calculate the book value of the mining equipment at the end of 8 years. (2)

$$A = 1650\,000(1 - 12,8\%)^8 \checkmark$$

$$= \underline{551\,589,29} \checkmark$$

- 3) The mining company plans on selling old mining equipment in 8 years time to help fund the purchase of new equipment. The remaining amount required will come from a sinking fund he sets up. Calculate the quarterly payments required if the mine makes quarterly payments starting immediately and the final payment is made at the end of 8 years. The interest rate on the money accumulated in the fund is 11% p.a. compounded quarterly. (6)

$$1\,867\,700,26 = x \left[ \frac{\left(1 + \frac{11\%}{4}\right)^{32} - 1}{\frac{11\%}{4}} \right] \checkmark$$

$$x = \underline{35\,472,35} \checkmark$$

- b) A loan of R250 000 is repaid by means of equal monthly installments of R2 350, starting one month after the loan was drawn. The interest on the outstanding balance of the loan is 9,7% p.a. compounded monthly. Calculate how many monthly payments it will take to repay the loan. (4)

$$250\,000 = 2\,350 \left[ \frac{1 - \left(1 + \frac{9,7\%}{12}\right)^{-n}}{\frac{9,7\%}{12}} \right]$$

$$n = 244,15$$

$$\therefore n = \underline{245}$$

[14]

**QUESTION 6:**

- a) Using the letters in the word **BOOKSHOP**, determine how many Eight letter words can be formed without repetition of the letters. (3)

$$\frac{8!}{3!} = \underline{6720}$$

- b) A laptop security lock code consists of a sequence of four digits. What is the probability that a person chose a code that has no duplicate digits in the sequence? (3)

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{10^4} = \frac{63}{125} = (0,504)$$

[6]

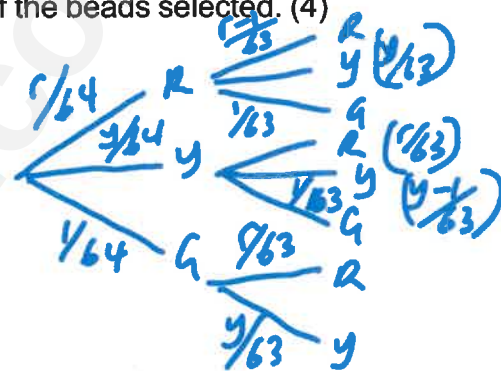
SECTION B-74 MARKSQUESTION 7:

- a) A bag contains 64 coloured beads. There are  $r$  red beads,  $y$  yellow beads and 1 green bead and  $r + y + 1 = 64$ .

Two beads are selected at random, one at a time without replacement.

- 1) Find the probability that the green bead is one of the beads selected. (4)

$$\begin{aligned}
 & RG + YG + GR + GY \\
 & \frac{r}{64} \times \frac{1}{63} + \frac{y}{64} \times \frac{1}{63} + \frac{1}{64} \times \frac{r}{63} + \frac{1}{64} \times \frac{y}{63} \\
 & = \frac{r}{2016} + \frac{y}{2016} \\
 & = \frac{1}{32} \quad (0,03125)
 \end{aligned}$$



- 2) The probability that both of the beads are red is  $\frac{5}{84}$ . Show that  $r$  satisfies the equation  $r^2 - r - 240 = 0$ . (3)

$$\begin{aligned}
 & \frac{r}{64} \times \frac{r-1}{63} = \frac{5}{84} \\
 & \frac{r^2 - r}{4032} = \frac{5}{84} \\
 & r^2 - r = 240 \\
 & \underline{r^2 - r - 240 = 0}
 \end{aligned}$$

- 3) Hence, calculate the possible value of  $y$ . (3)

$$r^2 - r - 240 = 0$$

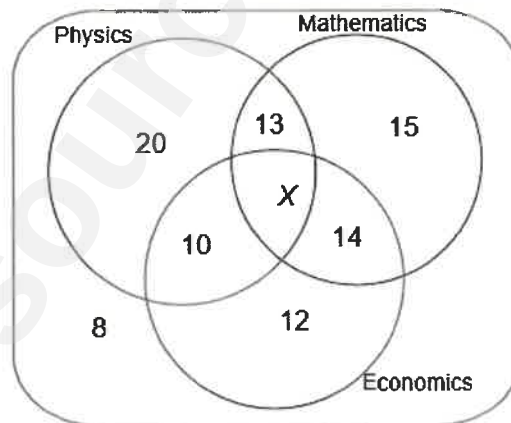
$$r = 16 \quad r \neq -15$$

$$y + r + 1 = 64$$

$$\underline{y = 47}$$

- b) A group of students were asked about the subjects they took at the Prelim examination. The numbers who have taken Physics, Mathematics and Economics are shown in the Venn diagram. The number of students who have taken all three subjects is  $x$ .

S is the event that the student has taken Physics  
M is the event that the student has taken Mathematics  
E is the event that the student has taken Economics



Given that the events M and E are independent.  
Calculate the value of  $x$ .

(4)

$$P(M) \cdot P(E) = P(M \cap E)$$

$$\frac{42+x}{92+x} \cdot \frac{36+x}{92+x} = \frac{x+14}{92+x}$$

$$1512 + 78x + x^2 = (x+14)(92+x)$$

$$1512 + 78x + x^2 = x^2 + 106x + 1288$$

$$224 = 28x$$

$$\underline{x = 8}$$

[14]

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**QUESTION 8:**

- a) An arithmetic sequence has first term  $a$  and common difference  $d$ .  
The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms.

- 1) Show that  $4a + 70d = 4a^2 + 20ad + 25d^2$  (4)

$$S_6 = 3(2a + 5d) = 6a + 15d \quad \checkmark \quad S_{36} = 18(2a + 35d) = 36a + 630d \quad \checkmark$$

$$\therefore 36a + 630d = (6a + 15d)^2 \quad \checkmark$$

$$36a + 630d = 36a^2 + 180ad + 225d^2 \quad \checkmark$$

$$4a + 70d = 4a^2 + 20ad + 25d^2$$

- 2) Given the sixth term of the sequence is 25, find the smallest possible value of  $a$  (5)

$$T_6 = a + 5d = 25$$

$$a = 25 - 5d \quad \checkmark$$

$$4(25 - 5d) + 70d = 4(25 - 5d)^2 + 20d(25 - 5d) + 25d^2 \quad \checkmark \text{ sub}$$

$$100 - 20d + 70d = 2500 - 1000d + 100d^2 + 500d - 100d^2 + 25d^2 \quad \checkmark$$

$$25d^2 - 50d + 2400 = 0 \quad \checkmark$$

$$d = 16 \quad d = 6$$

$$a = -55 \quad a = -5$$

$$\therefore a = -55$$

b) In a geometric series the common ratio is  $r$  and sum to  $n$  terms is  $S_n$ .

Given  $S_\infty = \frac{8}{7} \times S_6$ .

Show that  $r = \pm \frac{1}{\sqrt{k}}$ , where  $k$  is an integer to be found.

(5)

$$\frac{a}{1-r} = \frac{8}{7} \cdot \frac{a(1-r^6)}{1-r} \quad \checkmark \checkmark$$

$$1 = \frac{8}{7}(1-r^6) \quad \checkmark$$

$$\frac{7}{8} = 1-r^6$$

$$r^6 = 1 - \frac{7}{8}$$

$$r^6 = \frac{1}{8} \quad \checkmark$$

$$\therefore r = \frac{1}{\sqrt{2}} \quad \checkmark \therefore \underline{k=2}$$

[14]

**QUESTION 9:**

a)

| Statement                                                                            | Always True | Sometimes True | Never True | Reason                                                                                                                                  |
|--------------------------------------------------------------------------------------|-------------|----------------|------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| The quadratic equation $ax^2 + bx + c = 0$ has 2 real roots.                         |             | ✓              |            | It only has 2 real roots when $b^2 - 4ac > 0$ .<br>When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots |
| When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive. | ✓           |                |            | $(x-3)^2 + 1$ ✓<br>$(x-3)^2 \geq 0$ ✓<br>$\therefore (x-3)^2 + 1 \geq 1$<br>$\therefore$ always positive                                |
| If $ax > b$ then $x > \frac{b}{a}$                                                   |             | ✓              |            | If $a < 0 \therefore ax > b$<br>$x < \frac{b}{a}$ ✓<br>If $a > 0 \therefore ax > b$<br>$x > \frac{b}{a}$ ✓                              |
| The difference between consecutive square numbers is odd                             | ✓           |                |            | $(n+1)^2 - n^2 = 2n+1$ ✓<br>always true as<br>$2n+1 = \text{even} + 1$ ✓<br>$= \text{odd}$                                              |

(9)

- b) An archer shoots an arrow. The height,  $H$  metres, of the arrow above the ground is modelled by the formula  $H = 1,8 + 0,4d - 0,002d^2$ ,  $d \geq 0$  where  $d$  is the horizontal distance of the arrow from the archer, measured in metres. Given that the arrow travels in a vertical plane until it hits the ground,

- 1) Find the horizontal distance travelled by the arrow (3)

$$\begin{aligned}
 1,8 + 0,4d - 0,002d^2 &= 0 \quad \checkmark \\
 0,002d^2 - 0,4d - 1,8 &= 0 \\
 d^2 - 200d - 900 &= 0 \quad \checkmark \\
 d = 204,4 \quad d \neq -4,4 \quad \checkmark \\
 \therefore H \cdot 0 &= 204,4
 \end{aligned}$$

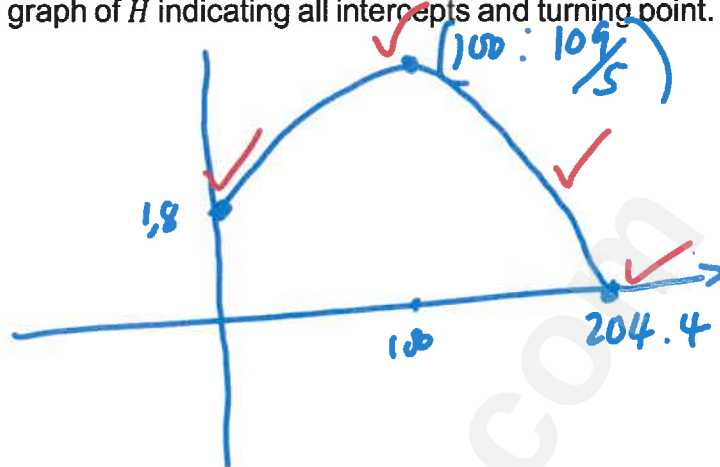
- 2) With reference to the model, interpret the significance of constant 1,8 in the formula. (1)

Initial height, <sup>above the ground</sup> before arrow is released.  $\checkmark$

- 3) Write  $H$  in the form  $A - B(d - C)^2$  where  $A, B$  and  $C$  are constants. (3)

$$-0,002(d - 100)^2 + \frac{109}{5} \quad \checkmark$$

- 4) Sketch the graph of  $H$  indicating all intercepts and turning point. (4)



- 5) Give a reason why  $H$  does not have an inverse. (1)

many to one function. ✓

- 6) If the domain of  $H$  is further restricted to  $x \leq k$ , state the greatest value of  $k$  for the inverse exists. (1)

$k = 100$  ✓

[22]

**QUESTION 10:**

Given a function  $g(x) = 4x^3 - 12x^2 - 15x + 50$ .

- a) Show that  $g(x)$  can be written in the form  $g(x) = (x + 2)(ax + b)^2$ , where  $a$  and  $b$  are integers to be found. (3)

$$g(x) = (x+2)(2x-5)^2$$

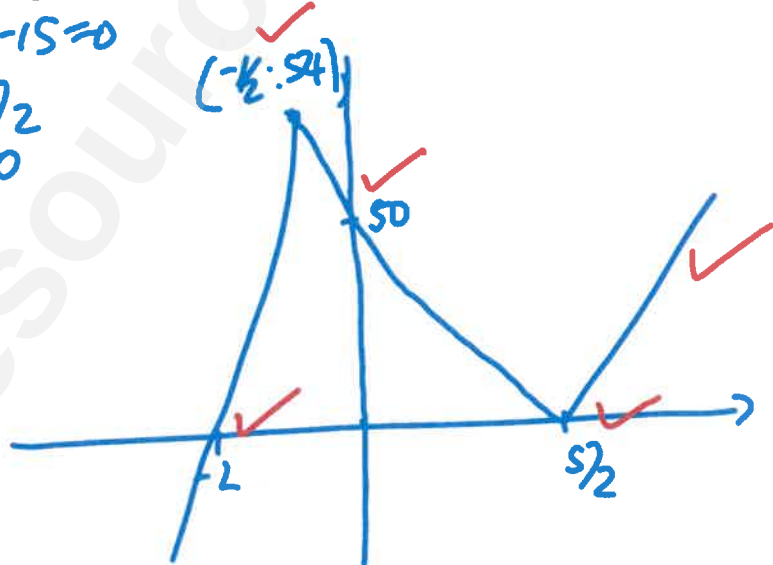
- b) Sketch  $g(x)$  indicating all intercepts with the axis and turning points. (5)

$$g'(x) = 12x^2 - 24x - 15 = 0$$

$$x = -\frac{b}{2a} : x = \frac{9}{2}$$

$$y = 54$$

$$y = 0$$



c) Calculate the coordinates of the point of inflection.

(3)

$$g''(x) = 24x - 24 = 0$$

$$x = 1$$

$$y = 27$$

$$PI (1; 27)$$

d) Use your graph to deduce the values of  $x$  for which:

1)  $g(2x) \leq 0$

(1)

$$\underline{x < -1}$$

2)  $g'(x) \leq 0$  or  $g''(x) > 0$

(3)

$$g'(x) \leq 0$$

$$\underline{-\frac{1}{2} \leq x \leq \frac{5}{2}}$$

$$g''(x) > 0$$

$$\underline{x > 1}$$

$$\therefore \underline{\underline{x \geq -\frac{1}{2}}}$$

[15]

**QUESTION 11:**

A lorry is driven between Johannesburg and Polokwane.

In a simple model, the cost of the journey  $RC$  (C rands) when the lorry is driven at a steady speed of  $v$  kilometres per hour is:

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

a) Find according to this model,

- 1) The value of  $v$  that minimizes the cost of the journey. (4)

$$\frac{dC}{dv} = -1500v^{-2} + \frac{2}{11} = 0$$

$$v^2 = 8250$$

$$v = \sqrt{8250} = 90,8 \text{ km/h}$$

- 2) The minimum cost of the journey. (2)

$$C = \frac{1500}{90,8} + \frac{2(90,8)}{11} + 60 = 193$$

- b) Prove by using  $\frac{d^2C}{dv^2}$  that the cost is minimized at the speed found in a(1). (1)

$$\frac{d^2C}{dv^2} = \frac{3000}{v^3} \text{ @ } v = 90,8$$

$$\frac{d^2C}{dv^2} = 0,004 > 0 \therefore \text{minimum cost}$$

- c) State one limitation of this model (1)

impossible to drive at constant speed for the whole journey.

[9]