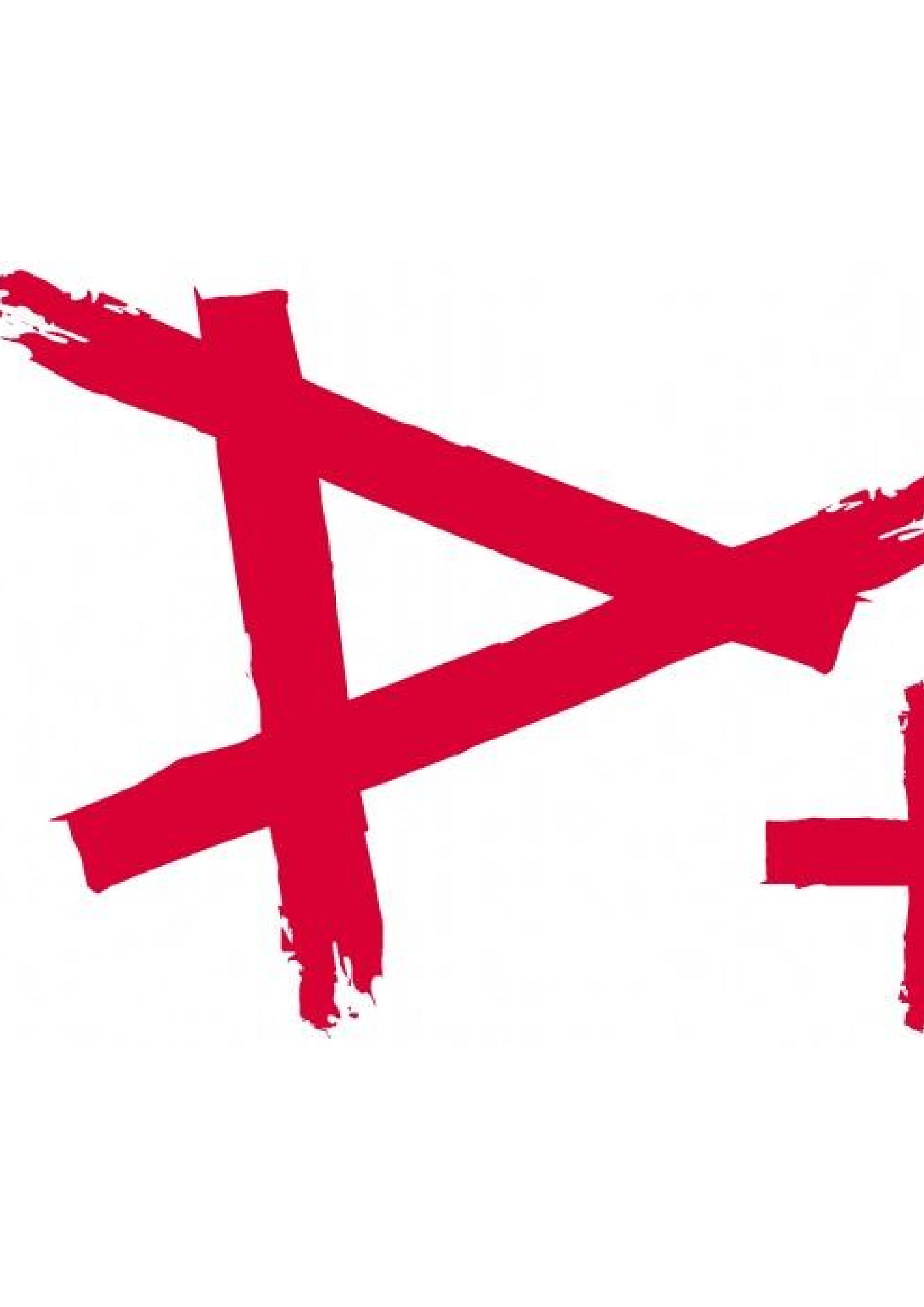






DURBAN GIRLS' COLLEGE

TRIALS EXAMINATION 2019

MATHEMATICS PAPER I

TIME: 3 HOURS

150 MARKS

EXAMINER: Mrs C Jacobsz

Moderators: Mrs D Bolton and Mr Raghunandan

Name: _____

Mathematics' teacher: _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

- This question paper consists of 20 pages (pages 19 and 20 are blank for you to use if necessary) and an information sheet of two pages.
- Read the questions carefully.
- **ALL** questions must be answered in the answer book provided.
- Diagrams are **NOT** necessarily drawn to scale.
- All answers must be given correct to **one decimal place** where necessary, unless stated otherwise.
- Approved calculators may be used, unless stated otherwise.
- All the necessary working details must be clearly shown.

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total	%
	17	12	12	17	10	11	15	9	14	9	10	14	150	

Paper 1	
Paper 2	
TOTAL	/300
	%

SECTION A [79 marks]

QUESTION 1

a) Solve for x :

1. $2x(3x + 4) = 0; x \in \mathbb{Z}$

$$2x = 0 \quad 3x + 4 = 0$$

$$x = 0 \quad \text{or} \quad x = -\frac{4}{3} \quad \checkmark \quad \text{must delete } x = -\frac{4}{3}$$

$$x = 0$$

(2)

2. $2x^2 - 4x + 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} \quad \checkmark m$$

(3)

$$x = \frac{-4 \pm \sqrt{8}}{4}$$

$$x = 1, 7 \quad \checkmark$$

$$\text{or } x = 0, 3$$

answers only 3, 3
Round off
incom

(3)

b) Solve for x and y in the following simultaneous equations:

$$3y + x = 2 \quad \text{and} \quad y^2 + x = xy + y$$

$$x = 2 - 3y \quad \checkmark$$

$$y^2 + (2 - 3y) = y(2 - 3y) + y \quad \checkmark m$$

$$y^2 + 2 - 3y = 2y - 3y^2 + y$$

$$4y^2 - 6y + 2 = 0$$

$$2y^2 - 3y + 1 = 0 \quad \checkmark$$

$$(2y - 1)(y - 1) = 0$$

$$y = \frac{1}{2} \quad \text{or} \quad y = 1 \quad \checkmark$$

$$x = 2 - 3\left(\frac{1}{2}\right) \quad \text{or} \quad x = 2 - 3(1) \quad \checkmark m$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = -1 \quad \checkmark$$

(6)

c) Given: $(x+2)(x-3) < -3x+2$

1. Solve for x .

$$x^2 - x - 6 < -3x + 2$$

$$x^2 + 2x - 8 < 0 \quad \Delta$$

$$(x+4)(x-2) < 0$$

$$CV: x = -4 \quad x = 2$$

$$\sqrt{m}$$

$$-4 < x < 2 \quad x \in \mathbb{R}$$

$$OT \quad x \in (-4, 2)$$

(4)

2. Hence, determine the sum of all the integers satisfying the inequality $x^2 + 2x - 8 < 0$.

$$= (-3) + (-2) + (-1) + 0 + 1 \quad \sqrt{m}$$

$$= -5 \quad \sqrt{m}$$

(2)

QUESTION 2

[17 marks]

a) The roots of a quadratic equation are $x = \frac{-1 \pm \sqrt{16-m^2}}{6}$.

For which value/s of m will the roots be equal?

(3)

$$\Delta = 0 \quad \sqrt{m}$$

$$16 - m^2 = 0$$

$$16 = m^2$$

$$\pm 4 = m$$

$$\sqrt{m}$$

(3)

b) Given the following equation: $\sqrt{x-2} = 4-x$

1. Without solving for x , show that the solution to the above equation lies in the interval $2 \leq x \leq 4$.

(3)

$$\begin{array}{lcl} x-2 \geq 0 & \checkmark & 4-x \geq 0 \quad \checkmark \\ x \geq 2 & & -x \geq -4 \\ & & x \leq 4 \end{array}$$

for both

$$2 \leq x \leq 4 \quad x \in \mathbb{R} \quad \} \quad \textcircled{3}$$

OR $x \in [2, 4]$

2. Now, solve the above equation.

(1)

$$\begin{aligned} \sqrt{x-2} &= 4-x \\ x-2 &= (4-x)^2 \quad \checkmark m \\ x-2 &= 16-8x+x^2 \quad \checkmark c \\ 0 &= x^2-9x+18 \quad \checkmark m \\ 0 &= (x-6)(x-3) \\ x=6 &\quad \text{or} \quad x=3 \\ \checkmark & \quad \checkmark \end{aligned}$$

must delete $x=6$

(5)

3. Without any further calculations, solve the equation $-\sqrt{x-2} = 4-x$.

(1)

$$x=6 \quad \checkmark \quad c.c.$$

(1)

[12 marks]

QUESTION 3

- a) Given the following sequence: 7; 10; 15; 22; 31; 42; ...
Determine the general term, T_n .

(4)

$$7, 10, 15, 22, 31, 42, \dots$$

$$\underbrace{3 \quad 5 \quad 7 \quad 9}_{2 \quad 2} \quad \checkmark \quad m \text{ quadratic}$$

$$2a = 2$$

$$3a + b = T_2 - T_1$$

$$a + b + c = T_1$$

$$a = 1$$

$$3(1) + b = 3$$

$$1 + 0 + c = 7$$

$$b = 0 \quad \checkmark$$

$$c = 6$$

$$T_n = n^2 + 6 \quad (\text{must write})$$

(4)

- b) The sum of the fourth term and the eighth term of an arithmetic series is 84, and the sum of the first 10 terms is 385. Calculate the first 3 terms of the series.

$$T_4 + T_8 = 84 \quad \checkmark$$

$$a + 3d + a + 7d = 84 \quad \checkmark$$

$$2a + 10d = 84$$

$$a + 5d = 42$$

$$a = 42 - 5d \quad \checkmark$$

$$S_{10} = 385$$

$$385 = \frac{10}{2} (2a + 9d) \quad \checkmark$$

$$385 = 5(2a + 9d)$$

$$77 = 2(42 - 5d) + 9d$$

$$77 = 84 - 10d + 9d$$

$$-7 = -d$$

$$7 = d \quad \checkmark$$

$$7 = a \quad \checkmark$$

$$7, 14, 21, \dots \quad \checkmark$$

[12 marks]

QUESTION 4

Riley wants to buy a car valued at R550 000.

- a) Calculate the book value of the car after 4 years if it depreciates on a reducing balance at an interest rate of 17,5% p.a.

$$A = 550\,000 \left(1 - 0,175\right)^4 \quad (2)$$

$$= R254\,787,71 \quad (3)$$

- b) If the effective interest rate is 17,5% p.a., calculate the nominal interest rate per annum, compounded monthly.

$$1 + 0,175 = \left(1 + \frac{i}{12}\right)^{12} \quad (3)$$

$$\sqrt[12]{\frac{1,175}{1}} = 1 + \frac{i}{12}$$

$$0,1623... = i$$

$$16,2\% = i \quad (3)$$

- c) If Riley invests R350 000, at an interest rate of 8,2% compounded half-yearly. How long will it take her to save until she has R550 000 to buy the car? Give your answer in years and months.

$$550\,000 = 350\,000 \left(1 + \frac{0,082}{2}\right)^{2n} \quad (1)$$

$$\frac{1}{2} = (1,041)^{2n}$$

$$\sqrt[2]{\log \frac{1}{2}} = 2n$$

$$5,62 = n$$

5 years 7 months

(4)

- d) 1. Riley decides that she can't wait that long for her money to earn interest. She decides to put the R350 000 down as a deposit and borrow the balance of R200 000 from her bank. She will repay the loan over 4 years by means of equal monthly instalments, starting 1 month after the granting of the loan. The interest rate is 12% per annum compounded monthly. Calculate her monthly repayments.

$$300\,000 = \frac{x}{\frac{0,12}{12}} \left[1 - \left(1 + \frac{0,12}{12} \right)^{-48} \right] \quad (4)$$

$$x = R5266,77 \quad (4)$$

2. After 3 years, Riley receives a bonus of R59 500 from work and decides to pay off the balance of her loan. Calculate if the R59 500 will be enough to settle her loan with the bank.

$$\text{Method 1: } 200\,000 \left(1 + \frac{0,12}{12} \right)^{36} - 5266,77 \left(\left(1 + \frac{0,12}{12} \right)^{36} - \frac{0,12}{12} \right) \quad (4)$$

$$OB = R59277,90$$

It will be enough $\{ \}$ 4

$$\text{Method 2: } OB = 5266,77 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-48} \right] \quad \text{Formula}$$

$$59277,90 = R59277,90 \quad \checkmark$$

It will be enough $\{ \}$

[17 marks]

QUESTION 5

a) Given: $g(x) = -(x - 2)^2 + 3$ and $p(x) = \frac{2}{x+1} + 2$.

1. Write down the coordinates of the turning point of graph g .

$(2, 3)$ (2)

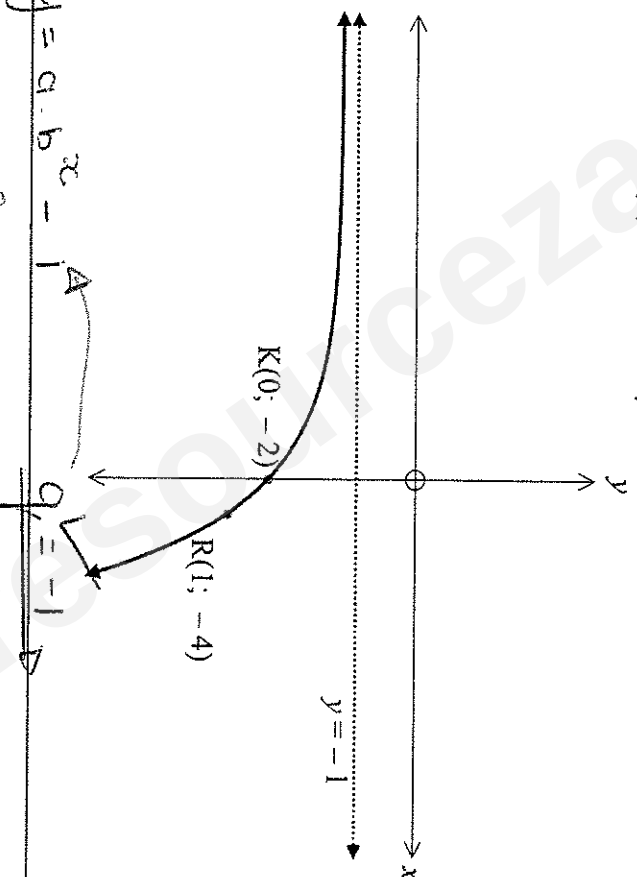
2. Write down the equations of the asymptotes of graph p .

$x = -1$ $\sqrt{a}$ $y = 2$ $\sqrt{a}$ (2)

3. Write down the range of graph m , if $m(x) = p(x) - 5$

$m(x) = \frac{2}{x+1} + 2 - 5 = \frac{2}{x+1} - 3$ $\sqrt{a}$ $y \in R$ $y = -3$ (2)

b) The graph of $f(x) = a \cdot b^x + q$ is sketched below. Points $K(0; -2)$ and $R(1; -4)$ are on the curve. Determine the value(s) of a, b and q .



$$y = a \cdot b^x - 1$$

$$-2 = a \cdot b^0 - 1$$

$$-2 = a - 1$$

$$-1 = a$$

$$y = -1 \cdot b^x - 1$$

$$-4 = -1 \cdot b^1 - 1$$

$$-3 = -b$$

$$3 = b$$

QUESTION 6

- a) Given, $f(x) = -x + x^2$ determine $f'(x)$, using first principles.

$$f'(x) = \lim_{h \rightarrow 0} \frac{-x + x^2 - (-x + h) + (x+h)^2 - (-x + x^2)}{h} \quad (5)$$

$$\begin{aligned} \text{cancel} &= \lim_{h \rightarrow 0} \frac{-x + h + x^2 + 2xh + h^2 + x - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-1 + 2x + h)}{h} \end{aligned}$$

$$= -1 + 2x$$

(5)

- b) $y = \sqrt{4x^3} + 2\sqrt{x} - \frac{2}{x}$, calculate $\frac{dy}{dx}$. Leave your answer with positive exponents.

$$y = 2x^{3/2} + 2x^{1/2} - 2x^{-1}$$

$$\frac{dy}{dx} = 3x^{1/2} + x^{-1/2} + 2x^{-2}$$

$$= 3x^{1/2} + \frac{1}{x^{1/2}} + \frac{2}{x^2}$$

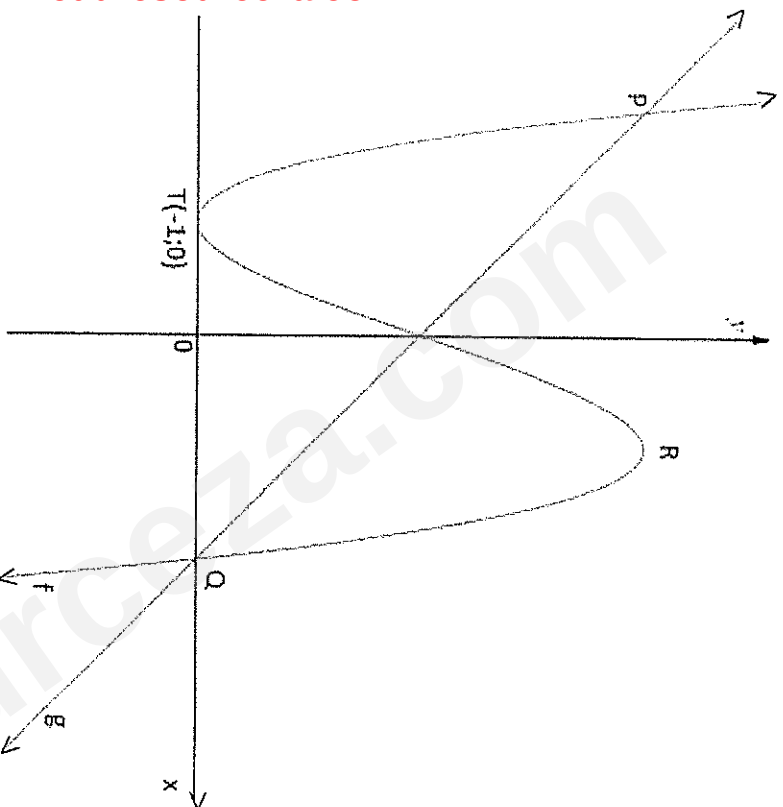
$$\sqrt{x} \quad \sqrt{x} \quad \sqrt{x}$$

(6)

SECTION B [71 marks]

QUESTION 7

The graphs of $f(x) = -x^3 + 3x + 2$ and $g(x) = -x + 2$ are drawn below. The graphs intersect at P and Q. Points R and T are the turning points of $f(x)$. The x-intercepts of $f(x)$ are at Q and $T(-1;0)$.



a) Determine the length of TQ.

$$x \text{ intercepts: } -x^3 + 3x + 2 = 0 \quad \checkmark m$$

$$x^3 - 3x - 2 = 0$$

$$(x+1)(x+1)(x-2) = 0 \quad \checkmark a$$

$$x = -1 \quad x = -1 \quad x = 2 \quad \checkmark$$

$$TQ = 3 \text{ units.} \quad \checkmark$$

(3)

$$\text{Method 2: } -x^3 + 3x + 2 = -x + 2$$

$$x = 2$$

$$TQ = 3$$

$$\text{Method 3: } -x + 2 = 0$$

$$-x = -2$$

$$x = 2$$

$$TQ = 3$$

b) 1. Determine the co-ordinates of R.

(3)

$$f'(x) = -3x^2 + 3 \quad \checkmark$$

$$-3x^2 + 3 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x = 1 \text{ or } x = -1$$

$$f(1) = -(1)^3 + 3(1) + 2$$

$$= 4$$

$$R\left(\frac{1}{-1}, 4\right) \quad \checkmark$$

(3)

2. Determine the co-ordinates of the turning points of $f(x) + 2$.

(2)

$$T(-1, 2) \quad \checkmark$$

$$R(1, 6) \quad \checkmark$$

(2)

c) For which values of x is $f'(x) > 0$?

(2)

$$x \in (-1, 1) \quad \checkmark$$

$$\text{OR } -1 < x < 1 \quad \checkmark$$

(2)

d) For which values of x is $f'(x) < 0$ and $f''(x) > 0$ simultaneously?

(2)

$$f'(x) < 0 \Rightarrow x < -1 \text{ or } x > 1 \quad \checkmark$$

$$f''(x) > 0 \text{ concave up } \Rightarrow x < -1 \quad \checkmark$$

(2)

Answer only : $x < -1$ 2 marks

e) What do you notice about the function of $g(x)$ and $g^{-1}(x)$? Explain your answer.

(3)

$$g: y = -x + 2$$

$$g^{-1}: x = -y + 2 \quad \checkmark$$

$$x - 2 = -y$$

$$-x + 2 = y \quad \checkmark$$

$$\circ \circ \circ \quad g(x) = g^{-1}(x) \Rightarrow \text{same equation because}$$

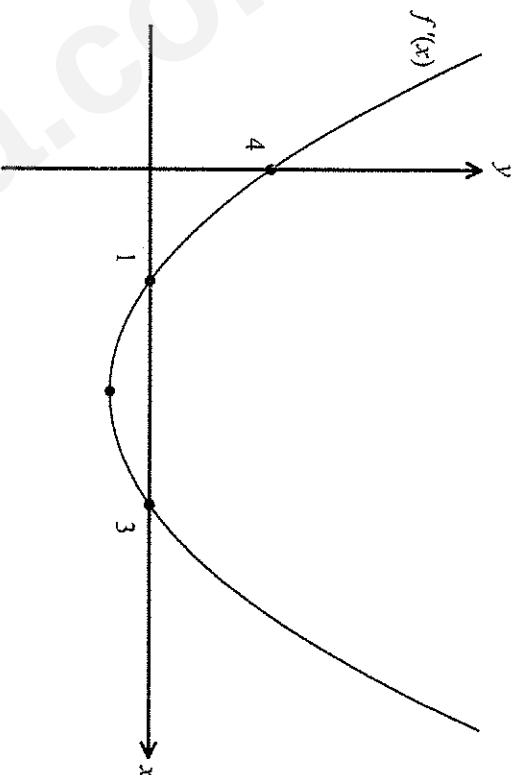
$$\text{the gradient of } g \text{ is } -1$$

(3)

[15 marks]

QUESTION 8

- a) The parabola in the figure below represents the curve of $f'(x)$. The parabola is the derivative of the cubic function $f(x) = ax^3 + bx^2 + cx + d$.



1. Write down the gradient of the tangent to $f(x)$ at the point where $x = 0$
 $m = 4$ ✓
2. Write down the x – coordinates of the turning points of the curve of $f(x)$
 $x = 1$ ✓ or $x = 3$ (2)
3. For what values of x is the graph of $f(x)$ decreasing?
 $1 < x < 3$ or $x \in (1, 3)$ (2)
- b) If you are given a cubic function, $g(x) = mx^3 + nx^2 + sx + t$, determine the x -coordinate of the point of inflection of $g(x)$ in terms of m and n .

$$g'(x) = 3mx^2 + 2nxc + s \quad \checkmark \quad \alpha$$

$$g''(x) = 6mx + 2n \quad \checkmark \quad \alpha$$

$$6mx + 2n = 0 \quad \checkmark \quad m \quad f''(x) = 0$$

$$6mx = -2n$$

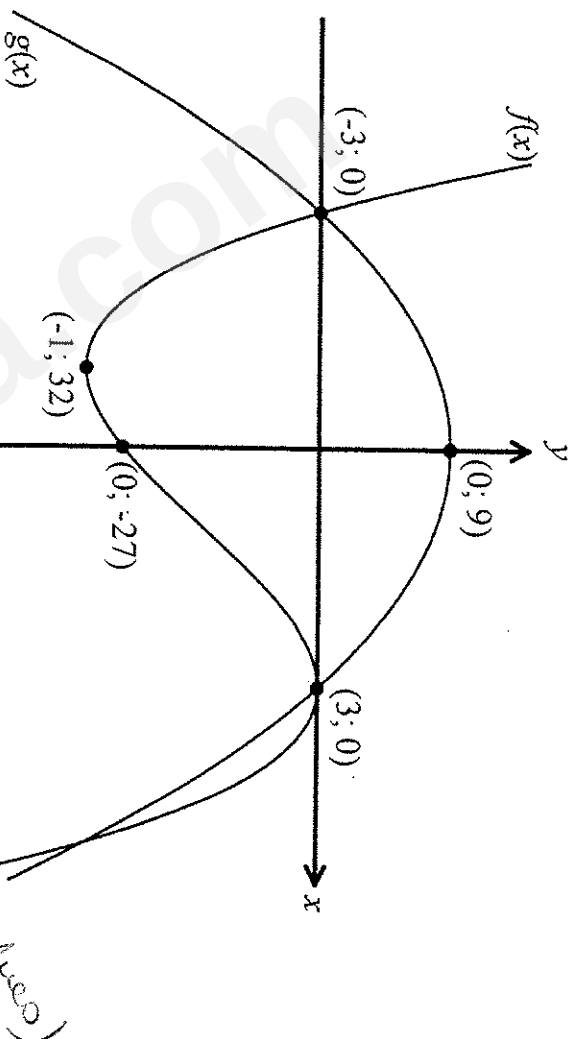
$$x = \frac{-2n}{6m}$$

(4)

$$x = \frac{-n}{3m} \quad \checkmark$$

QUESTION 9

The figure below represents the graphs of $f(x)$ and $g(x)$.



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a) Calculate the value of $g(3) - f(0)$

$$0 - (-27) = 27$$

(2)

b) Write down a common factor of $f(x)$ and $g(x)$

$$(x+3) \quad \text{or} \quad (x-3)$$

any one

c) State the values of x for which $g(x) \cdot f(x) \geq 0$.

$$\text{Method 1: } x = -3 \quad \text{or} \quad x \in [3, \infty)$$

$$\text{Method 2: } x = -3 \quad \text{or}$$

$$x > 3$$

(2)

d) State a possible restriction so that $g(x)$ has an inverse that is a function.

$$x > 0$$

or

$$x \leq 0$$

✓a

(only one answer)

(1)

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e) Determine the equation of $g^{-1}(x)$ in the form $y = \dots$ based on your restriction in (d). (5)

Method 1: $x > 0$

$g: y = a(x^2 - 9)$ ✓ *finding equation*

$(0, 9) \rightarrow 9 = a(\dots - 9)$

$9 = -9a$

$-1 = a$

$g(x) = -1(x^2 - 9) \Rightarrow y = -x^2 + 9$ ✓

$x = -1y^2 + 9$ ✓

$x - 9 = -y^2$

$\pm \sqrt{-x + 9} = y$ ✓

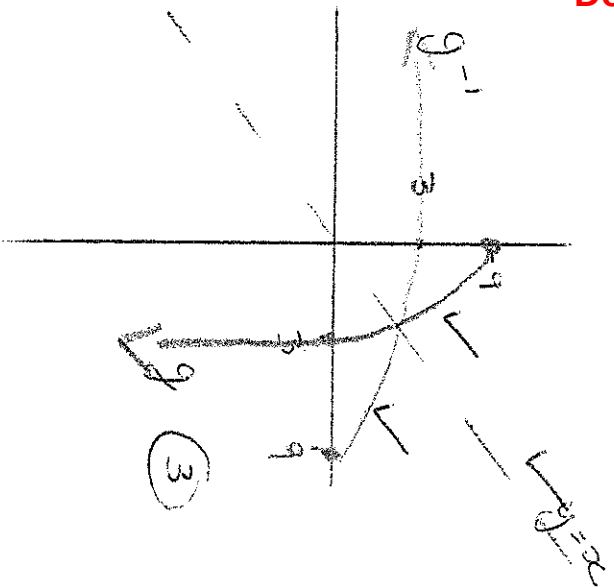
if $x > 0 \quad y = \sqrt{-x + 9}$ ✓ *ca*

Method 2: $x \leq 0 \Rightarrow y = -\sqrt{-x + 9}$

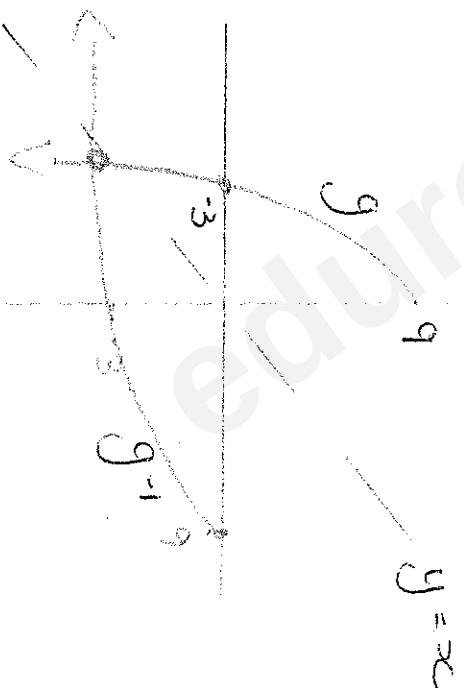
f) Sketch $g(x)$ and $g^{-1}(x)$, using your answers from (d) and (e) above.

Clearly showing at least two points, which must include any intercepts with the axes. Any asymptotes must also be clearly shown.

Method 1: $x > 0$

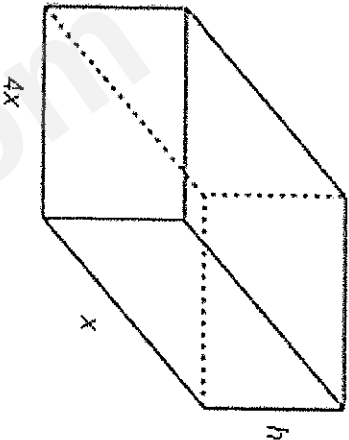


Method 2: $x \leq 0$



QUESTION 10

An open rectangular box is made of a very thin sheet of metal. The volume is 128 cm^3 and the base of the box has a width of $x \text{ cm}$ and a length of $(4x) \text{ cm}$.



- a) Show that the total surface area of the box can be written as $(4x^2 + \frac{320}{x}) \text{ cm}^2$. (5)

$$128 = x \cdot 4x \cdot h$$

$$\frac{32}{x^2} = h$$

$$A = 4x \cdot \cancel{x} + 2xh + 2(4x)(\cancel{h})$$

$$= 4x^2 + 2xh + 8xh$$

$$= 4x^2 + 10xh$$

$$= 4x^2 + 10x \left(\frac{32}{x^2} \right)$$

$$= 4x^2 + \frac{320}{x}$$

(5)

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- b) Calculate the height of the box for which the surface area is a minimum. $A = 4x^2 + 320x^{-1}$

$$\frac{dA}{dx} = 8x - \frac{320}{x^2} = 0$$

$$8x^3 - 320 = 0$$

$$8x^3 - 320 = 0$$

$$x^3 = 40$$

$$x = 3.4 \text{ cm}$$

$$h = \frac{32}{(3.4)^2} = 2.8 \text{ cm}$$

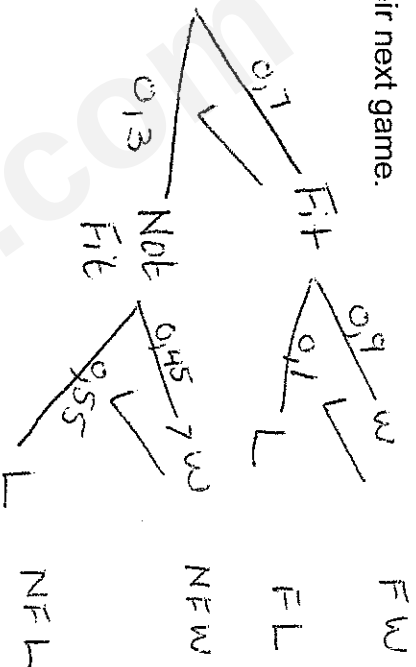
(4)

[9 marks]

QUESTION 11

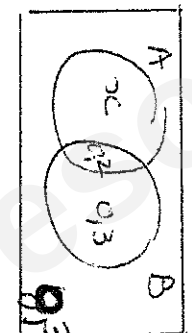
- a) The probability that the Springbok Team has all its players fit to play is 70%. The probability that they will win a game if all their players are fit is 90%. When they are not fit, the probability of them winning becomes 45%.

Draw a tree diagram to illustrate this situation and hence calculate the probability of them winning their next game.



(6)

$$\begin{aligned}
 P(\text{Win}) &= 0,7 \times 0,9 + 0,3 \times 0,45 \\
 &= 0,765 \\
 &= 0,8 \quad \checkmark
 \end{aligned}$$



- b) Given that A and B are independent events. Calculate the value of x and y if:
- $P(B \text{ only}) = 0,3$
 - $P(A \text{ and } B) = 0,2$
 - $P(A \text{ only}) = x$
 - $P(A' \text{ or } B') = y$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$0,2 = (x + 0,2) \times (0,5)$$

$$0,2 = x \quad \checkmark$$

$$y = 0,8 \quad \checkmark$$

(4)

QUESTION 12

a) Observe the pattern below:

A
 B B B
 C C C C C
 D D D D D D
 E E E E E E E

If the total number of letters in the pattern is 361, which letter will the last row consist of? (4)

$$1 + 3 + 5 + 7 + \dots \cdot \checkmark$$

$$S_n = \frac{n}{2} (a(1) + (n-1)a)$$

$$361 = \frac{n}{2} (2 + 2n - 2) \quad \checkmark m$$

$$722 = n(2n)$$

$$722 = 2n^2$$

$$361 = n^2$$

$$19 = n$$

$\checkmark a$

The letter is C $\checkmark ca$

(4)

b) Consider the following geometric series: $5 + \frac{5}{2} + \frac{5}{4} + \dots$

1. If A is the sum to infinity of the given series, evaluate A .

$$S_{\infty} = \frac{5}{1 - \frac{1}{2}} \checkmark$$

$$= 10$$

$$A = 10 \checkmark$$

(3)

2. If B is the sum of the first n terms of the given series, find B in terms of n .

$$B = S_n = 5 \left(\left(\frac{1}{2} \right)^n - 1 \right) \checkmark$$

$$\left(\frac{1}{2} - 1 \right)$$

$$= 5 \left(\left(\frac{1}{2} \right)^n - 1 \right) \checkmark$$

$$-1/2$$

$$B = -10 \cdot \left(\frac{1}{2} \right)^n + 10 \checkmark$$

(3)

3. Find the nearest integer value of n for which $A - B > \frac{5}{1024}$

$$10 - \left(-10 \cdot \left(\frac{1}{2} \right)^n + 10 \right) > \frac{5}{1024}$$

$$> \frac{5}{1024}$$

$\checkmark$

$$10 + 10 \cdot 2^{-n}$$

$$-10$$

$$>$$

$$\frac{5}{1024}$$

$$10 \cdot 2^{-n}$$

$$>$$

$$\frac{5}{1024}$$

$$2^{-n}$$

$$>$$

$$\frac{5}{1024} \times \frac{1}{10}$$

$$2^{-n}$$

$$>$$

$$\frac{1}{2048}$$

$\checkmark$

$$2^{-n} > \left(\frac{1}{2} \right)^{11}$$

$$+n > 11$$

$$n < 11 \checkmark$$

$$n = 10 \checkmark$$

(4)

[14 marks]