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**EXAMINATION
DECONSTRUCTION
WORKSHOP MEMO
2024**

**GRADE 12
PAPER 1**

ALGEBRA

Solving of equations and inequalities:

St Anne's 2022

a) Solve for x and show all relevant working

(2)

1) $2x^2 - 17x = 0$

$$x(2x - 17) = 0 \quad \checkmark m(\text{fact})$$

$$\therefore x = 0 \quad \text{or} \quad 2x - 17 = 0 \quad \checkmark A(\text{both})$$

$$x = \frac{17}{2} \quad (\text{OR } 8,5)$$

2) $3x^2 + 10x - 4 = 0$ (correct to 1 decimal place)

(3)

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(-4)}}{2(3)} \quad \checkmark m(\text{quadr})$$

$$x = \frac{-10 \pm \sqrt{148}}{6} \quad \checkmark A$$

$$x = 0,4 \quad \text{OR} \quad x = -3,7 \quad \checkmark A(\text{both})$$

3) $8 \cdot 2^x = \left(\frac{1}{4}\right)^{x-2}$

(3)

$$2^3 \cdot 2^x = (2^{-2})^{x-2} \quad \checkmark m(\text{prime bases})$$

$$2^{3+x} = 2^{-2x+4} \quad \checkmark A$$

$$\therefore 3 + x = -2x + 4$$

$$3x = 1$$

$$x = \frac{1}{3} \quad \checkmark A$$

4) $(x - 5)^2 < x - 5$ (4)

$$x^2 - 10x + 25 - x + 5 < 0 \quad \checkmark m(x \text{ out and one side})$$

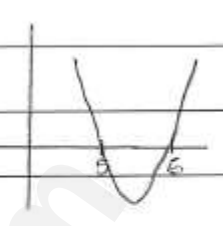
$$x^2 - 11x + 30 < 0$$

$$(x - 6)(x - 5) < 0 \quad \checkmark (fact)$$

C. Values : $x = 5$ OR 6

$\begin{array}{ccccccc} & + & 0 & - & 0 & + & \\ & & 5 & & 6 & & \end{array}$ $\checkmark m$

SOLN : $5 < x < 6$ $\checkmark n$



5) $\log_3 x = \frac{\log 5}{\log 3}$ (2)

$$\log_3 x = \log_3 5 \quad \checkmark \quad \text{OR} \quad \log_3 x = 1.46497...$$

$$\therefore x = 5 \quad \checkmark n \quad \quad \quad \therefore 3^{1.46497...} = 5$$

6) $(3^x + 9)(2^x - 27) = 0$ (4)

$$9(2^x - 27) = 0 \quad \checkmark m(eq. to 0)$$

$$3^x + 9 = 0 \quad \text{OR} \quad 2^x - 27 = 0$$

$$3^x = -9 \quad \checkmark n \quad \quad \quad 2^x = 27$$

No solution $\checkmark n$ $\quad \quad \quad \log_2 27 = x \quad \checkmark m(\log 6)$

$$\therefore x = 4, 8 \quad \checkmark n$$

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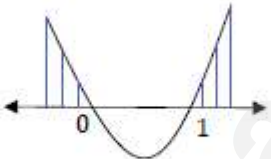
a) Solve for x .

1) $(x + 1)(x - 5) = 0$ (2)

$$(x + 1)(x - 5) = 0$$

$$x = -1 \text{ OR } x = 5$$

$\checkmark x = -1$
 $\checkmark x = 5$

- 2) $2x^2 - 5x = 1$ (in surd form) (4)
- $2x^2 - 5x = 1$ (in surd form)
- $2x^2 - 5x - 1 = 0$
- $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)}$
- $x = \frac{5 \pm \sqrt{33}}{4}$
- ✓ Standard form
✓ Substitute
✓ $\sqrt{33}$
✓ $\pm$ (both solutions)
- 3) $x^2 - x > 0$ (3)
- $x^2 - x > 0$
- $x(x - 1) > 0$
- CVs at 0 ; 1
- $x < 0$ or $x > 1$
- 
- ✓ Factors
✓ CVs
✓ Interval
- 4) $\sqrt{x-3} + 5 = x$ (5)
- $\sqrt{x-3} + 5 = x$
- $\sqrt{x-3} = x - 5$
- $x - 3 = x^2 - 10x + 25$
- $0 = x^2 - 11x + 28$
- $0 = (x - 4)(x - 7)$
- $x \neq 4$ or $x = 7$
- ✓ Isolate surd
✓ Square both sides (m)
✓ Standard form
✓ $x = 7$
✓ $x \neq 4$

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a) Solve for x , correct to ONE decimal place where necessary.

$$1) \quad x(x - 2) = 8 \quad (3)$$

$$\begin{aligned} x^2 - 2x - 8 &= 0 \checkmark \\ (x + 2)(x - 4) &= 0 \checkmark \\ x &= -2 \quad \text{or} \quad x = 8 \checkmark \end{aligned}$$

$$2) \quad x^2 - px = -2 \quad \text{in terms of } p. \quad (3)$$

$$\begin{aligned} x^2 - px &= -2 \\ x^2 - 6x + 2 &= 0 \\ x &= \frac{p \pm \sqrt{(-p)^2 - 4(1)(2)}}{2(1)} \checkmark \checkmark \\ x &= \frac{p \pm \sqrt{p^2 - 8}}{2} \checkmark \end{aligned}$$

$$3) \quad 2\sqrt{x - 1} + 1 = x \quad (4)$$

$$\begin{aligned} 2\sqrt{x - 1} &= x - 1 \\ 4(x - 1) &= (x - 1)^2 \checkmark \\ 4x - 4 &= x^2 - 2x + 1 \\ x^2 - 6x + 5 &= 0 \checkmark \\ (x - 5)(x - 1) &= 0 \\ x &= 5 \quad \checkmark \quad \text{or} \quad x = 1 \checkmark \end{aligned}$$

$$4) \quad 3^{x+3} - 3^{x+2} + 3^0 = 487 \quad (4)$$

$$\begin{aligned} 3^{x+3} - 3^{x+2} &= 486 \\ 3^x(3^3 - 3^2) &= 486 \checkmark \\ 3^x(18) &= 486 \checkmark \\ 3^x &= 27 \\ 3^x &= 3^3 \checkmark \\ x &= 3 \checkmark \end{aligned}$$

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a) Solve for x :

$$1) \quad x(\pi - x) = 0 \quad \text{where } x \text{ is a rational number} \quad (2)$$

$$x = 0 \text{ only } \checkmark \checkmark$$

$$2) \quad -3x^2 + 8x = -7 \quad (\text{round off to 2 decimal places}) \quad (4)$$

$$\begin{aligned} -3x^2 + 8x + 7 &= 0 \checkmark \\ x &= \frac{-8 \pm \sqrt{8^2 - 4(-3)(7)}}{2(-3)} \checkmark \\ x &= 3.36 \checkmark \quad \text{or} \quad x = -0.69 \checkmark \end{aligned}$$

3) $\sqrt{11-x} - x = 1$ (5)

$$\begin{aligned}\sqrt{11-x} &= 1+x \\ 11-x &= 1+2x+x^2 \\ 0 &= x^2+3x-10 \\ 0 &= (x+5)(x-2) \\ x &\neq -5 \text{ OR } x=2 \quad \checkmark_{\text{only}}\end{aligned}$$

4) $2 \cdot 3^{x-2} = 54$ (3)

$$\begin{aligned}3^{x-2} &= 27 \quad \checkmark \\ 3^{x-2} &= 3^3 \quad \checkmark \\ x-2 &= 3 \quad \checkmark \\ x &= 5 \quad \checkmark\end{aligned}$$

5) $4x^2 - 5x \leq 0$ (4)

$$\begin{aligned}x(4x-5) &\leq 0 \quad \checkmark \\ \text{CV: } x=0 \text{ OR } x=5/4 \quad \checkmark \\ \therefore 0 &\leq x \leq 5/4 \quad \checkmark\end{aligned}$$

Simultaneous Equations:

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a) Given: $\log_2 x = y$ and $2^{y+1} + x = 12$ Solve simultaneously for x and y . (5)

$$\begin{aligned}2^y &= x \quad \checkmark_{\text{m(exp form)}} \quad (1) \text{ and } 12 - 2^{y+1} = x \quad (2) \\ (1) &= (2) \quad \therefore 2^y = 12 - 2^{y+1} \quad \checkmark_{\text{m(equate)}} \\ 2^y + 2^y \cdot 2^1 &= 12 \\ \checkmark_{\text{factor}} 2^y (1+2) &= 12 \\ 2^y &= 4 \\ 2^y &= 2^2 \quad \therefore y=2 \quad \checkmark_A \\ \text{Subst back into (1)} \\ \therefore 2^2 &= x \\ x &= 4 \quad \checkmark_{\text{CA}} \\ \text{Solution: } (4; 2) &\end{aligned}$$

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- a) Solve the following equations simultaneously:

$$x^2 - xy + y^2 = 16 \quad \text{and} \quad 2x - y = 6$$

(6)

$x^2 - xy + y^2 = 16 \quad \text{and} \quad 2x - y = 4$ $2x - 4 = y$ $x^2 - x(2x - 4) + (2x - 4)^2 = 16$ $x^2 - 2x^2 + 4x + 4x^2 - 16x + 16 - 16 = 0$ $3x^2 - 12x = 0$ $x(x - 4) = 0$ $x = 0 \quad \text{or} \quad x = 4$ $y = -4 \quad y = 4$	✓ Isolate y ✓ Substitute ✓ Standard form ✓ Factors ✓ x values ✓ y values
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(6)

- b) If
- $2^{x+1} + 2^x = 3^{y+2} - 3^y$
- , calculate the values of
- x
- and
- y
- , where
- x
- and
- y
- are integers.

(5)

$2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x \cdot 2 + 2^x = 3^y \cdot 3^2 - 3^y$ $2^x(2 + 1) = 3^y(3^2 - 1)$ $3 \cdot 2^x = 3^y(8)$ $2^{x-3} = 3^{y-1}$ $x - 3 = 0 \quad \text{and} \quad y - 1 = 0$ $x = 3 \quad \text{and} \quad y = 1$	✓ Split powers ✓ Factors ✓ $2^{x-3} = 3^{y-1}$ ✓ Exponents = 0 ✓ Values of x and y
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(5)

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- a) Solve for
- x
- and
- y
- if
- $(2x - 1)^2 + (2y + 3x)^2 = 0$

(4)

$$2x - 1 = 0 \checkmark$$

$$x = \frac{1}{2} \checkmark$$

$$2y + 3(\frac{1}{2}) = 0 \checkmark$$

$$y = -\frac{3}{4} \checkmark$$

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- a) Determine the values of
- x
- and
- y
- if:

$$2x + y = 3 \quad \text{and} \quad y^2 = x^2 + y + x$$

(5)

$y = 3 - 2x \checkmark$ $(3 - 2x)^2 = x^2 + (3 - 2x) + x \quad \checkmark \text{subst}$ $9 - 12x + 4x^2 = x^2 + 3 - 2x + x$ $3x^2 - 11x + 6 = 0 \checkmark$ $x = \frac{2}{3} \quad x = 3 \quad \checkmark \text{both } x$ $y = \frac{5}{3} \quad y = -3 \quad \checkmark \text{both } y$
--

Nature of Roots:

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- a) The roots of a quadratic equation are $\frac{5 \pm \sqrt{17+4m^2}}{2m}$ where $m \neq 0$

Explain why this equation will have real roots for all $m \in \mathbb{R}$

where $m \neq 0$

(2)

Explain why this equation will have real roots for all $m \in \mathbb{R}$ where $m \neq 0$.

$$\Delta = 17 + 4m^2 \quad \checkmark \text{ (using } \Delta \text{)} \quad m^2 \geq 0$$

$$\therefore 4m^2 \geq 0 \quad \text{and} \quad \therefore 4m^2 + 17 \geq 0$$

$$\therefore \Delta \text{ is always positive} \quad \checkmark$$

$$\therefore \text{Roots are real.}$$

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Section B

- a) Given: $f(x) = x^2 + 8x + 16$

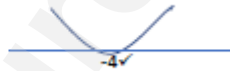
- 1) Solve for x for which $f(x) > 0$

(3)

$$x^2 + 8x + 16 > 0$$

$$(x+4)^2 > 0 \quad \checkmark$$

$$x \in \mathbb{R} - \{-4\} \quad \checkmark$$



- 2) for which value(s) of p will $f(x) = p$ have two unequal negative roots?

(3)

$$0 < p < 16$$

✓ ✓

Number Patterns

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a) Given: $T_n = 2^{1-n}$

- 1) Write down the first TWO terms of the sequence (2)

$$T_1 = 2^{1-1} = 2^0 = 1$$

$$T_2 = 2^{1-2} = 2^{-1} = \frac{1}{2}$$

- 2) Determine
- S_4
- of the sequence. (leave your answer as a fraction) (2)

$$T_3 = \frac{1}{4}$$

$$T_4 = \frac{1}{8}$$

$$\therefore S_4 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$= \frac{15}{8}$$

b) Determine:

$$\sum_{k=-1}^{20} (3k + 2)$$

(4)

$$k = -1 : 3(-1) + 2 = -1$$

$$k = 0 : 3(0) + 2 = 2$$

$$k = 1 : 3(1) + 2 = 5$$

$$\therefore -1 + 2 + 5 + \dots \dots \dots a = -1$$

$$d = 3$$

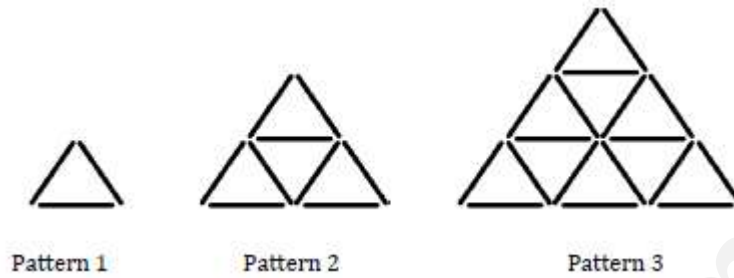
$$S_{22} = \frac{22}{2} [2(-1) + 21(3)]$$

$$= 11(61)$$

$$= 671$$

$$n = 20 - (-1) + 1 = 22 \text{ terms}$$

- c) The first three patterns formed by the arrangement of toothpicks are shown below:



- 1) How many toothpicks will the fourth pattern have? (1)

30 ✓ⁿ

- 2) Determine the n^{th} term of the sequence. (4)

$$\begin{array}{r}
 3 \quad 9 \quad 18 \quad 30 \\
 \swarrow \quad \swarrow \quad \swarrow \\
 6 \quad 9 \quad 12 \\
 \swarrow \quad \swarrow \\
 3 \quad 3
 \end{array}$$

$$\begin{array}{l}
 2a = 3 \\
 a = \frac{3}{2} \checkmark
 \end{array}
 \quad
 \begin{array}{l}
 3\left(\frac{3}{2}\right) + b = 6 \\
 \therefore b = 6 - \frac{9}{2} \\
 b = \frac{3}{2} \checkmark
 \end{array}
 \quad
 \begin{array}{l}
 \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) + c = 3 \\
 c = 0 \checkmark
 \end{array}$$

$$\therefore T_n = \frac{3}{2}n^2 + \frac{3}{2}n$$

- 3) How many toothpicks will be needed to form the 15th pattern? (1)

$$\begin{array}{l}
 T_{15} = \frac{3}{2}(15)^2 + \frac{3}{2}(15) \\
 = 360 \checkmark^{\text{n}}
 \end{array}$$

SECTION B

- d) The sum to infinity of a geometric sequence is 4 times the common ratio and the first term is three times the common ratio. Find the second term. (4)

$$S_{\infty} = 4r \quad \text{and} \quad T_1 = a = 3r$$

$$\frac{a}{1-r} = 4r \dots (1) \quad a = 3r \dots (2) \quad \checkmark \text{ m(set-up equations to find } r)$$

Subst (2) into (1)

$$\therefore \frac{3r}{1-r} = 4r \quad \checkmark \text{ m(solve simult.)}$$

$$\therefore 3r = 4r(1-r)$$

$$\therefore 4r^2 - 4r + 3r = 0$$

$$4r^2 - r = 0$$

$$r(4r - 1) = 0$$

$$r \neq 0 \quad \text{or} \quad r = \frac{1}{4} \quad \checkmark \text{ A (for } r = \frac{1}{4} \text{) and } T_2$$

$$\therefore T_2 = \frac{3}{4} \left(\frac{1}{4} \right) = \frac{3}{16} \quad (4)$$

- e) Determine the smallest value of n for which

$$\sum_{k=1}^n \frac{1}{4} (2)^{k-1} > 9 \quad (5)$$

$$k=1: \frac{1}{4} (2^0) = \frac{1}{4}$$

$$k=2: \frac{1}{4} (2^1) = \frac{1}{2} \quad \checkmark \text{ m(sub)}$$

$$k=3: \frac{1}{4} (2^2) = 1$$

$$k=4: \frac{1}{4} (2^3) = 2$$

$\therefore$ Geometric series with $a = \frac{1}{4}$ $r = 2$ $S_n > 9$

$$\therefore \frac{\frac{1}{4} (2^n - 1)}{2 - 1} > 9 \quad \checkmark \text{ m(sum)}$$

$$\therefore 2^n - 1 > 36$$

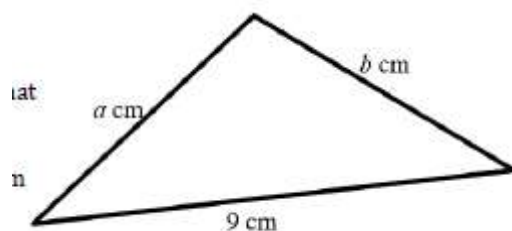
$$2^n > 37 \quad \checkmark \text{ A}$$

$$2^5 = 32 \quad \times$$

$$2^6 = 64 > 37$$

$\therefore$ 6 terms must be added to give a sum > 9 . $\checkmark \text{ A}$

- f) Refer to the figure (not drawn to scale):
 The length of the sides of a triangle are such that they form a geometric sequence.
 The longest side has a length of 9 cm.
 The other two sides are of length a cm and b cm with $a > b$



- 1) Show that $a^2 = 9b$ (2)

Sequence is: $b; a; 9$ ✓ⁿ
 then $\frac{a}{b} = \frac{9}{a}$ ✓^m (ratio)
 $\therefore a^2 = 9b$

- 2) Given the perimeter of the triangle is 19 cm, determine the length of the shortest side. (5)

Perimeter: $a + b + 9 = 19$ ✓ⁿ ... ①
 Subst $b = \frac{a^2}{9}$ into ①
 $\therefore a + \frac{a^2}{9} + 9 = 19$ ✓^m (solve simult)
 $\therefore a + \frac{a^2}{9} = 10$
 $9a + a^2 = 90$ ✓^{eqn}
 $a^2 + 9a - 90 = 0$
 $(a+15)(a-6) = 0$ ✓^m (fact)
 $\therefore a \neq -15$ or $a = 6$
 $\therefore \text{Shortest side} = b$
 $\therefore b = \frac{6^2}{9} = 4$ ✓ⁿ

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a) Consider the quadratic number pattern: 66; 67; 62; 51;

- 1) Determine the general term of this pattern in the form
- $T_n = \dots$
- . (4)

66	67	62	51	✓ 2 nd Difference ✓ a ✓ b ✓ c (4)
	1	-5	-11	
		-6	-6	
$2a = -6$	$3a + b = 1$	$a + b + c = 66$		
$a = -3$	$-9 + b = 1$	$-3 + 10 + c = 66$		
	$b = 10$	$c = 59$		
$T_n = -3n^2 + 10n + 59$				

- 2) Between which two terms of this pattern will there be a difference of -83? (4)

First differences:	✓ Subst. a and d ✓ $T_n = -83$ ✓ $n = 15$ ✓ Answer (4)
$T_n = 1 + (n - 1)(-6)$	
$T_n = -6n + 7$	
$6n + 7 = -83$	
$-6n = -90$	
$n = 15$	
$\therefore$ Between the 15 th and 16 th terms	

b) Given the arithmetic sequence: 3; 10; 17; (to 250 terms)

- 1) Write down the next term in the sequence. (1)

24

- 2) Calculate the 250
- th
- term, the last term of the sequence (2)

$T_n = 3 + (n - 1)(7)$	✓ a and d ✓ Answer (2)
$T_{250} = 3 + (249)(7)$	
$T_{250} = 1\,746$	

- 3) How many terms in the sequence are divisible by 3?
(Show all working)

(3)

The terms which are divisible by 3 are:

$$3; 24; 45; \dots; 1746$$

$$T_n = 1746; a = 3; d = 21$$

$$3 + (n-1)(21) = 1746$$

$$n = 84$$

$$\checkmark a = 3; d = 21$$

$$\checkmark \text{Subst in } T_n$$

$$\checkmark \text{Answer}$$

(3)

- c) $\sum$ (Sigma) notation describes the "sum" of a certain number of terms.
 $\prod$ (Pi) notation describes the "product" of a certain number of terms.
 Determine the value of:

$$\sum_{n=2}^5 \left(\frac{n}{5}\right) + \prod_{n=2}^5 \left(1 - \frac{1}{n}\right)$$

(3)

$$\sum_{n=2}^5 \left(\frac{n}{5}\right) + \prod_{n=2}^5 \left(1 - \frac{1}{n}\right)$$

$$= \left(\frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}\right) + \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)$$

$$= \frac{14}{5} + \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$$

$$= \frac{14}{5} + \frac{1}{5}$$

$$= 3$$

$$\checkmark \text{Expand } \prod$$

$$\checkmark \frac{14}{5}$$

$$\checkmark \frac{1}{5}$$

(3)

- d) The first three terms of a geometric sequence are 36; 12 and 4.
 Determine n if:

$$T_n = \frac{4}{243}$$

(3)

$$36 \left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$

$$\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^7$$

$$n = 8$$

$$\checkmark \text{Subst. } a, r, T_n$$

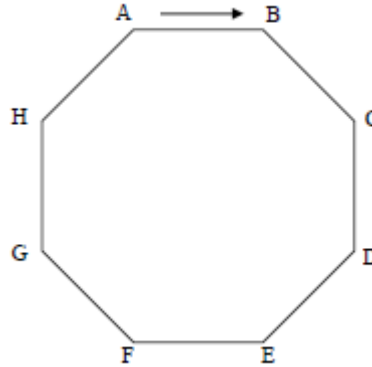
$$\checkmark \text{Same base/logs}$$

$$\checkmark n = 8$$

(3)

- e) In the figure, a robot is stationary at vertex A of a regular octagon with side 11cm. The robot begins to move around the perimeter of the octagon in a clockwise direction.

The robot covers 10cm in the first second, and there after covers $\frac{99}{100}$ of the previous distance covered per second.



- 1) What is the limit of the distance that the robot could travel? (2)

$$a = 10; r = \frac{99}{100}$$

$$S_{\infty} = \frac{10}{1 - \frac{99}{100}}$$

$$S_{\infty} = 1000 \text{ cm}$$

✓ Subst. $a; r$ in

S_{∞}

✓ Answer

(2)

- 2) The robot ran out of battery power after only 850cm. On which side of the octagon did the robot finish? (3)

$$850 \div 88 = 9,66$$

$\therefore$ 9 full cycles and 58cm (or 5,28 sides)

$\therefore$ land on GF

✓ $\div 88$

✓ Explain

✓ CD

(3)

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- a) If 2; x ; 50 form a geometric sequence, determine the value of x , if $x < 0$ (2)

$$\frac{x}{2} = \frac{50}{x}$$

$$x^2 = 100$$

$$(x = 10) \text{ n/a} \quad \text{or} \quad x = -10$$

- b) Calculate the value of
- n
- if:

$$\sum_{k=1}^n (-5k + 21) = -630 \quad (6)$$

$$16; 11; 6; \checkmark$$

$$= \frac{n}{2}[2(16) + (n-1)(-5)] = -630; \checkmark$$

$$= \frac{n}{2}[32 - 5n + 5] = -630;$$

$$n[-5n + 37] = -1260$$

$$-5n^2 + 37n + 1260 = 0$$

$$5n^2 - 37n - 1260 = 0 \quad \checkmark \quad \text{CA}$$

$$(n-20)(5n+63) = 0 \quad \checkmark \quad \text{CA}$$

$$n=20 \quad \checkmark \quad \text{CA}$$

$$\text{or } n = -\frac{63}{5} \text{ (reject)} \quad \checkmark \quad \text{CA}$$

- c)
- $3; x; 15; y$
- form a quadratic number pattern.

If the 2nd common difference is 2, determine the value of x and y . (5)

$$3; \quad x; \quad 15; \quad y$$

$$\begin{array}{c} x-3; 15-x; y-15 \quad \checkmark \quad \text{A} \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 2 \quad \quad 2 \end{array}$$

$$15 - x - (x - 3) = 2 \quad \checkmark \quad \text{CA}$$

$$15 - x - x + 3 = 2$$

$$-2x = -16$$

$$x = 8; \quad \checkmark \quad \text{CA}$$

$$y - 15 - (15 - 8) = 2; \quad \checkmark \quad \text{CA}$$

$$y - 15 - 15 + 8 = 2;$$

$$y = 24 \quad \checkmark \quad \text{CA}$$

SECTION B

- d) Consider the geometric sequence:

$$2(2x-1) + (2x-1)^2 + \frac{1}{2}(2x-1)^3 \dots \text{ where } x \neq \frac{1}{2}$$

- 1) Calculate the value of
- x
- for which the sequence converges. (4)

$$r = \frac{(2x-1)^2}{2(2x-1)} = \frac{1}{2}(2x-1) \quad \checkmark$$

$$\text{FOR CONVERGENCE: } -1 < r < 1 \quad \checkmark$$

$$-1 < \frac{1}{2}(2x-1) < 1 \quad \checkmark$$

$$-2 < 2x-1 < 2$$

$$-1 < 2x < 3$$

$$-\frac{1}{2} < x < \frac{3}{2} \quad \checkmark$$

- 2) Calculate the sum to infinity of the sequence if $x = \frac{1}{4}$. (3)

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \quad \checkmark \quad \text{A} \\ &= \frac{-1}{1+\frac{1}{4}} \quad \checkmark \quad \text{CA} \\ &= -\frac{4}{5} \quad \checkmark \quad \text{CA} \end{aligned}$$

- e) Consider the following sequence of numbers:

2 ; 10 ; 2 ; 14 ; 2 ; 18 ; 2 ; 22 ;

- 1) Write down the next TWO terms of the sequence, given that the pattern continues (2)

2; 26 ✓✓

- 2) Calculate the sum of the first 100 terms of the sequence. (4)

$$\begin{aligned} 50 \times 2 &= 100 \quad \checkmark \quad \text{A} \\ S_n &= \frac{50}{2} [2(10) + 49(4)] \quad \checkmark \quad \text{A} \\ &= 5400 \quad \checkmark \quad \text{A} \\ \text{Sum} &= 100 + 5400 = 5500 \quad \checkmark \quad \text{CA} \end{aligned}$$

Rustenburg Girls High 2022

- a) The first differences of a quadratic sequence are 102 ; 108 ; 114 ; ...

- 1) Determine which two consecutive terms have a difference of 2022. (3)

$$\begin{aligned} 6n + 96 &= 2022 \quad \checkmark \\ n &= 321 \quad \checkmark \\ \therefore \text{between } 321^{\text{st}} \text{ \& } 322^{\text{nd}} \end{aligned}$$

- 2) Determine the n^{th} term of the sequence if it is further given that the third term of the quadratic sequence is equal to 310. (4)

$$\begin{aligned} 2a + b &= 6 \quad \checkmark \\ a &= 3 \quad \checkmark \end{aligned} \quad \begin{aligned} 3a + b &= 102 \quad \checkmark \\ b &= 93 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 3n^2 + 93n + c &= T_n \\ 3(3)^2 + 93(3) + c &= 310 \quad \checkmark \\ \therefore c &= 4 \quad \checkmark \end{aligned}$$

$$T_n = 3n^2 + 93n + 4$$

- b) Given an arithmetic sequence: $\frac{2}{5}$; $\frac{3}{5}$; 0,8 ; 1 ;

- 1) Determine the value of the n^{th} term. (2)

$$T_n = \frac{1}{5}n - \frac{1}{5} \quad \checkmark \quad \checkmark$$

- 2) Calculate the sum of the first 30 terms. (3)

$$\begin{aligned} S_{30} &= \frac{30}{2} \left[2 \cdot \frac{2}{5} + 29 \cdot \frac{1}{5} \right] \quad \checkmark \\ &= 99 \quad \checkmark \end{aligned}$$

- c) Given: $2 + 6 + 18 + 54 + \dots + 118098$ (the pattern continues in the same manner)

Calculate the number of terms in the series.

(5)

$$\begin{aligned} r &= 3 \quad \checkmark \\ T_n &= 2 \cdot 3^{n-1} \quad \checkmark \\ 2 \cdot 3^{n-1} &= 118098 \quad \checkmark \\ 3^{n-1} &= 3^{10} \quad \checkmark \\ n &= 11 \quad \checkmark \end{aligned}$$

- d) The sum of the first two terms of a converging geometric series with positive terms is four times the sum of the next two terms.

- 1) Show that $r = \frac{1}{2}$.

(4)

$$\begin{aligned} a + ar &= 4(ar^2 + ar^3) \quad \checkmark \\ 1 + r &= 4r^2 + 4r^3 \quad \checkmark \\ 1 + r &= 4r^2(1 + r) \quad \checkmark \\ 4r^2 &= 1 \quad \checkmark \\ r &= \frac{1}{2} \quad \checkmark \end{aligned}$$

- 2) Calculate the value of the first terms if the sum to infinity of this series is 3

(2)

$$\begin{aligned} \frac{a}{1 - \frac{1}{2}} &= 3 \quad \checkmark \\ a &= \frac{3}{2} \quad \checkmark \end{aligned}$$

Financial Mathematics

St Annes 2022

- (a) Alice starts saving for retirement and pays R800 into a savings account at the end of each month. After she makes her last payment she has R884 942,47 in her retirement fund. She receives an interest rate of 6,5% per annum compounded monthly.

1. Determine the number of payments that she made.

$$\begin{aligned}
 x &= R800 & 884\,942,47 &= 800 \left[\left(1 + \frac{0,065}{12}\right)^n - 1 \right] \\
 FV &= 884\,942,47 & & \\
 i &= \frac{0,065}{12} & 4793,438379 &= 800 \left[\left(\frac{2413}{2400}\right)^n - 1 \right] \\
 n &=? & 6,9917\dots &= \left(\frac{2413}{2400}\right)^n \\
 & & \therefore \log_{\left(\frac{2413}{2400}\right)} 6,9917\dots &= n \\
 & & n &= 360 \text{ payments} \\
 & & \text{(if you use 12n then } n = 30 \text{ years} \times 12 = 360 \text{)} &
 \end{aligned}
 \tag{4}$$

2. Convert the nominal interest rate to an effective interest rate.

$$\begin{aligned}
 1 + i_{\text{eff}} &= \left(1 + \frac{i_{\text{nom}}}{12}\right)^{12} \\
 \therefore 1 + i_{\text{eff}} &= \left(1 + \frac{0,065}{12}\right)^{12} \\
 i_{\text{eff}} &= \left(\frac{2413}{2400}\right)^{12} - 1 \\
 i_{\text{eff}} &= 0,06677\dots \\
 \therefore \text{eff. rate is } 6,7\% \text{ pa}
 \end{aligned}
 \tag{2}$$

- (b) Thandi took a mortgage loan of R 850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8% p.a, compounded monthly.

1. Show that her monthly instalment was R 6237.

$$\begin{aligned}
 850\,000 &= x \left[1 - \left(1 + \frac{0,08}{12}\right)^{-360} \right] & P_v &= R850\,000 \\
 & & i &= \frac{0,08}{12} \\
 \therefore x &= R\,6237 & n &= 30 \times 12 \\
 & & x &=?
 \end{aligned}
 \tag{3}$$

2. Calculate the outstanding balance on her loan at the end of the first year (Round off your answer to the nearest rand).

$ \begin{aligned} OB &= F_v - F_{v \text{ ANNUITY}} \\ &= 850\,000 \left(1 + \frac{0,08}{12}\right)^{12} - 6237 \left[\left(1 + \frac{0,08}{12}\right)^{12} - 1 \right] \\ &= 920\,549,5808 - R\,77650,18859 \\ &= R\,842\,899,39 \\ &= R\,842\,899 \text{ (to nearest rand)} \end{aligned} $	OR using P value with 29 years to go $ \begin{aligned} P_v &= 6237 \left[1 - \left(1 + \frac{0,08}{12}\right)^{-39} \right] \\ &= R\,842\,899,56 \end{aligned} $
--	---

(3)

3. Hence, or otherwise, calculate how much of the R 74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged.

$$\begin{aligned} \text{Loan amount reduced by } R 350\,000 - R 84\,287 \\ = R 7101 \quad \checkmark^m \\ \text{But she has paid } 6237 \times 12 = R 74\,844 \text{ in the 1st year} \\ \therefore \text{Payments made towards interest charged} \\ 74\,844 - 7101 \\ = R 67\,743 \quad \checkmark^{CA} \end{aligned}$$

(3)

Northcliff High School 2022

- a) Kate invested R1 430,77 in a fund for 18 months. Interest was compounded quarterly. Calculate the annual interest rate (compounded quarterly) if the money grew to R1 711,41.

(3)

$$A = P(1 + i)^n$$

$$1711,41 = 1430,77 \left(1 + \frac{i}{4}\right)^6$$

$$1 + \frac{i}{4} = 1,0303 \dots$$

$$i = 0,121203 \dots$$

$$r = 12,12\%$$

✓ Subst. $\frac{i}{4}$ and 6

✓ Manipulation

✓ Rate (%)

(F)

- b) Mbali saves R450 at the beginning of each month for four years. She earns interest at 7,5 % p.a. compounded monthly. How much money does she have at the end of four years if her last deposit was made one month before the end of four years?

(4)

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = \frac{450 \left[\left(1 + \frac{0,075}{12}\right)^{48} - 1 \right]}{\frac{0,075}{12}} \left(1 + \frac{0,075}{12}\right)$$

$$F = R25099,1389 \left(1 + \frac{0,075}{12}\right)$$

$$F = R25\,256,01$$

- ✓ $n = 48$
- ✓ $i = \frac{0,025}{12}$ in FV
- ✓ $\left(1 + \frac{0,075}{12}\right)$
- ✓ Answer (no CA)

(4)

- c) A couple have found their dream house and need to secure a loan for R1 750 000. They plan to repay the loan monthly over a period of 20 years starting one month after the granting of the loan. The interest is fixed at 9% p.a. compounded monthly.

- 1) Calculate their monthly repayments on a R1 750 000 loan.

(4)

$$p = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$1750000 = \frac{x \left[1 - \left(1 + \frac{0,09}{12}\right)^{-240} \right]}{\frac{0,09}{12}}$$

$$x = \frac{1750000 \times \frac{0,09}{12}}{1 - \left(1 + \frac{0,09}{12}\right)^{-240}}$$

$$x = R15\,745,20$$

- ✓ Sub
- R1 750 000
- into F
- ✓ i and n
- ✓ Manipulation (implied)
- ✓ Answer

(4)

- 2) Unfortunately, the couple had not considered having to pay an additional R97 887 in Transfer Fees. They can only afford to borrow the additional amount and keep their monthly repayments the same, if the bank will extend the loan period.

Determine how much borrowing an additional R97 887 would cost the couple.

(4)

$$97887 + 1750000 = R1\,847\,887$$

$$1\,847\,887 = \frac{15745,20 \left[1 - \left(1 + \frac{0,09}{12} \right)^{-n} \right]}{\frac{0,09}{12}}$$

$$n = 284$$

$\therefore$ 44 extra payments

$$44 \times 15\,745,20 = R692\,788,80$$

- ✓ New total loan
 - ✓ Sub x and new P
 - ✓ Using logs
 - ✓ $44 \times 15\,745,20$
- (4)

Crawford International 2022

- a) Zinhle plans to save R350 000 towards buying a new car in 3 years' time. She makes 3 equal deposits at the end of each year into a savings account. The interest paid on the money in the savings account is 11% p.a. compounded quarterly. Calculate how much money she will need to deposit on each occasion. (4)

$$x \left(1 + \frac{11\%}{4} \right)^8 + x \left(1 + \frac{11\%}{4} \right)^4 + x = 350\,000$$

$$x = R104\,259,70$$

SECTION B

- b) Justin invests a certain amount of money (P) at an interest rate of $r\%$ p.a on a straight line basis. After two years, his investment grows to R17 024 and after six years, his investment grows to R25 472. Determine P , the initial amount invested. (4)

Determine P , the initial amount invested.

$$\frac{25\,472 - 17\,024}{4} = 2112 \quad \checkmark \checkmark$$

$$y = 2112x + P$$

$$17\,024 = 2112(2) + P \quad \checkmark$$

$$P = R12\,800 \quad \checkmark$$

- c) Mr. Govender and Mrs. Williams have both taken loans from ABSA Bank. Repayments are made consistently after one month of granting of the loan. The details are tabulated below.

	Mr Govender	Mrs Williams
Loan Amount	R75 000	P
Monthly payment	R1 200	R 1 180,42
Interest rate	15% p.a compounded monthly	13% p.a compounded monthly
Balance outstanding after 62 months.	K	K

If the interest rates and monthly repayments remain constant, determine:

- 1) K, the balance outstanding on Mr Govender's loan after 62 months. (5)

$$\begin{aligned} \text{Balance outstanding} &= 75000 \left(1 + \frac{15\%}{12}\right)^{62} - \frac{1200 \left[\left(1 + \frac{15\%}{12}\right)^{62} - 1\right]}{\frac{15\%}{12}} \\ &= R50\,636,01 \end{aligned}$$

- 2) Immediately after 62 months, Mrs Williams received the following summary from ABSA Bank.

Balance Outstanding	R50 636
Total interest paid	R44 764

Using this information, calculate P, the initial amount that Mrs Williams borrowed from ABSA Bank. (5)

$$\begin{aligned} 62 \times 1180,42 - (79057 - 50636) \\ = R44\,764,10 \end{aligned}$$

Rustenburg Girls High 2022

- a) An investor indicates that they will be able to triple the value of the investment at the end of 6 years. The interest rate is fixed and compounded monthly. Calculate the annual interest rate that the investor offers. (4)

$$\begin{aligned} 3P &= P \left(1 + \frac{i}{12}\right)^{6 \times 12} \\ 3 &= \left(1 + \frac{i}{12}\right)^{72} \\ i &= \left(\sqrt[72]{3} - 1\right) \times 12 \\ i &= 18,45\% \end{aligned}$$

- b) Samuel decided to buy a car costing R192 000. He takes out a loan for 5 years at an interest rate of 12 % p.a. compounded monthly. Payments are made at the end of each month

- 1) Calculate the monthly repayments over a period of 5 years. (4)

$$192\,000 = \frac{x \left[1 - \left(1 + \frac{12\%}{12} \right)^{-60} \right]}{\frac{12\%}{12}}$$

✓ correct f-l-a

$$\therefore x = 4270,93 \quad \checkmark$$

- 2) After Samuel had made 45 payments, he decides to settle the balance on the loan. Calculate the lump sum that he will need to pay off the loan after he has made the 45th payment. (4)

$$DB_{45} = 192\,000 \times 1,01^{45} - \frac{4270,93 \left[1,01^{45} - 1 \right]}{0,01}$$

$$= 59\,216,95 \quad \checkmark$$

Alternative

$$DB_{45} = \frac{4270,93 \left[1 - 1,01^{-15} \right]}{0,01}$$

✓ a = -15 ✓ f-l-a

$$= 59\,216,69 \quad \checkmark$$

- 3) Determine the amount of interest paid if Samuel settled his loan as mentioned in (b)(2) (3)

$$\text{Total paid} = 59\,216,69 + 4270,93 \times 45$$

$$\text{Interest} = 251\,408,54 - 192\,000$$

$$= 59\,408,54 \quad \checkmark \quad \text{Alt: } 59\,408,9$$

Functions:

St Anne's 2022

a) Given: $f(x) = \sqrt{2x+1}$

- 1) Give the domain of
- $f(x)$
- (1)

$$2x+1 \geq 0 \quad \therefore x \geq -\frac{1}{2}$$

- 2) If
- $g(x) = 2x - 1$
- , solve for
- $f(x) = g(x)$
- (4)

$$\begin{aligned} 2x-1 &= \sqrt{2x+1} \\ (2x-1)^2 &= 2x+1 \quad \checkmark \text{ (square)} \\ 4x^2 - 4x + 1 - 2x - 1 &= 0 \\ 4x^2 - 6x &= 0 \quad \checkmark \\ 2x(2x-3) &= 0 \quad \checkmark \text{ (factor)} \\ \therefore x &= 0 \quad \text{or} \quad x = \frac{3}{2} \\ \text{Check: LHS: } -1 & \quad \text{LHS: } 2 \\ \text{RHS: } 1 & \quad \text{RHS: } \sqrt{2(\frac{3}{2})+1} = 2 \\ \therefore \text{SOLN: } x &= \frac{3}{2} \text{ only. } \checkmark \end{aligned}$$

- b) The equation of the graphs of
- $g(x) = \frac{3x-3}{x-1}$
- and
- $h(x) = -\frac{3}{x-1} + 3$
- are given.

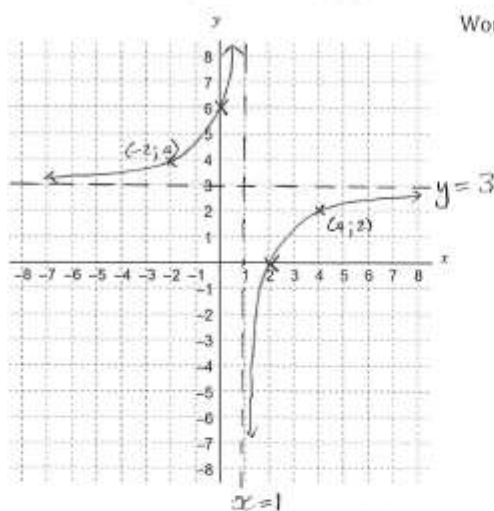
- 1) Show that
- $g(x)$
- is an asymptote of
- $h(x)$
- and state whether it is the vertical or horizontal asymptote. (2)

$$\begin{aligned} g(x) &= \frac{3(x-1)}{x-1} = 3 \\ \therefore g(x) &= 3 \text{ is the horizontal asymptote of } h(x) \end{aligned}$$

- 2) Write down the domain and range of
- $h(x)$
- . (2)

$$\begin{aligned} \text{Domain: } x &\in \mathbb{R}; x \neq 1 \quad \checkmark \\ \text{Range: } y &\in \mathbb{R}; y \neq 3 \quad \checkmark \end{aligned}$$

- 3) Sketch the graph of $h(x)$. Show clearly all intercepts with the axes, the asymptotes and two points on each arm of the graph



Working space

$$\begin{aligned} \text{y int: let } x &= 0 \\ \therefore y &= \frac{-3}{-1} + 3 = 6 \\ &\quad (0; 6) \end{aligned}$$

$$\begin{aligned} \text{x int: let } y &= 0 \\ 0 &= \frac{-3}{x-1} + 3 \\ 3 &= 3(x-1) \\ 3x &= 6 \\ x &= 2 \quad (2; 0) \end{aligned}$$

$$\begin{aligned} \text{Choose } x &= -2 \\ y &= \frac{-3}{-3} + 3 = 4 \\ &\quad (-2; 4) \end{aligned}$$

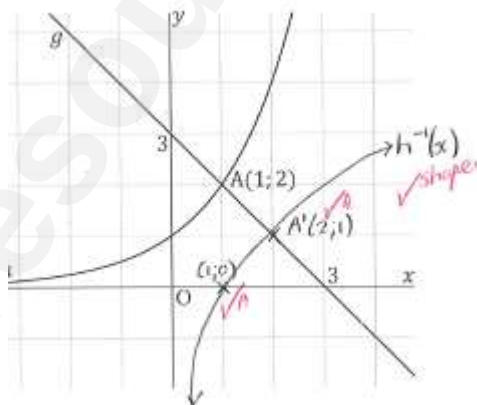
$$\begin{aligned} \text{Choose } x &= 4 \\ y &= \frac{-3}{3} + 3 = 2 \\ &\quad (4; 2) \end{aligned}$$

✓ asymptotes
✓ shape
✓ intercepts
✓ 2 other points

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(4)

- c) The graph shows the function $g(x) = -x + 3$ and $h(x) = b^x$. The two functions intersect at $A(1; 2)$.



- 1) Determine the value of b .

(2)

$$\begin{aligned} \text{Subst } (1; 2) \text{ into } h(x) &\quad \checkmark^m \\ \therefore 2 &= b^1 \\ b &= 2 \quad \checkmark^A \end{aligned}$$

- 2) If the point $A(1; 2)$ lies on h , determine the new coordinate, A' when h is reflected about the line $y = x$.

(1)

$$A'(2; 1) \quad \checkmark^A$$

- 3) Write down the inverse function of $h(x)$ in the form

(2)

$$h^{-1}(x) = \dots$$

Inverse $h(x)$: $x = 2^y$ *✓ h (switch)*

$$\therefore y = \log_2 x \quad \therefore h^{-1}(x) = \log_2 x$$

✓ A (must be in $h^{-1}(x)$ form)

(2)

- 4) On the graph above, sketch h^{-1} labelling any intercepts with the axes as well as point A' (3)
- 5) Use the graphs above to determine the values of x for which $-2^x \cdot x + 6 \cdot 2^{x-1} < 0$. (3)

$$-2^x \cdot x + 6 \cdot 2^x \cdot 2^{-1} < 0$$

$$\checkmark 2^x (-x + 6(\frac{1}{2})) < 0$$

$$2^x (-x + 3) < 0$$

$$\text{for } x > 3 \quad \checkmark \text{ (Read from x-int of } g)$$

Northcliff High School 2022

a) Given: $f(x) = \frac{-6}{x+2}$

- 1) Write down the equations of the vertical and horizontal asymptotes. (2)

$$x = -2 \quad \text{and} \quad y = 0$$

$$\checkmark x = -2$$

$$\checkmark y = 0$$

- 2) Calculate the coordinates of the y-intercept of f . (2)

$$y = \frac{-6}{0+2}$$

$$y = -3$$

$$\therefore (0; -3)$$

$$\checkmark x = 0$$

$$\checkmark y = -3$$

P1 for coordinates

(2)

- 3) If $A(-5; t)$ lies on the graph of f , determine the value(s) of t . (1)

$$t = \frac{-6}{-5+2}$$

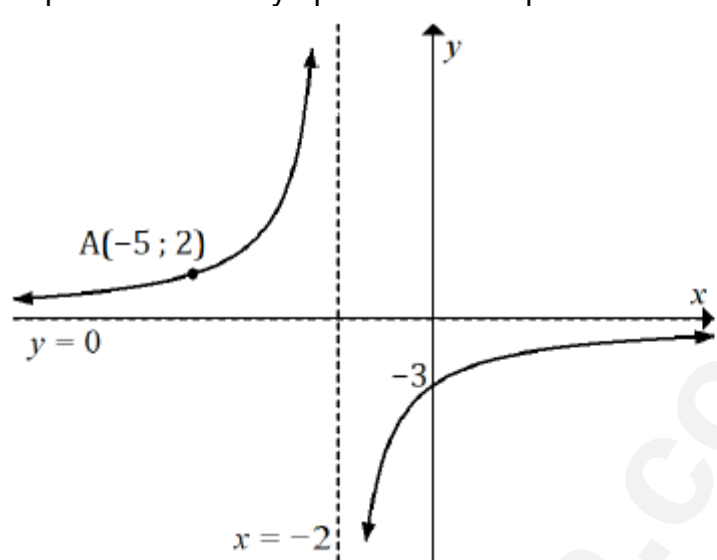
$$t = 2$$

$$\checkmark t = 2$$

(1)

- 4) Draw a neat sketch graph of f . Label the y -intercept, the equation of the asymptotes and the point A.

(3)



✓ y - intercept
and A
✓ Shape
✓ Asymptotes
(approach)

(3)

- 5) The graph of $g(x) = \frac{x-4}{x+2}$ is a vertical translation of the graph of f by k units. Determine the value of k , showing working.

(2)

$$g(x) = \frac{x-4}{x+2}$$

✓ Method

✓ $k = 1$

$$g(x) = \frac{-6 + x + 2}{x + 2}$$

(2)

$$g(x) = \frac{-6}{x+2} + \frac{x+2}{x+2}$$

$$g(x) = \frac{-6}{x+2} + 1$$

$$\therefore k = 1$$

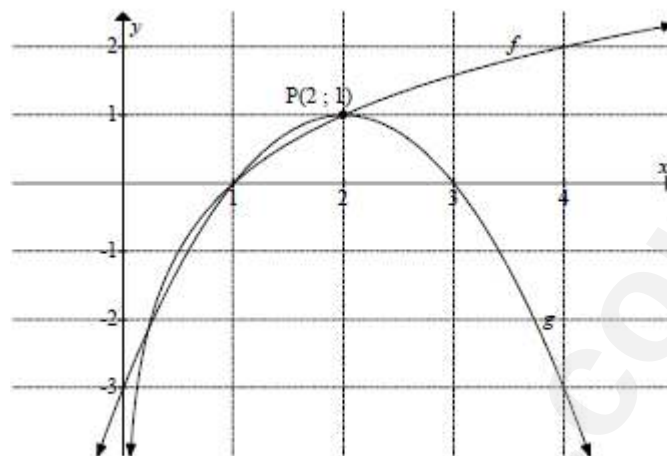
ALTERNATIVE

$$\begin{array}{r} x+2 \overline{) x-4} \\ \underline{x+2} \\ -6 \end{array}$$

$$\therefore \frac{x-4}{x+2} = 1 + \frac{-6}{x+2}$$

$$\therefore k = 1$$

- b) The diagram below shows the graphs of $f(x) = \log_a x$ and $g(x) = -x^2 + 4x - 3$.
Point $P(2;1)$, the turning point of g , represents a point of intersection between f and g .



- 1) Determine the value of a . (2)

$$1 = \log_a 2$$

$$a^1 = 2$$

$\checkmark$ Subst.
 $\checkmark$ a
- 2) Determine the equation of the inverse of f , in the form $y = \dots$ (2)

$$f: y = \log_2 x$$

$$f^{-1}: x = \log_2 y$$

$$2^x = y$$

$\checkmark$ Swop x and y
 $\checkmark$ Standard form
- 3) For which values of x would $1 < f^{-1} \leq 4$? (2)

Values of y on f for which $1 \leq x \leq 4$

$$\therefore 0 \leq x \leq 2$$

$\checkmark \checkmark$ Interval
- 4) Write down a possible restriction for g such that g^{-1} would be a function, while keeping the original range of g the same. (1)

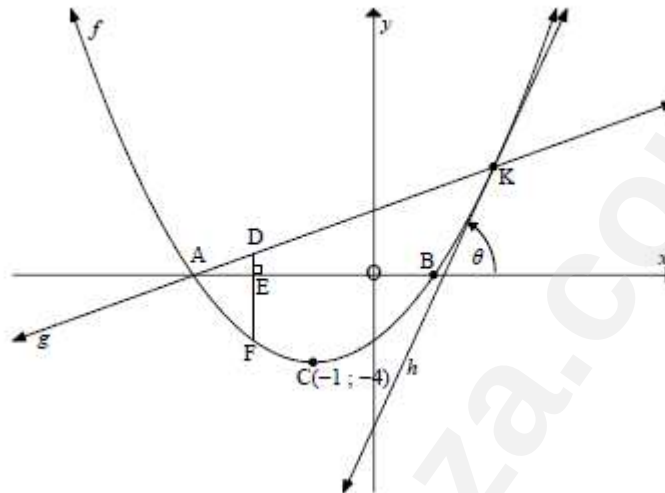
$$x \leq 2 \text{ or } x \geq 2$$

$\checkmark$ Either
 (accept
 $x > 2$ etc)
- 5) For which values of x would $f'(x) \cdot g'(x) < 0$. (1)

$$x > 2$$

$\checkmark$ Answer

- c) In the diagram below, $f(x) = x^2 + 2x - 3$ where $C(-1; -4)$ is the turning point of f and A and B the x -intercepts. The graph of $g(x) = x + 3$ passes through A and K , where K is the point of intersection of f and g . Line h is a tangent to f at K , and the angle of inclination for h is θ . D lies on g , with F on f and E on the x -axis such that $DEF \perp OE$.



- | | |
|--|---|
| <p>1) Write down the range of f.</p> <p>$y \geq -4$</p> | <p>(1)</p> <p>✓ Answer</p> |
| <p>2) Determine the length of AB.</p> <p>$0 = x^2 + 2x - 3$</p> <p>$0 = (x + 3)(x - 1)$</p> <p>$x = -3$ or $x = 1$</p> <p>$AB = 4$ units</p> | <p>(4)</p> <p>✓ $y = 0$</p> <p>✓ Factors</p> <p>✓ x values</p> <p>✓ Length</p> <p>(4)</p> |
| <p>3) Show that $K(2; 5)$.</p> <p>$x^2 + 2x - 3 = x + 3$</p> <p>$x^2 + x - 6 = 9$</p> <p>$(x + 3)(x - 2) = 0$</p> <p>$x = -3$ or $x = 2$</p> <p>$f(2) = (2)^2 + 2(2) - 3$</p> <p>$f(2) = 5$</p> <p>$\therefore K(2; 5)$</p> | <p>(4)</p> <p>✓ Equate</p> <p>✓ Standard form</p> <p>✓ Factors</p> <p>✓ Subst. back</p> <p>(4)</p> |

- 4) Calculate the value of θ , the angle of inclination of h . (4)

$$f'(x) = 2x + 2$$

$$f'(2) = 2(2) + 2 = 6$$

$$\tan \theta = 6$$

$$\theta = 80,54^\circ$$

✓ Derivative

✓ Subst. $x = 2$ ✓ Subst. $\tan \theta$ ✓ θ

(4)

- 5) Determine the length of OE when DEF is at a maximum. (4)

$$DF = L(x) = x + 3 - (x^2 + 2x - 3)$$

$$L(x) = -x^2 - x + 6$$

$$\text{Max when: } L'(x) = 0$$

$$0 = -2x - 1$$

$$x = -\frac{1}{2}$$

$$\therefore OE = \frac{1}{2} \text{ unit}$$

✓ $g(x) - f(x)$

✓✓ Max method

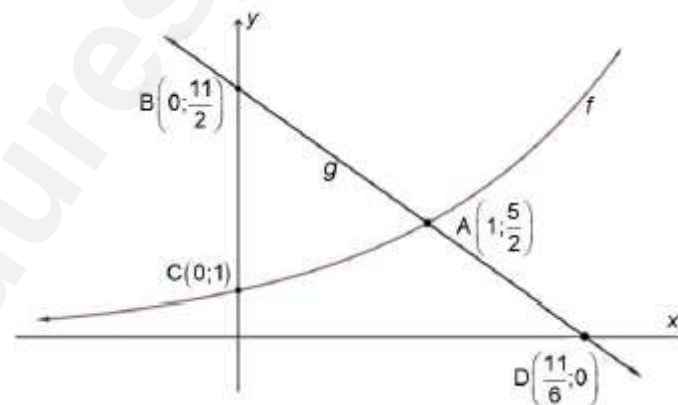
(AOS or Deriv)

✓ OE length

(4)

Crawford International 2022

- a) The graphs of $f(x) = \left(\frac{5}{2}\right)^x$ and $g(x) = mx + c$ are drawn. $A\left(1; \frac{5}{2}\right)$ is the point of intersection of the two graphs. $B\left(0; \frac{11}{2}\right)$ is the y-intercept of g , and $C(0; 1)$ is the y-intercept of f .



- 1) Determine the values of m and c (2)

$$m = -3 \checkmark \text{ a}$$

$$c = \frac{11}{2} \checkmark \text{ a}$$

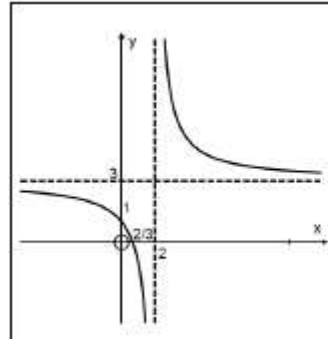
- 2) Determine f^{-1} , the inverse of f in the form $y = \dots$ (2)

$$y = \log_{\frac{5}{2}} x \checkmark \checkmark$$

- 3) Hence, write down the domain of $f^{-1}(x - 2)$ (2)

$$x > 2 \checkmark \checkmark$$

- b) 1) On the grid below, sketch the graph of $f(x) = \frac{4}{x-2} + 3$ showing all intercepts and asymptotes clearly.



✓a asymptotes

✓a x int

✓a y int

✓a quadrants (shape)

(4)

- 2) Determine the equation of the axis of symmetry of f which has a negative gradient. (3)

$$y = -x + c$$

$$\text{subst}(2,3)$$

$$3 = -2 + c$$

$$5 = c$$

$$y = -x + 5$$

✓a neg gradient

✓m subst

✓a c value in equation

- c) Given:

$$p(x) = \frac{3x + 1}{x - 2}$$

- Rewrite p in the form $p(x) = \frac{a}{x-2} + n$ where a and n are integers. (4)

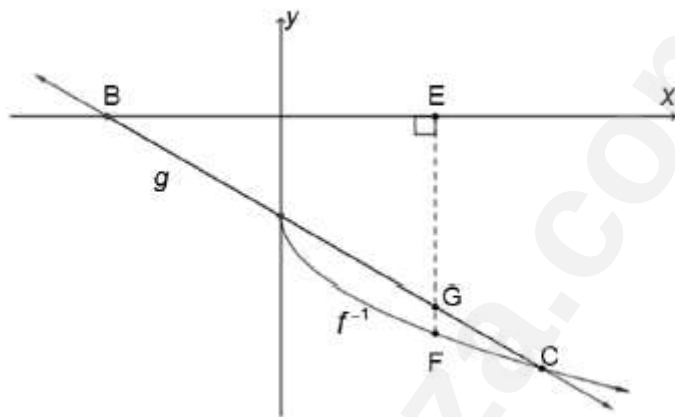
$$\frac{3x+1}{x-2} = \frac{3(x-2)+7}{x-2} = \frac{7}{x-2} + 3$$

✓✓

✓✓

- d) In the figure below, the graphs of f^{-1} with equation $f^{-1} = -\sqrt{x} - 2$, the inverse f , and g with equation $g(x) = \frac{-1}{3}x - 2$ are drawn.

- The graph of g intersects the x -axis at B and the y -axis at A .
- $E(x_e; 0)$ is a point on the x -axis.
- $C(x_c; y_c)$ is point of intersection of f^{-1} and g .
- G and F are points on g and f^{-1} respectively.
- EGF is parallel to the y -axis such that $0 < x_e < x_c$



- 1) Is f^{-1} a function? Justify your answer. (2)

Yes ✓ a, it passes the "vertical line test" (1:1 correspondence) ✓ a

- 2) Determine the equation of the function f , in the form $f(x) = \dots$ (4)

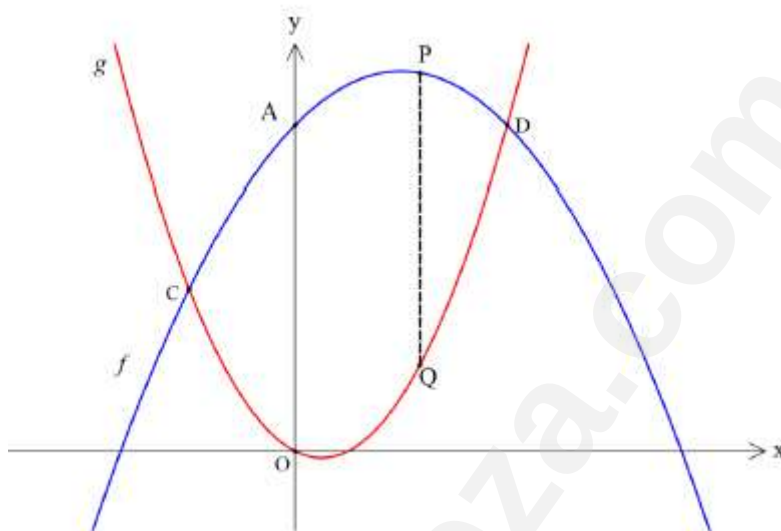
$$\begin{aligned}
 y &= -\sqrt{x} - 2 \\
 \therefore x &= -\sqrt{y} - 2 \quad \checkmark \text{ m} \\
 \sqrt{y} &= -2 - x \quad \checkmark \text{ m (squaring)} \\
 y &= (-2 - x)^2 \\
 \therefore f(x) &= (-2 - x)^2 = y \quad \checkmark \text{ a; } x \leq -2 \quad \checkmark \text{ a}
 \end{aligned}$$

- 3) The position of the vertical line GF is a variable such that $x_A < x_E < x_C$. Determine the length of the vertical line segment GF , in terms of x . (2)

$$\begin{aligned}
 GF &= y_G - y_F \\
 &= -\frac{1}{3}x - 2 - (-\sqrt{x} - 2) \quad \checkmark \text{ m} \quad \checkmark \text{ a} \\
 &= -\frac{1}{3}x + \sqrt{x}
 \end{aligned}$$

Rustenburg Girls High 2022

- a) In the diagram, the graph of $f(x) = -x^2 + 2x + 6$ and $g(x) = 2x^2 - x$ are drawn below. They intersect at C and D. A is the y-intersect of f. P and Q are two points on f and g respectively such that PQ is parallel to the y-axis.



- 1) Write down the co-ordinates of A (1)

$$A(0;6) \checkmark$$

- 2) Calculate the coordinates of C and D. (5)

$$\begin{aligned} -x^2 + 2x + 6 &= 2x^2 - x \checkmark \\ -3x^2 + 3x + 6 &= 0 \checkmark \\ x^2 - x - 2 &= 0 \checkmark \\ (x-2)(x+1) &= 0 \checkmark \\ x &= 2 \text{ or } x = -1 \checkmark \text{ with } x \\ y &= 6 \quad y = 3 \checkmark \text{ with } y \end{aligned}$$

- 3) Write down the values of x for which $f(x) \leq g(x)$ (2)

$$x \leq -1/3 \checkmark \text{ or } x \geq 2 \checkmark$$

- 4) Calculate the maximum length of PQ where line PQ is between C and D. (4)

$$\begin{aligned} PQ &= (-x^2 + 2x + 6) - (2x^2 - x) \checkmark \\ PQ &= -3x^2 + 3x + 6 \checkmark \\ x_{\text{TP}} &= 0,5 \checkmark \\ \therefore \text{max } PQ &= 6,75 = 27/4 \checkmark \end{aligned}$$

- 5) Calculate the value of x where the instantaneous gradient of f is equal to 3. (4)

$$\begin{aligned} f'(x) &= -2x + 2 \checkmark \\ -2x + 2 &= 3 \checkmark \\ x &= -1/2 \checkmark \end{aligned}$$

6) Determine the values of k for which $f(x) = k$ has:

i) Equal roots (3)

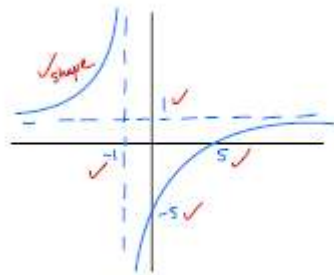
$$\begin{aligned} \text{TP of } f & \quad x=1 \checkmark \\ y &= 7 \checkmark \\ \therefore k &= 7 \checkmark \end{aligned}$$

ii) Two positive unequal roots (2)

$$6 < k < 7 \checkmark$$

b) Given: $g(x) = -\frac{6}{x+1} + 1$

1) Draw the graph of g . Show all the intercepts with the axes as well as the asymptotes of the graph (5)



2) Determine the equation of the symmetry line of g with positive gradient. (2)

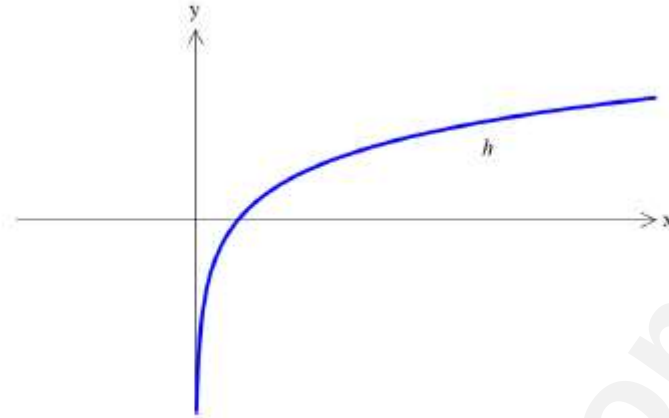
$$y = x + 2 \checkmark \checkmark$$

3) Determine:

$$\sum_{x=0}^2 g(x) \quad (2)$$

$$\begin{aligned} &= g(0) + g(1) + g(2) \checkmark \\ &= -5 - 2 - 1 \\ &= -8 \checkmark \end{aligned}$$

- c) In the diagram, the graph of $h(x) = \log_3 x$ is drawn.



- 1) Write down the equation of h^{-1} , the inverse of h , in the form: $y = \dots$. (2)
 $y = 3^x$ ✓✓
- 2) Write down the domain and range of h^{-1} . (2)
 $x \in \mathbb{R}$ ✓ $y > 0$ ✓
- 3) Calculate the values of x for which $2h(x) < -6$. (4)
 $2 \log_3 x = -6$
 $\log_3 x = -3$ ✓
 $x = 3^{-3}$ ✓
 $\therefore 0 < x < \frac{1}{27}$ ✓✓
- 4) $t(x) = \log_b x$, is the reflection of h in the x -axis. Write down the value of b . (1)
 $b = 1/3$ ✓

Calculus:

St Anne's 2022

- a) 1) Using first principles, find $f'(x)$ if $f(x) = x^2 - 3x$ (4)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 3(x+h)] - (x^2 - 3x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \quad (-1) \text{ for incorrect notation.} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h} \\
 &= \lim_{h \rightarrow 0} (2x + h - 3) = 2x - 3 \quad (4)
 \end{aligned}$$

- 2) Determine the equation of the tangent to the graph of f at the point where $f'(x) = -1$. (4)

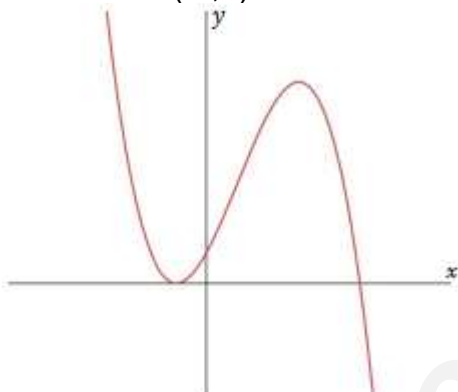
$$\begin{aligned}
 2x - 3 &= -1 \quad \text{m(equate)} \\
 \therefore x &= 1 \\
 \text{So } f(1) &= 1^2 - 3(1) \\
 &= -2 \quad \text{m(sub)} \\
 \text{Point } (1, -2) & \\
 \text{Eqn of tang} & \\
 y - (-2) &= -1(x - 1) \\
 y &= -x - 1 \quad \text{m(sub)}
 \end{aligned}$$

- b) Determine $\frac{dy}{dx}$ if $y = \sqrt{4x^3} + 2\sqrt{x} - \frac{2}{x}$ (leave your answer with positive exponents.) (4)

$$\begin{aligned}
 y &= 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2x^{-1} \quad \text{m(simplify)} \\
 \therefore \frac{dy}{dx} &= 3x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 2x^{-2} \quad \text{m(diff)} \\
 &= 3x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} + \frac{2}{x^2} \quad (-1) \text{ if ans left as -ve exponents}
 \end{aligned}$$

SECTION B

- c) Given: $f(x) = -x^3 + 3x^2 + 9x + 5 = -(x + a)^2(x - 5)$ and f has a stationary point on the x-axis at $(-1; 0)$.



- 1) Write down the value of a (1)

$$a = 1$$

- 2) Determine the coordinates of the other stationary point of $f(x)$. (4)

$$\begin{aligned} \text{at stat. pt: } f'(x) &= 0 \\ \therefore -3x^2 + 6x + 9 &= 0 \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x = 3 \text{ is other stat pt} &\therefore f(3) = -3^3 + 3(3)^2 + 9(3) + 5 \\ &= 32 \\ \therefore (3; 32) \end{aligned}$$

- 3) Write down the value(s) of x for which $f(x) \cdot f''(x) > 0$. (3)

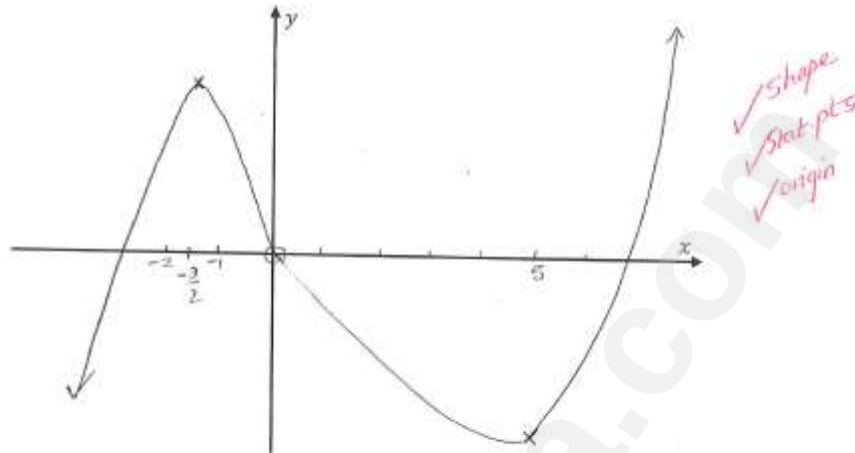
$$\begin{aligned} \text{Point of inflection is at } x &= 1 \\ x < 1 \text{ or } x > 5 \end{aligned}$$

- 4) For which value(s) of k will $f(x) + k$ have two real roots? (2)

$$k = -32 \text{ or if } k = 0$$

- d) Given: $f(x) = ax^3 + bx^2 + cx$ with $f'\left(\frac{-3}{2}\right) = f'(5) = 0$ and $f(x)$ decreasing for $\frac{-3}{2} < x < 5$

1) Use this information to draw a rough sketch of $y = f(x)$



(3)

2) Determine the value(s) of x for which:

i) $f'(x) > 0$

(2)

$$x < -\frac{3}{2} \text{ and } x > 5$$

ii) $f(x)$ is concave upwards

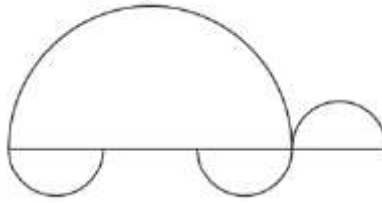
(2)

$$\text{inflection pt: } x = -\frac{\frac{3}{2} + 5}{2} = \frac{7}{4}$$

$$x > \frac{7}{4}$$

- e) A company manufactures metal tortoises to decorate one of their products. The tortoise is made up of a large semi-circle with a radius of R , and three smaller semicircles with radius r .

The values of R and r may vary, but $R + r = 300\text{mm}$.



- 1) Show that the area of the figure (in mm^2) is given by:

$$A = 2\pi R^2 - 900\pi R + 135000\pi$$

(4)

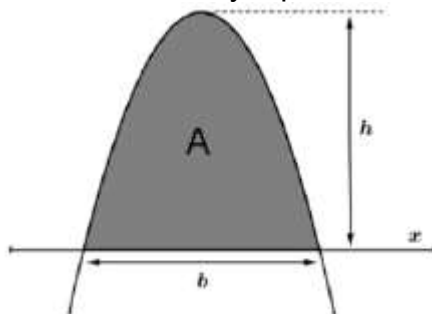
$$\begin{aligned} \text{Area} &= \text{large semicircle} + 3 \text{ small semicircles} & R + r &= 300 \quad \checkmark n \\ &= \frac{1}{2}(\pi R^2) + 3\left(\frac{1}{2}\pi r^2\right) & \checkmark m(\text{idea}) & \therefore r = 300 - R \dots \textcircled{1} \\ &= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi r^2 \dots \textcircled{2} \\ &\text{Sub } \textcircled{1} \text{ into } \textcircled{2} & \checkmark m(\text{sub}) & \\ \therefore A &= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi (300 - R)^2 & \checkmark n & \\ &= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi (90000 - 600R + R^2) \\ &= \frac{1}{2}\pi R^2 + 135000\pi - 900\pi R + \frac{3}{2}\pi R^2 \\ &= 2\pi R^2 - 900\pi R + 135000\pi \end{aligned}$$

- 2) Determine the value of R if A , the area of the tortoise, is at a minimum.

(3)

$$\begin{aligned} \text{for min area, } \frac{dA}{dR} &= 0 \quad \checkmark m(\text{idea}) \\ \therefore 4\pi R - 900\pi &= 0 \quad \checkmark n \\ 4R - 900 &= 0 \\ R &= \frac{900}{4} \\ R &= 225\text{mm} \quad \checkmark n \end{aligned}$$

- f) The diagram shows the area bound by a parabola and a horizontal line.

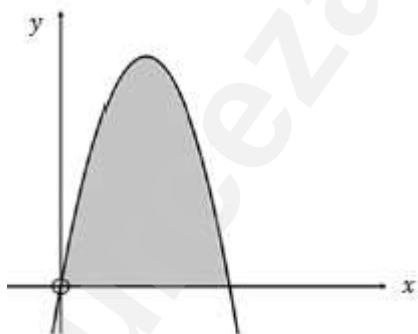


Let b be the horizontal base of the area and h the height ie: h is the vertical distance from the turning point to the base. The area A is given by the formula:

$$A = \frac{2}{3}bh$$

The equation of the curves below is $y = 4x - x^2$. Determine the area of the **shaded region** for each of the following:

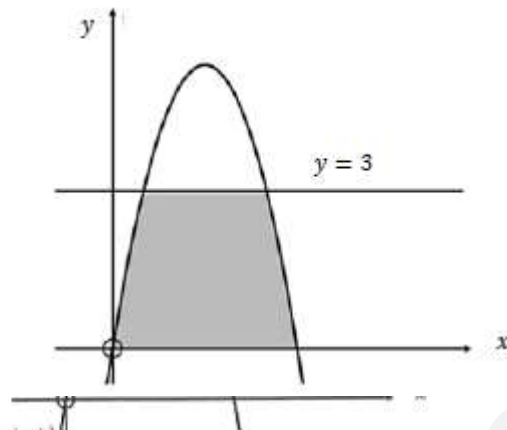
1)



(4)

$$\begin{aligned}
 y &= 4x - x^2 & \text{roots: } 0 &= 4x - x^2 \\
 \text{TPE} \Rightarrow x &= -\frac{4}{2(-1)} = 2 & 0 &= x(4 - x) \\
 \therefore y &= 4(2) - (2)^2 & \therefore x &= 0 \text{ or } x = 4 \\
 y &= 4 & \therefore \text{base} &= 4u \\
 \text{TPE } (2; 4) & \therefore \text{height } (h) &= 4u \\
 \therefore \text{Area shaded} &= \frac{2}{3}(4)(4) \\
 &= \frac{32}{3} \text{ u}^2 \quad (\text{or } 10,7 \text{ u}^2)
 \end{aligned}$$

2)



(5)

$$\text{at } y = 3 : 3 = 4x - x^2$$

$$\therefore x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \text{ or } x = 1$$

$$\therefore \text{base unshaded} = 2u^2$$

$$\text{and height} = 4 - 3 = 1u$$

$$\therefore \text{Area unshaded}$$

$$= \frac{2}{3} (2)(1)$$

$$= \frac{4}{3} u^2$$

$$\therefore \text{Area shaded} = \text{whole area} - \text{unshaded area}$$

$$= \frac{32}{3} - \frac{4}{3}$$

$$= \frac{28}{3} u^2 \quad (\text{OR } 9.3u^2)$$

Northcliff High School 2022

- a) Determine $f'(x)$ from first principles if $f(x) = -3x^2 + 7$ (5)

$$f(x) = -3x^2 + 7$$

$$f(x+h) = -3(x+h)^2 + 7$$

$$= -3(x^2 + 2xh + h^2) + 7$$

$$= -3x^2 - 6xh - 3h^2 + 7$$

$$f(x+h) - f(x) = -3x^2 - 6xh - 3h^2 + 7 - (-3x^2 + 7)$$

$$= -6xh - 3h^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$$

$$= \lim_{h \rightarrow 0} (-6x - 3h)$$

$$= -6x$$

$$\checkmark f(x+h)$$

$$\checkmark f(x+h) - f(x)$$

$$\checkmark \text{Subst. in } f'(x) \text{ formula}$$

$$\checkmark \text{Factorise}$$

$$\checkmark f'(x)$$

ANS ONLY – No Marks

(5)

- b) Determine:

1) $\frac{dy}{dx}$ if $y = (x^2 - 1)(x + 4)$ (2)

$$y = (x^2 - 1)(x + 4)$$

$$y = x^3 + 4x^2 - x - 4$$

$$\frac{dy}{dx} = 3x^2 + 8x - 1$$

$$\checkmark \text{Distribution}$$

$$\checkmark \text{Answer}$$

(2)

2) $g'(x)$ if $g(x) = \frac{1+3\sqrt{x}}{x^2}$ (4)

$$g(x) = \frac{1+3\sqrt{x}}{x^2}$$

$$g(x) = \frac{1+3x^{\frac{1}{2}}}{x^2}$$

$$g(x) = x^{-2} + 3x^{-\frac{3}{2}}$$

$$g'(x) = -2x^{-3} - \frac{9}{2}x^{-\frac{5}{2}}$$

$$\checkmark 3x^{\frac{1}{2}}$$

$$\checkmark \text{Split into terms (exponential terms)}$$

$$\checkmark -2x^{-3}$$

$$\checkmark -\frac{9}{2}x^{-\frac{5}{2}}$$

(4)

- c) Tessa was required in a test to find the derivative of a function $f(x)$. Instead of the DERIVATIVE, she (correctly) found the INVERSE

Her answer was:

$$f^{-1} = \sqrt{\frac{x-4}{5}}$$

Determine the correct answer to the question.

(3)

$$f^{-1}: y = \sqrt[3]{\frac{x-4}{5}}$$

$$f: x = \sqrt[3]{\frac{y-4}{5}}$$

$$x^3 = \frac{y-4}{5}$$

$$y = 5x^3 + 4$$

$$f'(x) = 15x^2$$

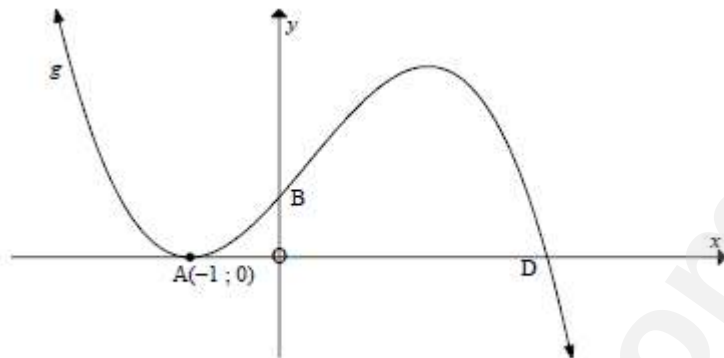
✓ Swop x and y

✓ y as subject

✓ $f'(x)$

(3)

- d) In the diagram below, g is the graph of $g(x) = -x^3 + bx^2 + cx + 3$. The graph has a stationary point at $A(-1; 0)$, a y -intercept at B and cuts the x -axis at D .



- 1) Show the $b = 1$ and $c = 5$

(4)

$$g(x) = -x^3 + bx^2 + cx + 3$$

$$g(x) = -(x+1)^2(x-3)$$

$$g(x) = -(x-3)(x^2 + 2x + 1)$$

$$g(x) = -(x^3 + 2x^2 + x - 3x^2 - 6x - 3)$$

$$g(x) = -x^3 + x^2 + 5x + 3$$

$$\therefore b = 1 \text{ or } c = 5$$

ALTERNATIVE

$$0 = -(-1)^3 + b(-1)^2 + c(-1) + 3$$

$$c = b + 4 \quad (1)$$

$$g'(x) = -3x^2 + 2bx + c$$

$$0 = -3(-1)^2 + 2b(-1) + c$$

$$0 = -3 - 2b + c \quad (2)$$

(1) into (2)

$$2b = -3 + b + 4$$

$$b = 1 ; c = 5$$

$$\checkmark (x+1)^2$$

$$\checkmark (x-3)$$

$\checkmark$ Expand

$\checkmark$ b and c
values

$\checkmark$ Subst.

$$g(-1) = 0$$

$\checkmark$ Subst.

$$g'(-1) = 0$$

$\checkmark$ Simultaneous

$\checkmark$ Both b and c

(4)

- 2) Determine the equation of the tangent to g at B. (3)

$$g'(x) = -3x^2 + 2x + 5$$

$$g'(0) = 5$$

$$\therefore y = 5x + 3$$

✓ $g'(x)$

✓ 5

✓ Equation

- 3) For which values of x is g concave down? (3)

$$g''(x) = -6x + 2$$

$$-6x + 2 < 0$$

$$x > \frac{1}{3}$$

✓ $g''(x)$ ✓ $g''(x) < 0$

✓ Answer

(3)

- 4) Determine the x -value of the turning of f if $f(x) = g'(x + 1)$ (3)

$$g'(x) = -3x^2 + 2x + 5$$

At turning point of g' , $x = \frac{1}{3}$

$\therefore x$ value of turning point of $f(x) = g'(x + 1)$ is g shifted 1 unit left.

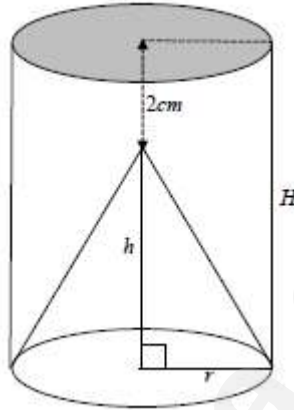
$$\therefore x = -\frac{2}{3}$$

✓ $x = \frac{1}{3}$

✓✓ Answer

ANS ONLY –
Full Marks

- e) In the given diagram, a cone is cut out of a cylindrical piece of wood. The cone and the cylinder have equal radii and the height of the cylinder (H) is 2cm more than the height of the cone (h).
 The ratio of the radius to the height of the cylinder is 1 : 3.
 The volume of a right circular cone = $\frac{1}{3}\pi r^2 h$.
 All measurements are in cm.



- 1) Show that the volume of wood left over, after the cone has been cut out of the cylinder is given by:

$$V = 2\pi r^3 + \frac{2}{3}\pi r^2 \quad (5)$$

$$V_{\text{CYLINDER}} = \pi r^2 H$$

$$V_{\text{CYLINDER}} = \pi r^2 (3r)$$

$$V_{\text{CYLINDER}} = 3\pi r^3$$

$$V_{\text{CONE}} = \frac{1}{3}\pi r^2 h$$

$$V_{\text{CONE}} = \frac{1}{3}\pi r^2 (3r - 2)$$

$$V_{\text{CONE}} = \pi r^3 - \frac{2}{3}\pi r^2$$

$$V_{\text{LEFT OVER WOOD}} = 3\pi r^3 - \left(\pi r^3 - \frac{2}{3}\pi r^2\right)$$

$$V_{\text{LEFT OVER WOOD}} = 2\pi r^3 + \frac{2}{3}\pi r^2$$

$$\checkmark H = 3r$$

$$\checkmark 3\pi r^3$$

$$\checkmark h = 3r - 2$$

$$\checkmark \text{Expand}$$

$$\checkmark \text{Subtract volumes}$$

(5)

- 2) Calculate the rate of change of volume of wood with respect to r when $r=9\text{cm}$. Include appropriate *units* with your final answer.

$$V'(9) = 6\pi(9)^2 + \frac{4}{3}\pi(9)$$

$$V'(9) = 498\pi$$

$$V'(9) = 1\,564,51\text{ cm}^3/\text{cm}$$

$$\checkmark V'(9)$$

$$\checkmark \text{Answer}$$

$$\checkmark \text{Units}$$

(3)

Crawford International 2022

a) Given: $m(x) = x^2 - 3x$ 1) Determine $m'(x)$ using first principles (5)

$$m(x) = x^2 - 3x$$

Determine $m'(x)$ using first principles.

$$\begin{aligned} m(x+h) &= (x+h)^2 - 3(x+h) \\ &= x^2 + 2xh + h^2 - 3x - 3h \quad \checkmark \end{aligned}$$

$$\begin{aligned} m'(x) &= \lim_{h \rightarrow 0} \frac{m(x+h) - m(x)}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h} \quad \checkmark \text{sub} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h} \\ &= 2x + 0 - 3 \\ &= 2x - 3 \quad \checkmark \end{aligned}$$

2) Hence, determine the gradient of the tangent at $x = -2$ (1)

$$\begin{aligned} m'(-2) &= 2(-2) - 3 \quad \checkmark \\ &= -7 \end{aligned}$$

b) Determine $f'(x)$ if:

$$f(x) = \left(\frac{3}{\sqrt{x}} + \frac{\sqrt{x}}{3} \right)^2 \quad (4)$$

$$\begin{aligned} f(x) &= \left(\frac{3}{\sqrt{x}} + \frac{\sqrt{x}}{3} \right)^2 \\ &= \frac{9}{x} + 2 + \frac{x}{9} \quad \checkmark \checkmark \\ &= 9x^{-1} + 2 + \frac{1}{9}x \end{aligned}$$

$$f'(x) = -9x^{-2} + \frac{1}{9} \quad \checkmark \checkmark$$

c) Determine $\frac{dy}{dx}$ if $y + xy = x^2 - 1$ (4)

$$y + xy = x^2 - 1$$

$$y(1+x) = x^2 - 1 \quad \checkmark$$

$$y = \frac{x^2 - 1}{1+x} \quad \checkmark$$

$$y = \frac{(x-1)(x+1)}{1+x}$$

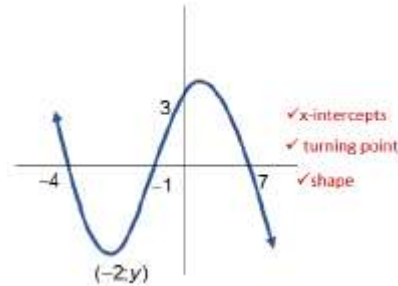
$$y = x - 1 \quad \checkmark$$

$$\frac{dy}{dx} = 1 \quad \checkmark$$

SECTION B

- d) Consider the function: $f(x) = ax^3 + bx^2 + cx + d$.
Draw a rough sketch graph of f where: (3)

- $a < 0$
- $d = 3$
- $f(-4) = f(7) = f(-1) = 0$
- $f'(-2) = f'(4) = 0$



- e) Calculate the values of t such that the line $y = 3x + t$ is a tangent to the curve $g(x) = -x^3 + x^2 + 8x - 12$. (7)

$$-3x^2 + 2x + 8 = 3 \quad \checkmark \checkmark$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0 \quad \checkmark$$

$$x = \frac{5}{3} \quad \text{or} \quad x = -1 \quad \checkmark$$

$$y = -\frac{14}{27} \quad \text{or} \quad y = -18 \quad \checkmark$$

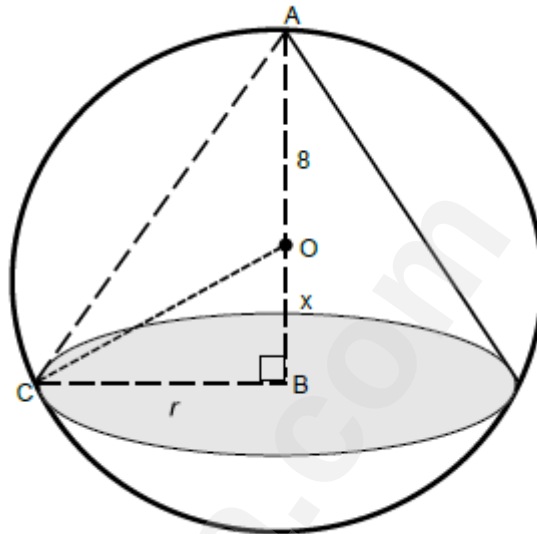
$$t = -\frac{14}{27} - 3\left(\frac{5}{3}\right) = -\frac{149}{27} \quad \checkmark$$

$$\text{or} \quad t = -18 - 3(-1) = -15 \quad \checkmark$$

- f) A cone with radius r cm and height AB is inscribed in a sphere with centre O and radius = 8cm. $OB = x$.

volume of a cone = $\frac{1}{3}\pi r^2 h$

volume of a sphere = $\frac{4}{3}\pi r^3$



- 1) Calculate the volume of the sphere in terms of π (1)

$$V = \frac{4}{3}\pi(8)^3 = \frac{2048}{3}\pi$$

- 2) Show that $r^2 = 64 - x^2$. (1)

$$OC = OA = 8\text{cm} \quad (\text{radii})$$

$$r^2 = 8^2 - x^2 \quad (\text{Thm of Pythag}) \quad \checkmark$$

$$r^2 = 64 - x^2$$

- 3) Determine the value of x for which the volume of the cone is a maximum. (9)

$$V = \frac{1}{3}\pi r^2 h$$

$$V(x) = \frac{1}{3}\pi(64 - x^2)(x + 8) \quad \checkmark \checkmark$$

$$V(x) = \frac{1}{3}\pi(64x + 512 - x^3 - 8x^2) \quad \checkmark$$

$$V(x) = \frac{64}{3}\pi x + \frac{512}{3}\pi - \frac{1}{3}\pi x^3 - \frac{8}{3}\pi x^2$$

$$V'(x) = 0 \quad \checkmark$$

$$\frac{64}{3}\pi - \pi x^2 - \frac{16}{3}\pi x = 0 \quad \checkmark \checkmark$$

$$\frac{64}{3} - x^2 - \frac{16}{3}x = 0$$

$$3x^2 + 16x - 64 = 0 \quad \checkmark$$

$$(3x - 8)(x + 8) = 0 \quad \checkmark$$

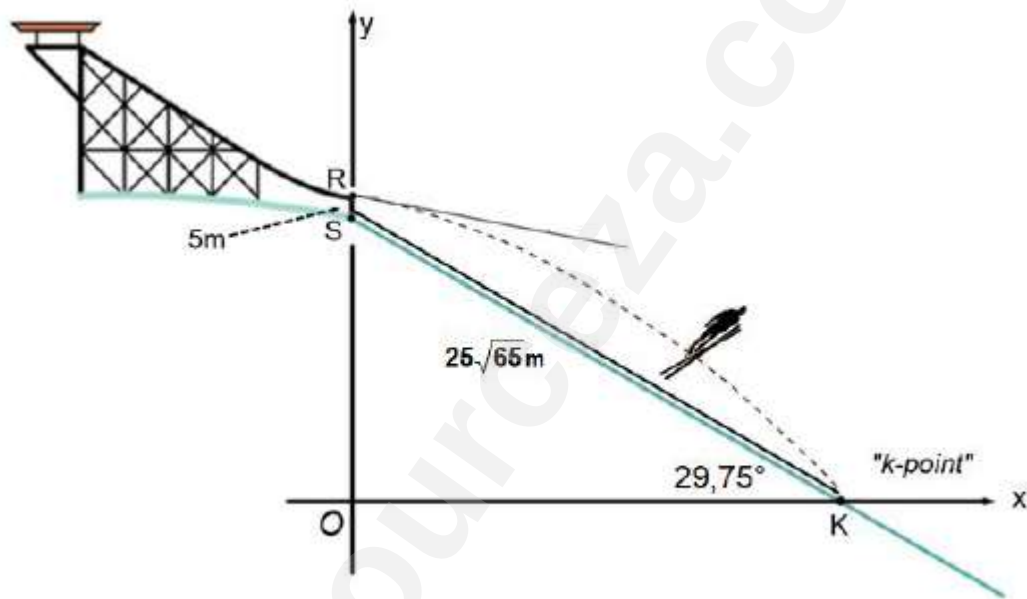
$$x = \frac{8}{3} \quad \checkmark \text{ or } x = -8$$

- g) Diego-the-Daredevil has spent his life pursuing his dream of participating in the Winter Olympics. His favourite event is the Ski-jump, so he does some research into the ramp design, as well as the mechanics and motion of the jump. During his research, Diego discovers that, contrary to popular belief, the ramp does not go up at the end.

The data suggests that:

- The trajectory of the skier, in flight, is parabolic in nature, $y = ax^2 + bx + c$; $x \geq 0$.
- Skiers land at the designated "k-point" $25\sqrt{65}$ metres down the slope.
- The slope is inclined at $29,75^\circ$ to the horizontal.
- There is a vertical drop of 5m from the end of the ramp to the top of the slope.
- The tangent to the curve at R is $y = -0,25x + k$ and $RS = 5$ metres.

Find an equation which models the skier's parabolic trajectory.



(8)

$$\sin 29,75^\circ = \frac{OS}{25\sqrt{65}} \checkmark \therefore OS = 100 \text{ m}$$

$$OR = 105 \text{ m} \checkmark$$

$$\cos 29,75^\circ = \frac{OK}{25\sqrt{65}} \checkmark \therefore OK = 175 \text{ m}$$

$$K(175; 0) \checkmark$$

$$y = ax^2 + bx + 105$$

$$2ax + b = -0,25 \checkmark$$

$$2a(0) + b = -0,25$$

$$b = -0,25 \checkmark$$

$$y = ax^2 - 0,25x + 105$$

$$0 = a(175)^2 - 0,25(175) + 105 \checkmark$$

$$a = -0,002 \checkmark$$

$$y = -0,002x^2 - 0,25x + 105$$

Rustenburg Girls High 2022

- a) Determine $f'(x)$ from first principles given $f(x) = x^2 - bx$, where b is a constant. (5)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - bh}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - b)}{h} \quad \text{factor} \\
 &= 2x - b
 \end{aligned}$$

- b) Determine:

1) $\frac{d}{dx} \left[x^4 - 3\sqrt[3]{x^2} + \frac{3}{4} \right]$ (3)

$$\begin{aligned}
 &= \frac{d}{dx} \left[x^4 - 3x^{2/3} + \frac{3}{4} \right] \\
 &= 4x^3 - 2x^{-1/3}
 \end{aligned}$$

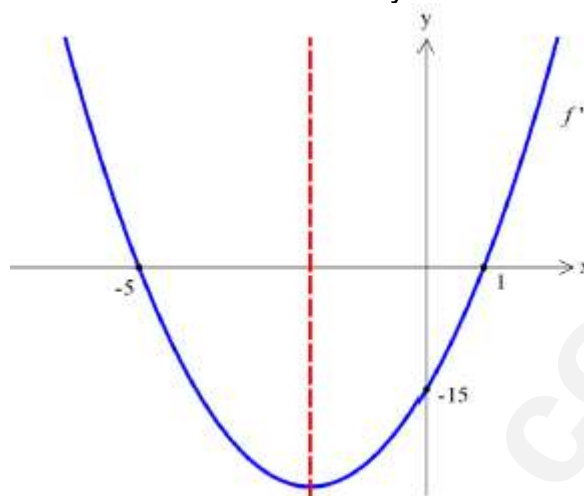
2) $\frac{dy}{dx}$ if $y = \left(x - \frac{2}{x} \right)^2$ (4)

$$\begin{aligned}
 y &= x^2 - 4 + \frac{4}{x^2} \\
 y &= x^2 - 4 + 4x^{-2} \\
 \frac{dy}{dx} &= 2x - 8x^{-3}
 \end{aligned}$$

- c) The graph of $f'(x)$, the derivative of f , is drawn below.

$$f(x) = ax^3 + bx^2 + cx + d; a \neq 0$$

f' intersects the x -axis at -5 and 1 and the y -axis at -15



- 1) Write down the x -value(s) of:

- i) The turning point of f and state whether they are local maximum or minimum. (3)

$$\begin{array}{ll} x = -5 \checkmark & \text{max} \\ x = 1 \checkmark & \text{min} \end{array} \quad \checkmark$$

- ii) The interval in which $f(x)$ is concave up. (2)

$$x > -2 \checkmark \quad (\text{accept } x \geq -2)$$

- iii) The point(s) where the gradient of f is equal to -15 . (2)

$$x = 0 \checkmark \quad \text{or} \quad x = -4 \checkmark$$

- 2) Show that the equation of f' is given by $f'(x) = 3x^2 + 12x - 15$ (3)

$$\begin{aligned} y &= a(x+5)(x-1) \quad \checkmark \text{Root form} \\ -15 &= a(0+5)(0-1) \quad \checkmark \text{subst} \\ a &= 3 \\ \therefore y &= 3(x^2 + 4x - 5) \quad \checkmark \\ y &= 3x^2 + 12x - 15 \end{aligned}$$

- 3) It is further given that $a + b + c + d = 5$

- i) Explain why $x = 1$ is not a root of $f(x)$ (2)

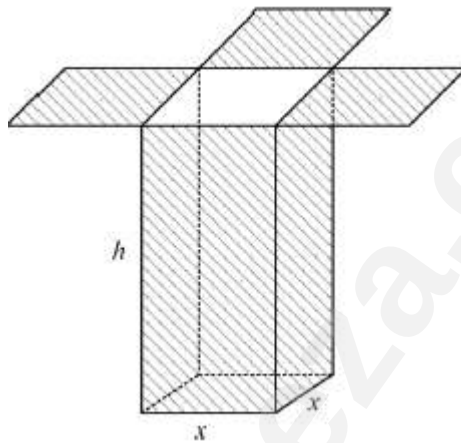
$$\begin{aligned} f(1) &= a + b + c + d \quad \checkmark \\ f(1) &= 5 \Rightarrow 1 \text{ is not a root} \quad \checkmark \end{aligned}$$

- ii) Hence, calculate the values of a ; b ; c and d . (3)

$$f(x) = x^3 + 6x^2 - 15x + d$$

$$a=1 \quad b=6 \quad c=-15 \quad d=13$$

- d) The rectangular milk carton has a square base which holds 1000 cm^3 (a litre) of milk. It has a specially designed fold-in top. The area of the cardboard used for the top is three times the area of the base. The height of the carton is h and a side of the base is x .



- 1) Show that the Total Surface Area of the carton in cm^2 is given by:

$$A(x) = 4x^2 + \frac{4000}{x} \quad (4)$$

$$1000 = h x^2 \quad \checkmark$$

$$h = \frac{1000}{x^2} \quad \checkmark$$

$$A = 4x^2 + 4xh \quad \checkmark$$

$$A = 4x^2 + 4x \cdot \frac{1000}{x^2} \quad \checkmark \text{ subst}$$

$$\therefore A = 4x^2 + \frac{4000}{x}$$

- 2) Determine the dimensions of the carton so that a minimum amount of cardboard is used. (5)

$$A = 4x^2 + 4000x^{-1}$$

$$A' = 8x - 4000x^{-2} \quad \checkmark f'(x)$$

$$8x - \frac{4000}{x^2} = 0 \quad \checkmark = 0$$

$$8x = \frac{4000}{x^2}$$

$$x^3 = 500 \quad \checkmark$$

$$x = 7,94 \quad \checkmark$$

$$h = 15,86 \quad \checkmark$$