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Functions, Logs and Inverses Cycle Test

GRADE:	12	DATE:	14 February 2024
TIME:	60 minutes	MARKS:	50
EXAMINER	A Gunning	MODERATOR:	S Singh

Name	MEMO <i>Version 1.</i>
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PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions in the space provided.
4. All calculations should be rounded off to two decimal places.
5. All relevant details which you have used to give your solutions MUST be clearly shown. Answers only will not necessarily be awarded full marks.
6. It is in your own interest to write legibly and to present your work neatly.

Question	1	2	3	4	5	6	Total
Actual							
Possible	9	7	2	5	7	10	50

QUESTION 1

(a) Determine the value of x , without using a calculator: (Show ALL working)

$$x = \log_2 16 + \log_3 1 - \log_{\frac{1}{3}} 27$$

$$= \log_2 2^4 + 0 - \log_{\frac{1}{3}} 3^3$$

$$= 4 \log_2 2 - 3 \log_{\frac{1}{3}} 3$$

$$= 4 - 3 \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-1}$$

$$= 4 + 3(1)$$

$$= 7$$

Prime factors

law 3

log a a

(5)
(4)(b) The class were asked to solve for x given $2^{x+1} = 3$ Mary's solution $\log_2 3 = x + 1$

$$\log_2 3 - 1 = x$$

Sam's solution

$$2^x = \frac{3}{2}$$

$$x = \log_2 \frac{3}{2}$$

Are they both correct? Explain.

Both are correct.
changed to log form
Mary got x by itself on 1 side

Sam divided both sides by 2
then changed to log form.

$$2^{x+1} = 2 \cdot 2^x$$

(4)

[8]

[9]

QUESTION 2

(a) Consider $f(x) = (x-1)^2 - 4$.

Is f a function? Why / why not?

(1)

yes $m-1$ ✓

K

(b) Is the inverse a function? Why / why not?

(1)

no $1-m$ ✓

K

(c) How could you restrict the domain of f so that when you find the inverse, the inverse will be a function?

(2)

either $x \geq 1$ ✓ or $x \leq 1$ ✓

RP

(d) Write down the inverse of $f(x)$ in the form $f^{-1}(x) = \dots$

stating clearly any restriction which are relevant.

(3)

Space

$$y = (x-1)^2 - 4$$

$$x = (y+1)^2 - 4$$

$$x+4 = (y+1)^2$$

$$y = \sqrt{x+4} + 1 \quad \text{for } x \geq 1$$

CP

move to page 4

$$\text{or } y = -\sqrt{x+4} + 1 \quad \text{for } x \leq 1$$

[7]

QUESTION 3

Given that $f(x) = \left(\frac{1}{2}\right)^x$

(a) Write down the equation of the asymptote for f .

(1)

$$y = 0$$

(b) Sketch the graphs $f(x)$ showing at least two points on the graph, which must include any intercepts with the axes. Any asymptotes must also be clearly shown.

RP

(5)(4) ✓

(c) On the same set of axes, draw the graph of f^{-1} , clearly showing the asymptote, and state the equation of f^{-1} .

at least 3 points

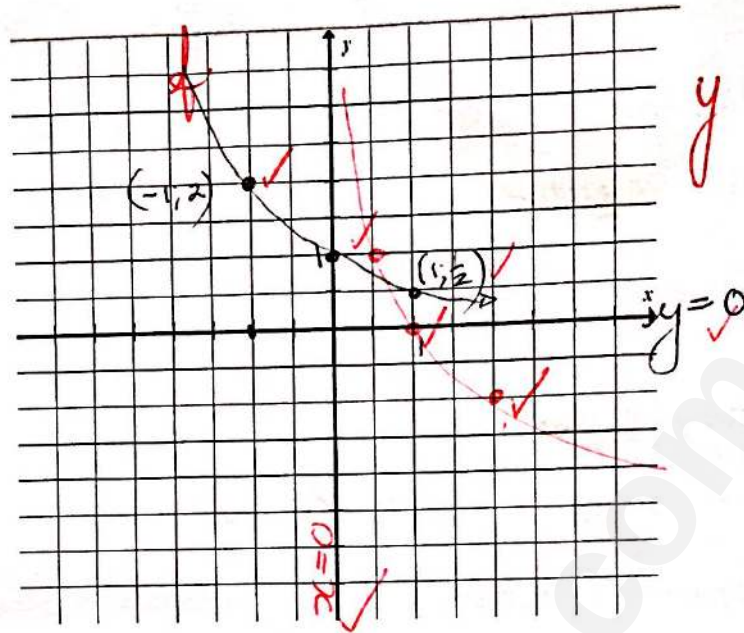
RP

(3)(4) ✓

$$y = \left(\frac{1}{2}\right)^x$$

$$x = \left(\frac{1}{2}\right)^y$$

$$y = \log_{\frac{1}{2}} x$$



$$y = \left(\frac{1}{2}\right)^x$$

shape

(d) Using your graph, state the values of x for which $f^{-1}(x) > 0$

(2)

$$x \in (0; 1)$$

(e) Write down the domain of $f^{-1}(x + 1)$

(2)

Horizontal shift left by 1.

$$\therefore x \in (-1; \infty)$$

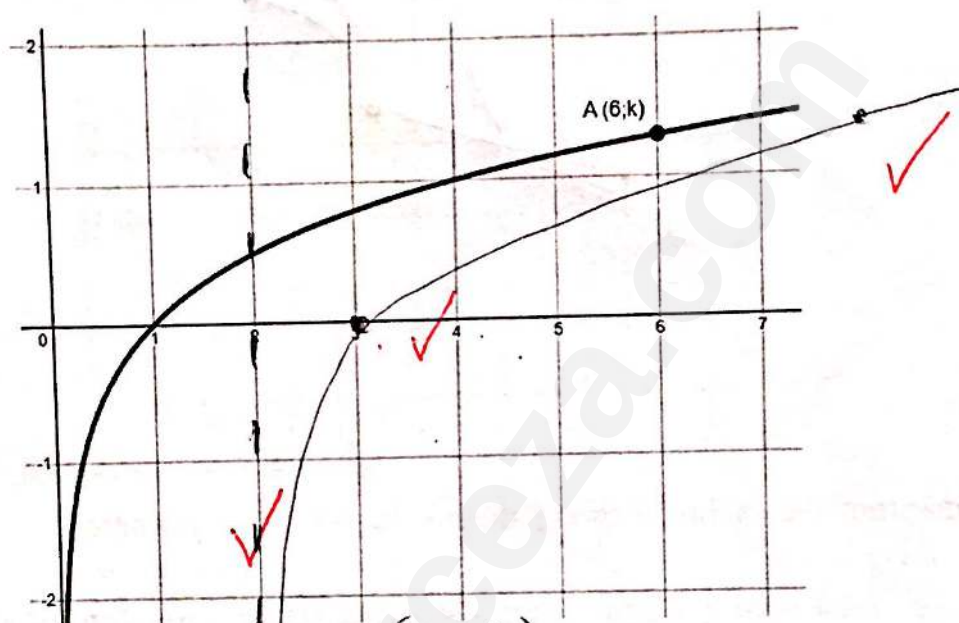
[13]

[12]

QUESTION 4

In the diagram below is the function $f(x) = \log_4 x$.

The coordinate $(6;k)$ is indicated on the graph.



- (1) Determine the value of k . *Sub (6;k)* (1)

$$y = \log_4 6 = 1.29$$

- (2) If $f(x)$ is translated so that its new equation becomes $h(x) = f(x-2)$, plot the function h on the same graph grid above to show clearly the transformation that has taken place.

horizontal shift Right by 2

- (3) State the equation of f^{-1} in the form $y = \dots$ (2)

$$y = \log_4 x$$

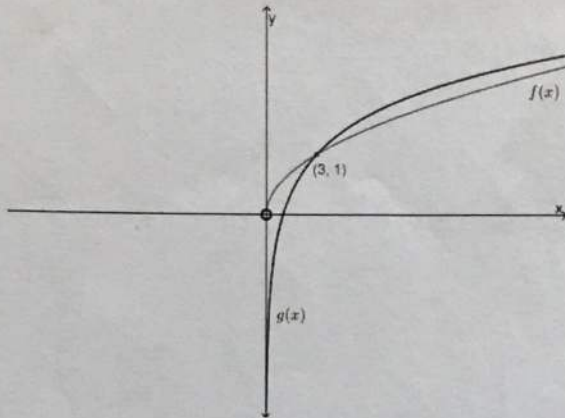
$$x = \log_4 y$$

$$y = 4^x$$

[5] ✓

QUESTION 6

In the diagram below are sketched the graphs of $f(x) = \sqrt{kx}$ and $g(x) = \log_a x$. O is the origin and the graphs of f and g intersect at $(3;1)$.



(a) Determine the values of k and a .

$$f(x) \quad y = \sqrt{kx}$$

Sub (3;1) $(1) = (\sqrt{3k})^2$ ✓
 $1 = 3k$ ✓
 $\frac{1}{3} = k$ ✓

$$g(x) = \log_a x$$

$1 = \log_a 3$ ✓ m.
 $a^1 = 3$ ✓ ca.

(5) (4) ✓ Sub.

CP

(b) Determine

$$f^{-1}(x)$$

$$y = \sqrt{\frac{x}{3}} \quad (3) \quad (2) \quad [7]$$

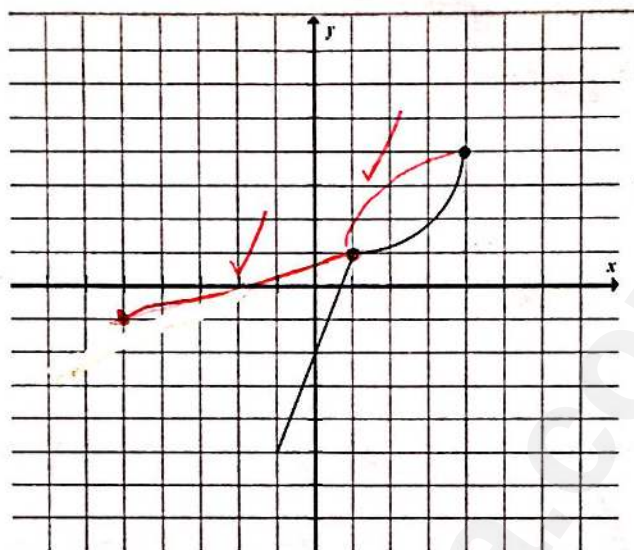
$$x = \sqrt{\frac{y}{3}} \quad \checkmark$$

$$x^2 = \frac{y}{3}$$

$$y = 3x^2 \quad \checkmark \quad \text{for } x \geq 0 \quad \checkmark$$

K.

last question
QUESTION 5



The diagram shows the function f defined for $-1 \leq x \leq 4$, where

$$f(x) = 3x - 2 \text{ for } -1 \leq x < 1 \text{ and } f(x) = \frac{4}{5-x} \text{ for } 1 < x \leq 4$$

(a) State the range of f

$y \in [-5; 4]$ but $y \neq 1$ (2) (3)

(b) On the diagram given above, sketch the graph of $f^{-1}(x)$

(3) (2)

(c) Write down the equation which defines $f^{-1}(x)$ in the form $f^{-1}(x) = \dots$

giving also the set of x values for which each expression is valid. (5)

$$y = 3x - 2 \quad -1 \leq x < 1$$

$$x = \frac{y+2}{3} \quad -1 \leq y < 1$$

$$\frac{x+2}{3} = y$$

$$x \in [-5; 4]$$

OR $y = \frac{4}{5-x}$

$$x = \frac{4}{5-y}$$

$$5-y = \frac{4}{x}$$

$$y = 5 - \frac{4}{x} \quad 1 < y \leq 4$$