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**PRELIM EXAM 2014**  
**GRADE 12 MATHEMATICS PAPER 1**  
**[MEMO]**

**Time: 3 hours**

**150 marks**

**Examiner: Mr S. Dzingwa**

**Moderator: Mr M. Phungula**

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**PLEASE READ THESE INSTRUCTIONS CAREFULLY**

1. This question paper consists of 10 pages.
  2. You must also receive a DIAGRAM SHEET for Question 6 a), Question 6 c) and Question 8 and also a **green** information sheet.
  3. Read the questions carefully.
  4. Answer all the questions.
  5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
  6. All the necessary working details must be clearly shown.
  7. It is in your own interest to write legibly and to present your work neatly.
  8. Round all answers to **2 decimal places** unless told to do otherwise.
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**SECTION A****QUESTION 1**a) Solve for  $x$  in the following equations:

1)  $x(x-3)=10$  (4)

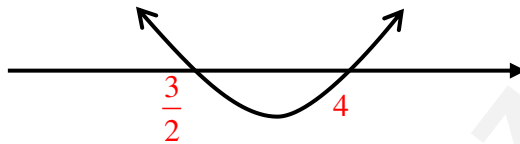
$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$\therefore x = 5 \text{ or } x = -2$$

2)  $2x^2 - 11x + 12 < 0$  (4)

$$(2x-3)(x-4) < 0$$



$$\therefore \frac{3}{2} < x < 4$$

3)  $2^{x+1} = 4\sqrt{2}$  (3)

$$2^{x+1} = 2^2 \times 2^{\frac{1}{2}}$$

ALT

$$2^{x+1} = 2^2 \times 2^{\frac{1}{2}}$$

$$2^{x+1} = 2^{2\frac{1}{2}}$$

$$2^{x+1} = 2^{2\frac{1}{2}}$$

$$\therefore x+1 = 2\frac{1}{2}$$

$$x+1 = \log_2 2^{2\frac{1}{2}}$$

$$\therefore x = 1\frac{1}{2}$$

$$\therefore x = 1\frac{1}{2}$$

b) Solve the simultaneous equations (7)

$$\begin{cases} 2x - y + 9 = 0 \dots\dots\dots(1) \\ x^2 + 2xy + y^2 = 9 \dots\dots\dots(2) \end{cases}$$

$$\begin{cases} x^2 + 2xy + y^2 = 9 \dots\dots\dots(2) \end{cases}$$

$$\text{From (1) } y = 2x + 9$$

$$\text{Subst in (2): } x^2 + 2x(2x+9) + (2x+9)^2 = 9$$

$$\therefore x^2 + 4x^2 + 18x + 4x^2 + 36x + 81 = 9$$

$$\therefore 9x^2 + 54x + 72 = 0$$

$$\therefore x^2 + 6x + 8 = 0$$

$$\therefore (x+2)(x+4) = 0$$

$$\therefore x = -2 \text{ or } x = -4$$

$$\therefore y = 5 \text{ or } y = 1$$

**[18]**

## QUESTION 2

Given that  $\log_2 x = k$ ,

a) write each of the following in terms of  $k$ ,

1)  $\log_2 \sqrt{x}$  (2)

$$\begin{aligned} &= \log_2 x^{\frac{1}{2}} \\ &= \frac{1}{2} \log_2 x \\ &= \frac{1}{2} k \end{aligned}$$

2)  $\log_2 8x$  (2)

$$\begin{aligned} &= \log_2 8 + \log_2 x \\ &= 3 \log_2 2 + \log_2 x \\ &= 3 + k \end{aligned}$$

b) Hence, or otherwise solve the equation  $\log_2 8x - \log_2 \sqrt{x} = \log_3 9$  (3)

$$\log_2 8x - \log_2 \sqrt{x} = \log_3 9$$

$$\therefore 3 + k - \frac{1}{2}k = 2 \log_3 3$$

$$\therefore 3 + k - \frac{1}{2}k = 2$$

$$\therefore \frac{1}{2}k = -1$$

$$\therefore k = -2$$

$$\Rightarrow \log_2 x = -2$$

$$\therefore x = 2^{-2} = \frac{1}{4}$$

[7]

## QUESTION 3

a) Given:  $\sum_{t=0}^{99} (3t-1)$

- 1) Write down the first THREE terms of the series. (3)

$$-1, 2 \text{ and } 5$$

- 2) Calculate the sum of the series. (4)

$$\begin{aligned} S_{99} &= \frac{100}{2} \{2(-1) + (100-1)3\} \\ &= 14750 \end{aligned}$$

- b) Consider the series  $3 + 9 + 17 + 27 + \dots$  to 50 terms.

- 1) Write down the 5<sup>th</sup> term. (1)

$$39$$

- 2) Write the series in sigma  $\left[\sum\right]$  notation. (5)

$$\begin{array}{ccccccc} 3 & + & 9 & + & 17 & + & 27 & + & \dots \\ \vdots & & \ddots & & \ddots & & \ddots & & \ddots \\ & 6 & & 8 & & 10 & & & \\ & \vdots & & \ddots & & \ddots & & \ddots & \\ & & 2 & & & 2 & & & \end{array}$$

$$\therefore T_n = 3 + (n-1)6 + \frac{(n-1)(n-2)2}{2}$$

$$\therefore T_n = n^2 + 3n - 1$$

$$\sum_{n=1}^{50} (n^2 + 3n - 1)$$

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## QUESTION 4

a) The sum,  $S_n$ , of the first  $n$  terms of a series is given by  $S_n = 3^n - 1$ .

1) Show that the fourth term of the series is 54. (3)

$$\begin{aligned} T_4 &= S_4 - S_3 \\ &= (3^4 - 1) - (3^3 - 1) \\ &= 80 - 26 \\ &= 54 \end{aligned}$$

2) Show that the  $n^{\text{th}}$  term of the series can be written in the form  $k(3^n)$ . (4)

$$\begin{aligned} T_4 &= S_4 - S_3 & \text{ALT} \\ \therefore T_n &= (3^n - 1) - (3^{n-1} - 1) & 2; 6; 18; 54 \\ &= 3^n - 1 - 3^{n-1} + 1 & \therefore r = 3 \\ &= 3^n - 3^{n-1} & \therefore T_n = 2(3)^{n-1} \\ &= 3^n (1 - 3^{-1}) & \therefore T_n = 2 \times 3^n \times 3^{-1} \\ &= 3^n \left( \frac{2}{3} \right) & \therefore T_n = \frac{2}{3}(3^n) \\ &= \frac{2}{3}(3^n) \end{aligned}$$

b) Prove that for any arithmetic sequence of which the first term is  $a$  and the constant difference is  $d$ , the sum to  $n$  terms can be expressed as:

$$S_n = \frac{n}{2} \{2a + (n-1)d\} \quad (5)$$

$$S_n = a + \{a+d\} + \dots + \{a+(n-2)d\} + \{a+(n-1)d\} \text{-----(1)}$$

$$S_n = \{a+(n-1)d\} + \{a+(n-2)d\} + \dots + \{a+d\} + a \text{-----(2)}$$

$$\therefore (1)+(2): \quad 2S_n = \{2a+(n-1)d\} + \{2a+(n-1)d\} + \dots + \{2a+(n-1)d\} + \{2a+(n-1)d\}$$

$$2S_n = n\{2a+(n-1)d\}$$

$$\therefore S_n = \frac{n}{2}\{2a+(n-1)d\}$$

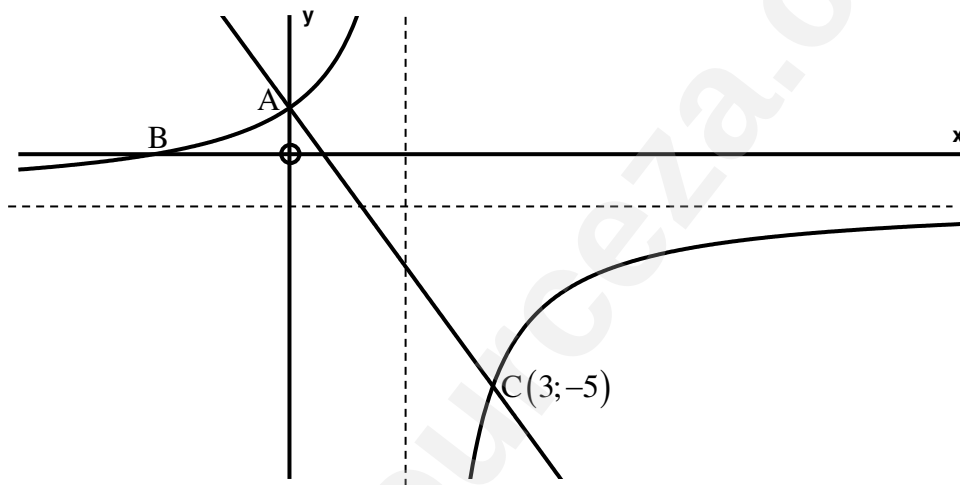
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## QUESTION 5

The figure shows the graphs of hyperbola  $f(x) = \frac{-4}{x-2} - 1$  and line  $g(x) = mx + k$ .

The two graphs intersect at points A and C.

The graph of  $f$  cuts the x-axis at point B.



- 1) Determine the equations of the asymptotes of the hyperbola. (2)

$$x = -2 \quad \text{and} \quad y = -1$$

- 2) Determine the coordinates of the points A and B. (4)

$$A(0;1) \quad \text{and} \quad B(-2;0)$$

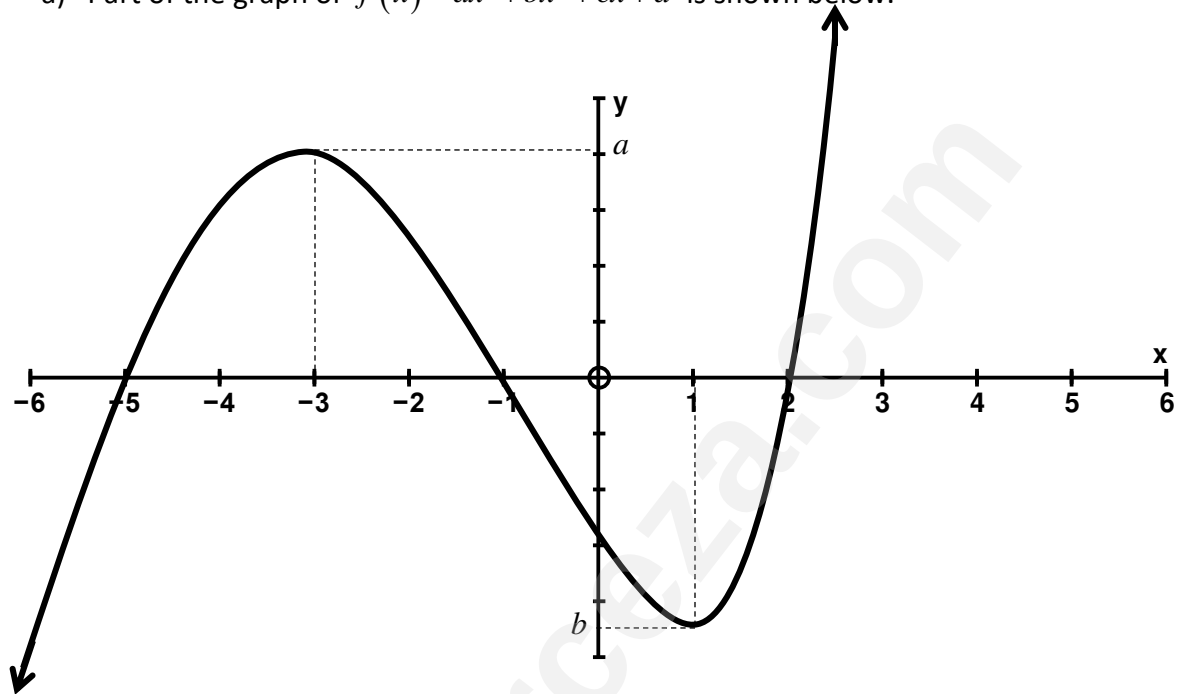
- 3) Determine the values of  $m$  and  $k$  if  $g(x) = mx + k$ . (3)

$$m = \frac{1}{2} \quad \text{and} \quad k = 1$$

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## QUESTION 6

a) Part of the graph of  $f(x) = ax^3 + bx^2 + cx + d$  is shown below.



1) For which values of  $x$  is  $f'(x) = 0$ ?  $x = -3$  or  $x = 1$  (2)

2) On the DIAGRAM SHEET provided draw a sketch of the derivative  $f'(x)$  indicating only the  $x$ -intercepts of the graph. (3)

3) The equation  $f(x) - k = 0$  has only one root. What are the possible values of  $k$ ?  $k < b$  or  $k > a$  (3)

b) Find the average gradient of the graph  $f(x) = x^2 - 1$  between the points where  $x = -1$  and  $x = 3$ . (4)

$$y = 0 \text{ and } y = 8$$

$$\therefore m = \frac{8-0}{3-(-1)} = 2$$

c) A cubic function  $f$  has the following properties:

- $f(-4) = f(-1) = f(5) = 0$
- $f'(-3) = f'(2) = 0$
- $f$  increases only at  $-3 < x < 2$

On the DIAGRAM SHEET provided sketch a possible graph of  $f$ , clearly indicating the  $x$ -intercepts and the coordinates of the turning points. (4)

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SECTION B

## QUESTION 7

a) Determine  $\lim_{x \rightarrow 4} \left( \frac{x^2}{x-2} \right)$  (2)

$$\begin{aligned} & \frac{4^2}{4-2} \\ &= 8 \end{aligned}$$

b) Determine  $f'(x)$  from **first principles** if  $f(x) = 3 - 5x^2$  (5)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{3 - 5(x+h)^2 - (3 - 5x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3 - 5x^2 - 10xh - 5h^2 - 3 + 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-10xh - 5h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-10x - 5h)}{h} \\ &= \lim_{h \rightarrow 0} (-10x - 5h) \\ &= -10x \end{aligned}$$

c) Determine  $D_x \left[ 7\sqrt{x} - 2\pi x^3 + \frac{5}{x} \right]$  (3)

$$\begin{aligned} &= 7x^{\frac{1}{2}} - 2\pi x^3 + 5x^{-1} \\ &= \frac{7}{2}x^{-\frac{1}{2}} - 6\pi x^2 - 5x^{-2} \\ &= \frac{7}{2x^{\frac{1}{2}}} - 6\pi x^2 - \frac{5}{x^2} \end{aligned}$$

d) If  $y = x^5$ , show that  $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ . (4)

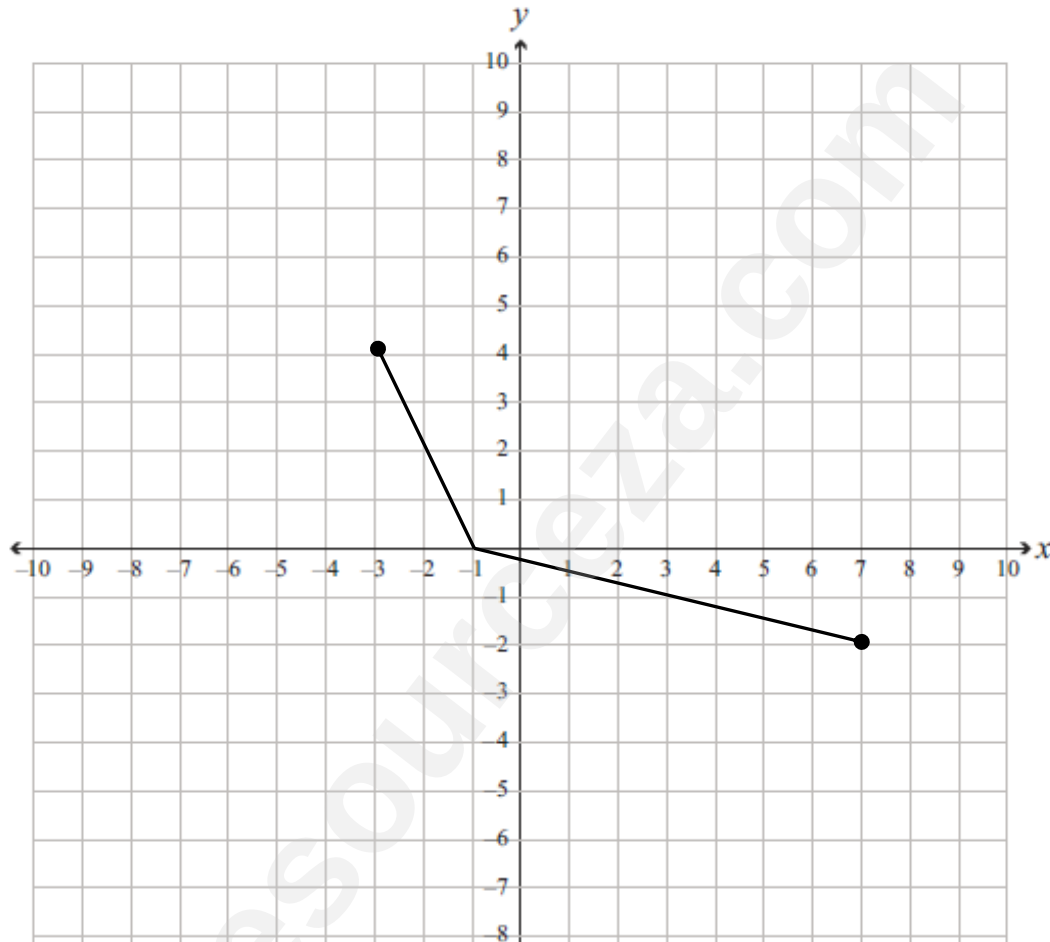
$$\frac{dy}{dx} = 5x^4 \quad \text{and} \quad x = y^{\frac{1}{5}}$$

$$\frac{dx}{dy} = \frac{1}{5}y^{-\frac{4}{5}}$$

$$\begin{aligned} \therefore \frac{dy}{dx} \times \frac{dx}{dy} &= 5x^4 \times \frac{1}{5}y^{-\frac{4}{5}} \\ &= x^4 \times y^{-\frac{4}{5}} \\ &= x^4 \times (x^5)^{-\frac{4}{5}} \\ &= x^4 \times x^{-4} \\ &= x^0 \\ &= 1 \end{aligned}$$

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## QUESTION 8 ANSWER THIS QUESTION ON THE DIAGRAM SHEET



The diagram shows a sketch of the graph of  $g$  over the domain  $-3 \leq x \leq 7$ .

a) Is  $g$  a function or a non – function? Motivate your answer. (2)

b) On the DIAGRAM SHEET provided sketch the graph of the inverse  $g^{-1}$  and state its domain. (4)

The relation  $h$  is defined by  $h(x) = 2g(x-1)$

c) Using your graphs, or otherwise, find the value of  $h(g(-3))$ . (3)

$$g(-3) = 4$$

$$\therefore h(4) = -2$$

[9]

## QUESTION 9

- a) Mr Phungula purchases a new vehicle for R200 000. The vehicle is expected to depreciate at a rate of 24% p.a. on a reducing balance. Mr Phungula expects to replace the vehicle after 5 years. A sinking fund is set up for this purpose. If the replacement cost of the vehicle increases by 18% p.a. compounded annually, calculate:

- 1) The value of the current vehicle after 5 years. (2)

$$A = 200\,000(1 - 24\%)^5$$
$$A = R50\,710,51$$

- 2) The cost of a new vehicle after 5 years. (2)

$$A = 200\,000(1 + 18\%)^5$$
$$A = R457\,551,55$$

- 3) The total required value of the sinking fund, if the old vehicle is sold and the proceeds contribute towards the new vehicle. (1)

$$R406\,841,04$$

- b) Sally takes out a 20 year loan of R100 000 to be paid back by means of equal monthly payments at an interest rate is 18% per annum compounded monthly.

- 1) Calculate the value of the monthly repayments. (3)

$$R100\ 000 = x \left( \frac{1 - \left(1 + \frac{18\%}{12}\right)^{-240}}{\frac{18\%}{12}} \right)$$

$$\therefore x = R1\ 543,31$$

- 2) Calculate the balance outstanding of the loan after the 20<sup>th</sup> payment. (3)

$$A = 1543,31 \left( \frac{1 - \left(1 + \frac{18\%}{12}\right)^{-220}}{\frac{18\%}{12}} \right)$$

$$A = R98\ 998,38$$

- 3) Sally has financial difficulties and is unable to make the 21<sup>st</sup>, 22<sup>nd</sup> and 23<sup>rd</sup> payments. The bank agrees to restructure the repayments. Determine the new monthly repayments. (5)

$$\text{New balance} = 98\ 999,38 \left(1 + \frac{18\%}{12}\right)^3$$

$$= R103\ 520,47$$

$$\therefore R103\ 520,47 = x \left( \frac{1 - \left(1 + \frac{18\%}{12}\right)^{-217}}{\frac{18\%}{12}} \right)$$

$$\therefore x = R1\ 616,71$$

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## QUESTION 10

- a) A survey conducted about the use of nuclear energy amongst 34 girls and 666 boys in one of the high schools in the Karoo District shows some of the results, as illustrated in the table below:

|                            | Girls (G)   | Boys (B)     | Total        |
|----------------------------|-------------|--------------|--------------|
| Nuclear energy (N)         | <b>a=30</b> | 270          | 300          |
| Not nuclear energy (not N) | 4           | <b>b=396</b> | <b>c=400</b> |
| Total                      | 34          | 666          | <b>d=700</b> |

- 1) Write down the values of a, b, c and d. (4)

$$a = 30 \quad b = 396 \quad c = 400 \quad \text{and} \quad d = 700$$

- 2) Is a person's choice of the use of nuclear energy independent of a person's gender? Support your answer with appropriate calculations. (6)

$$P(N) = \frac{300}{700} = \frac{3}{7}$$

$$P(G) = \frac{34}{700} = \frac{17}{350}$$

$$P(N \text{ and } G) = \frac{30}{700} = \frac{3}{70} = 0,043$$

$$P(N) \times P(G) = \frac{3}{7} \times \frac{17}{350} = \frac{51}{2450} = 0,021$$

$$P(N \text{ and } G) \neq P(N) \times P(G)$$

$\therefore$  not independent

- 3) If there are 35 000 high school learners in the Karoo District, how many learners will be against the use of nuclear energy if the information in the table is used to make a prediction? (2)

$$\frac{400}{700} \times \frac{35000}{1} = 20\,000 \text{ learners}$$

- 4) Is the prediction valid? Give a reason for your answer. (2)

*No. The sample size is very small, only 0,02% of a large learner population.*

*OR*

*Yes. The sample was chosen at random which should be a good representation of the learner population.*

b) Given:

$$P(A) = \frac{2}{3}$$

$$P(B) = \frac{1}{4}$$

$$P(A \cup B) = \frac{11}{12}$$

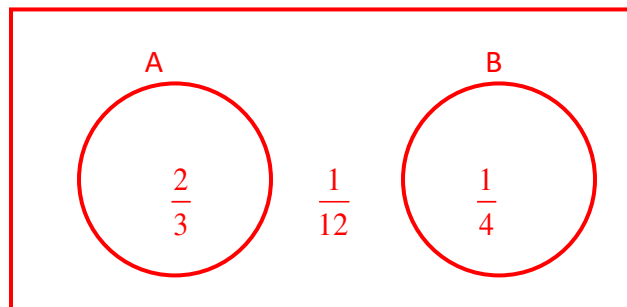
- 1) Are events A and B mutually exclusive? Use the appropriate rules and calculations to motivate your answer. (3)

*For mutually exclusive events:  $P(A) + P(B) = P(A \cup B)$*

$$\begin{aligned} P(A) + P(B) &= \frac{2}{3} + \frac{1}{4} \\ &= \frac{8+3}{12} \\ &= \frac{11}{12} \\ &= P(A \cup B) \end{aligned}$$

*$\therefore$  mutually exclusive*

- 2) Represent the data by using a Venn diagram. (3)



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## QUESTION 11

- a) Maria won the javelin field event on the sports day.

Her coach recorded the heights of the javelin during her winning throw.

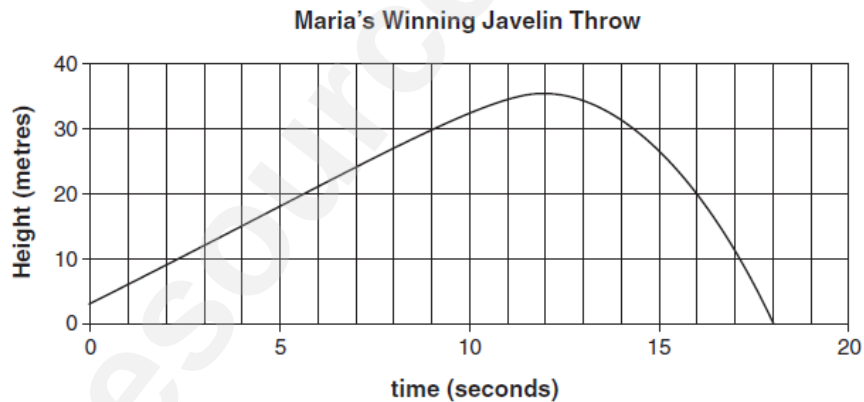
The javelin is 2 metres above the ground when it is released.

During the first 10 seconds of the javelin's flight, its height increases at a **constant rate**. The rate is 3 metres per second.

The remainder of the javelin's flight can be modeled by a **parabola**.

The javelin reaches a maximum height of 36 metres after 12 seconds.

The coach's data is graphed on the set of axes below.



- 1) Find the line equation modeled by the flight during the first 10 seconds. (2)

$$h(t) = 3t + 2 \quad \text{accept } y = 3x + 2$$

- 2) Find the parabola equation of the flight. (4)

$$h(t) = a(t - 12)^2 + 36$$

$$0 = a(18 - 12)^2 + 36$$

$$\therefore a = -1$$

$$\therefore h(t) = -1(t - 12)^2 + 36 \quad \text{accept } y = -(x - 12)^2 + 36$$

- 3) Video cameras were placed 27 metres above the ground. Calculate the TWO times when the javelin was level with the video cameras. (4)

$$27 = 3t + 2 \quad \text{and} \quad 27 = (t - 12)^2 + 36$$

$$t = \frac{25}{3} = 8\frac{1}{3} \text{ sec} \quad \text{and} \quad (t - 12)^2 = 9$$

$$\therefore t \neq 9 \text{ sec} \quad \text{but} \quad t = 15 \text{ sec}$$

- b) The common difference of an arithmetic sequence is 4.

If  $S_n$  is the sum of the sequence, prove that  $\sqrt{S_{2n} - 2S_n} = 2n$ . (6)

$$S_{2n} = \frac{2n}{2} \{2a + (2n - 1)4\}$$

$$\therefore S_{2n} = 2an + 8n^2 - 4n \dots \dots \dots A$$

$$S_n = \frac{n}{2} \{2a + (n - 1)4\}$$

$$\therefore 2S_n = 2an + 4n^2 - 4n \dots \dots \dots B$$

$$A - B: S_{2n} - 2S_n = 4n^2$$

$$\therefore \sqrt{S_{2n} - 2S_n} = \sqrt{4n^2}$$

$$\therefore \sqrt{S_{2n} - 2S_n} = 2n$$

[16]

END OF PAPER