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**MATHEMATICS PAPER 1**

**VCAA 2017**

**150 marks**

**Time: 3 hours**

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:**

1. This question paper consists of 13 pages and an Information Sheet of 2 pages. Please check that your paper is complete.
  2. Read the questions carefully.
  3. Answer all the questions.
  4. Number your answers exactly as the questions are numbered.
  5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
  6. Round off your answers to one decimal digit where necessary.
  7. All the necessary working details must be clearly shown.
  8. It is in your own interest to write legibly and to present your work neatly.
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## SECTION A

## QUESTION 1

(a) Solve for  $x$  :

$$(1) \quad (x^2 - 1)(x + 1) = 0 \quad (2)$$

$$(2) \quad 3^{2x+1} - 8 \cdot 3^x - 3 = 0 \quad (5)$$

(b) Given :  $(3m - 1)(4 - m) > 0$ 

$$(1) \quad \text{Solve for } m. \quad (2)$$

$$(2) \quad \text{Hence, find the sum of all the integers that satisfy the inequality.} \quad (2)$$

(c) Determine the value(s) of  $k$  for which the function

$$f(x) = 3k^3 + x^2(7 - 3k) + x(35 - 15k) + 9k - 21$$

$$\text{leaves a remainder of } -1 \text{ when it is divided by } x - k. \quad (5)$$

(d) Solve for  $a$  and  $b$  simultaneously :

$$\log_2(a + 4) = 3 \quad \text{and} \quad ab - b^2 = -12 \quad (5)$$

(e) The solution to a quadratic equation is  $x = 3 - \sqrt{1 + 2p}$ .Find a value for  $p$ ,  $-2 \leq p \leq 6$ ;  $p \in \mathbb{Z}$ , if  $x$  is :

- (1) Zero
  - (2) Irrational
  - (3) Non-real
- (3)

**[24]**

**QUESTION 2**

(a) If  $4 ; x ; 50$  form an Arithmetic sequence, determine the value of  $x$ . (1)

(b) Calculate the value of  $a$  if  $\sum_{k=1}^3 a \cdot 2^{k-1} = 28$ . (2)

(c) Determine the sum of  $3 + 4 + 5 + 6 + \dots + 83$ ,  
if all the multiples of 8 are excluded. (5)

(d)  $3 ; x ; 15 ; y$  form a quadratic number pattern.

If the 2<sup>nd</sup> common difference is 2, determine the value of  $x$  and  $y$ . (4)

**[12]****QUESTION 3**

(a) If  $f(x) = 2x - x^2$ , determine  $f'(x)$  from FIRST PRINCIPLES. (5)

(b) (1) Determine  $\frac{dy}{dx}$  if  $xy = 3$ . (2)

(2) Determine  $D_x \left[ \frac{\sqrt[3]{x^2} - 2x + 1}{x^2} \right]$  (4)

(c) Draw a rough sketch of  $g(x) = ax^3 + bx^2 + cx + d$  where :

- $a > 0$
- $g'(x) > 0$  when  $x < -1$  and  $x > \frac{5}{3}$
- $g'(-1) = 0$
- one of the  $x$ -intercepts is  $-1$
- $y$ -intercept  $= -3$  (4)

**[15]**

#### QUESTION 4

- (a) Nthabiseng is studying for three years. The first year will cost R15 600, after which every year the fees will increase by 14% per annum simple interest.
- (1) Nthabiseng takes out a loan to pay for her studies. The value of the loan is the sum of the cost of the 3 annual fees. Show that she has to borrow R53 352 in order to pay for her studies. (3)
- (2) If Nthabiseng can afford to pay back the bank R1000 every month, and the bank charges her 5% per annum interest compounded monthly, how long will it take Nthabiseng to pay back the bank? (4)
- (3) If Nthabiseng decides to save R500 at the end of every month for the next three years, and invests the money in a savings account with a 13,8% per annum interest rate compounded monthly, determine how much money she will have at the end of the third year. (4)
- (b) A student buys a laptop and pays an initial deposit of R7 000, R8 000 after 2 years and a further R10 000 two years later. Interest is 18% per annum compounded monthly. Calculate the original price of the laptop. (4)

**[15]**

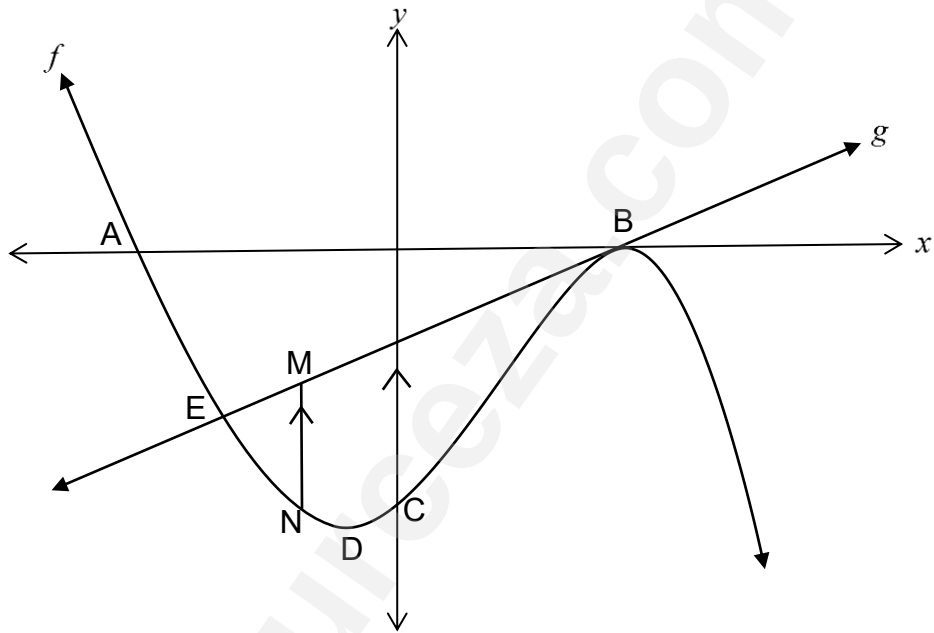
## QUESTION 5

The graphs of  $f(x) = -x^3 + 3x^2 + 9x - 27$  and  $g(x) = tx + q$  are sketched below.

A(-3;0) and B (3;0) are the  $x$ -intercepts and C the  $y$ -intercept of  $f$ .

The turning points of  $f$  are D(-1;-32) and B.

$g$  and  $f$  intersect at E (-2;-10) and B.



- (a) Determine the equation of  $g$ . (2)
- (b) For what value(s) of  $x$  is :
- (1)  $f$  increasing. (2)
  - (2)  $f$  concave up. (2)
  - (3)  $x \cdot f'(x) < 0$ . (3)
  - (4) MN a maximum, if M is a point on  $g$  between E and B, N is a point on  $f$  and MN is parallel to the  $y$ -axis. (5)

[14]

## QUESTION 6

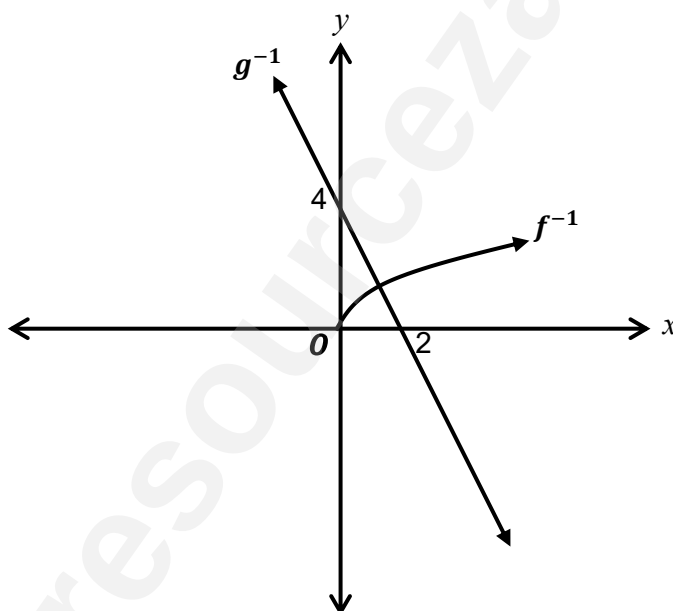
(a)  $h: x \rightarrow 2^{-x}$

$t: x \rightarrow \frac{-2}{x} ; x < 0$

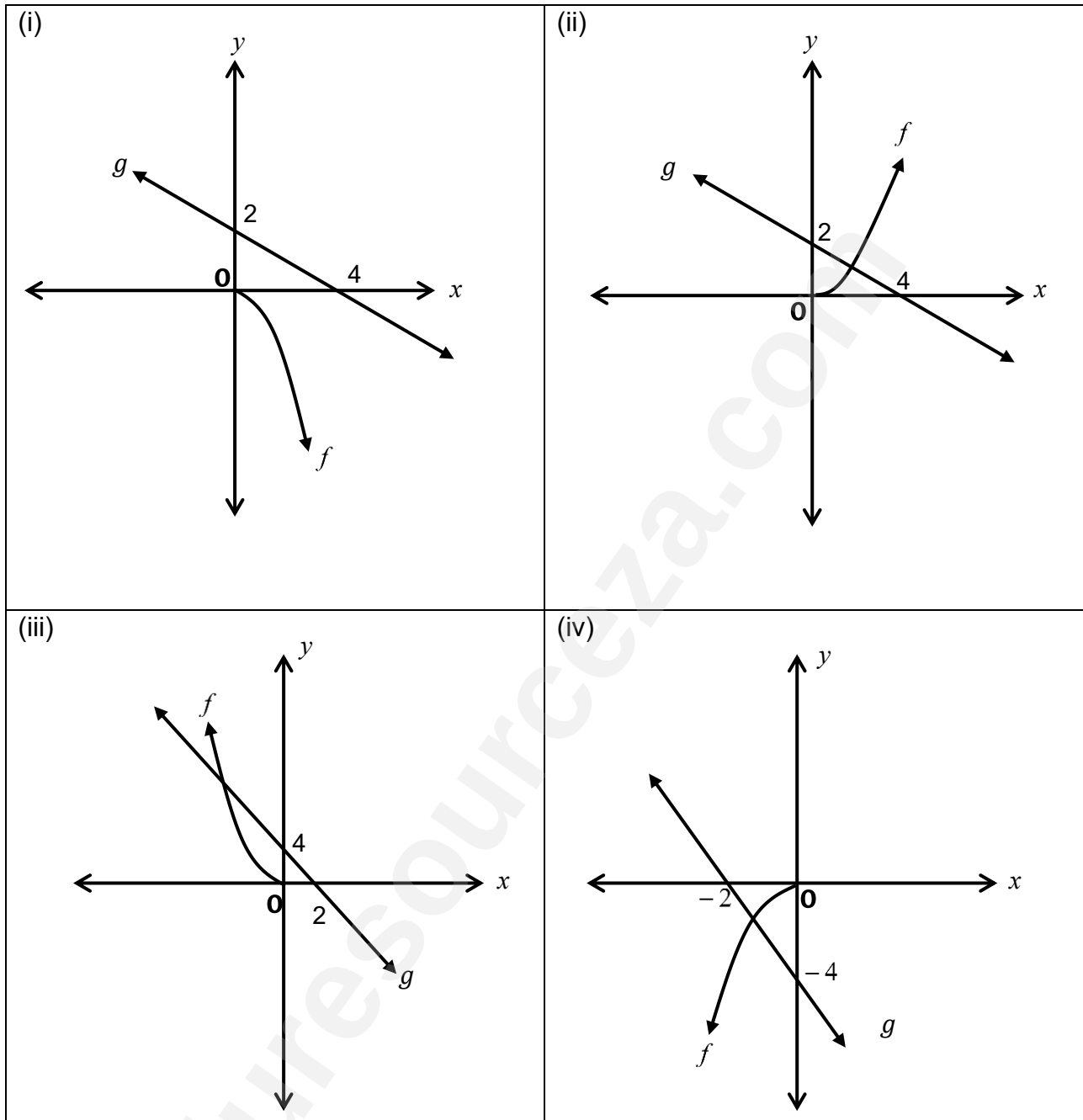
- (1) Sketch the graphs of  $h$  and  $t$  on the system of axes provided on page 11. Clearly indicate the intercepts with the axes and two points on each graph. (5)

- (2) Explain how you would use your graphs to solve  $2^{x+1} = -x$ . Use the letter A to show where the solution can be read off. (4)

- (b) Given the graphs of  $f^{-1}$  and  $g^{-1}$  as follows:



Which of the following is a representation of the graphs of  $f$  and  $g$ ?  
Choose the correct answer by only writing down the number of your choice.



(2)

[11]

## SECTION B

## QUESTION 7

- (a) A team of workers are erecting a fence along a main road.  
They pitch camp at one end of the fence and erect 0,65km of new fencing each day. Every night they return to their camp from where they started.
- (1) How far will they travel on day 30? (2)
- (2) How far will they travel in total in 30 days? (5)
- (b) If  $t+1$  ;  $t-1$  ;  $2t-5$  are the first three terms of a convergent geometric series. Determine the value of  $t$ . (5)

[12]

## QUESTION 8

- (a) Determine the value of  $p$  if  $y = 2x + p$  is a tangent to the curve

$$y = 2x^2 - 6x + 20. \quad (5)$$

- (b) Consider the functions  $t$  and  $n$  where

$$t(x) = 3x - 5 \text{ and}$$

$$n(x) = x - 2$$

- (1) If  $h(x) = \frac{t(x)}{n(x)}$  ;  $x \neq 2$ , show that  $h(x)$  can be written in the form

$$y = \frac{a}{x+p} + q, \text{ where } a = 1, p = -2 \text{ and } q = 3. \quad (3)$$

- (2) Sketch the graph of  $h(x)$  for  $-3 \leq x \leq 7$  and  $-2 \leq y \leq 8$ , including the asymptotes and clearly indicating the intercepts with the axes. Use the system of axes provided on page 11. (4)

- (3) By writing  $h(x)$  in the form  $y = \frac{a}{x+p} + q$ , it can be seen that a vertical asymptote exists at  $x = -p$ . Hence rewrite  $h(x)$  in the form

$$x = \frac{cy+d}{y+e}, \text{ to show that the range of } h(x) \text{ is } Range = R - \{3\}. \quad (5)$$

[17]

### QUESTION 9

A bridge is built in the shape of a parabolic arch. The bridge arch has a span of 196 metres and a maximum height of 30 metres.

Find the height of the arch at 15 metres from its centre.

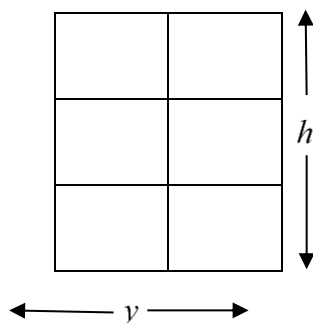
(Correct to 1 decimal place)

[5]



**QUESTION 10**

A window frame with dimensions  $y$  by  $h$  is illustrated below.  
The frame consists of six smaller frames.



- (a) If 12m of material is used to make the entire frame, show that  

$$y = \frac{1}{4}(12 - 3h).$$
 (2)
- (b) Show that the area of the window is given by Area,  $A = 3h - \frac{3}{4}h^2$ . (3)
- (c) Find  $\frac{dA}{dh}$  and hence the dimensions,  $h$  and  $y$ , of the frame so that the area of the window is a maximum. (5)

**[10]****QUESTION 11**

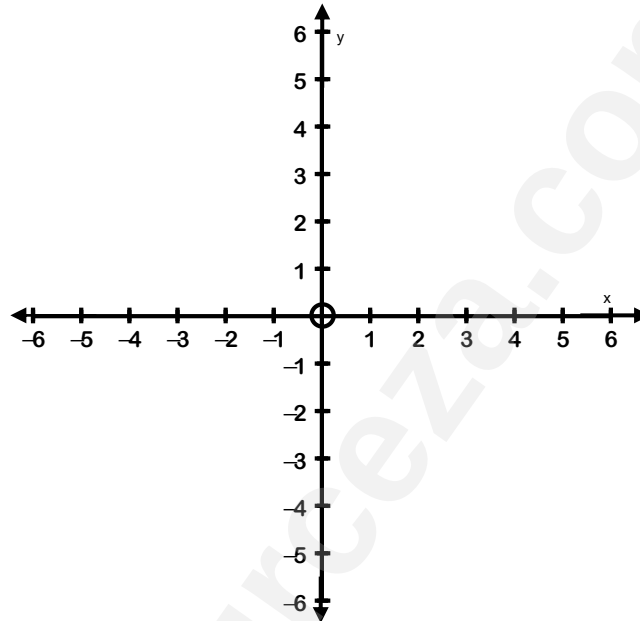
- (a) Two events  $A$  and  $B$  are such that  $P(A \cap B) = k$ ,  
 $P(A \cap B') = 0,6k$  and  $P(A' \cap B) = k - 0,125$ .
- (1) Draw a Venn diagram to illustrate this situation. (2)
- (2) Determine the value of  $k$  when  $A$  and  $B$  are independent events. (4)
- (b) 10 swimmers compete in a 100 m backstroke race. In how many different ways can the first 6 places in the event be arranged, assuming that there are no dead heats? (4)
- (c) How many four digit numbers have no repeat digits, do not contain zero, and have a sum of digits equal to 28? (5)

**[15]**

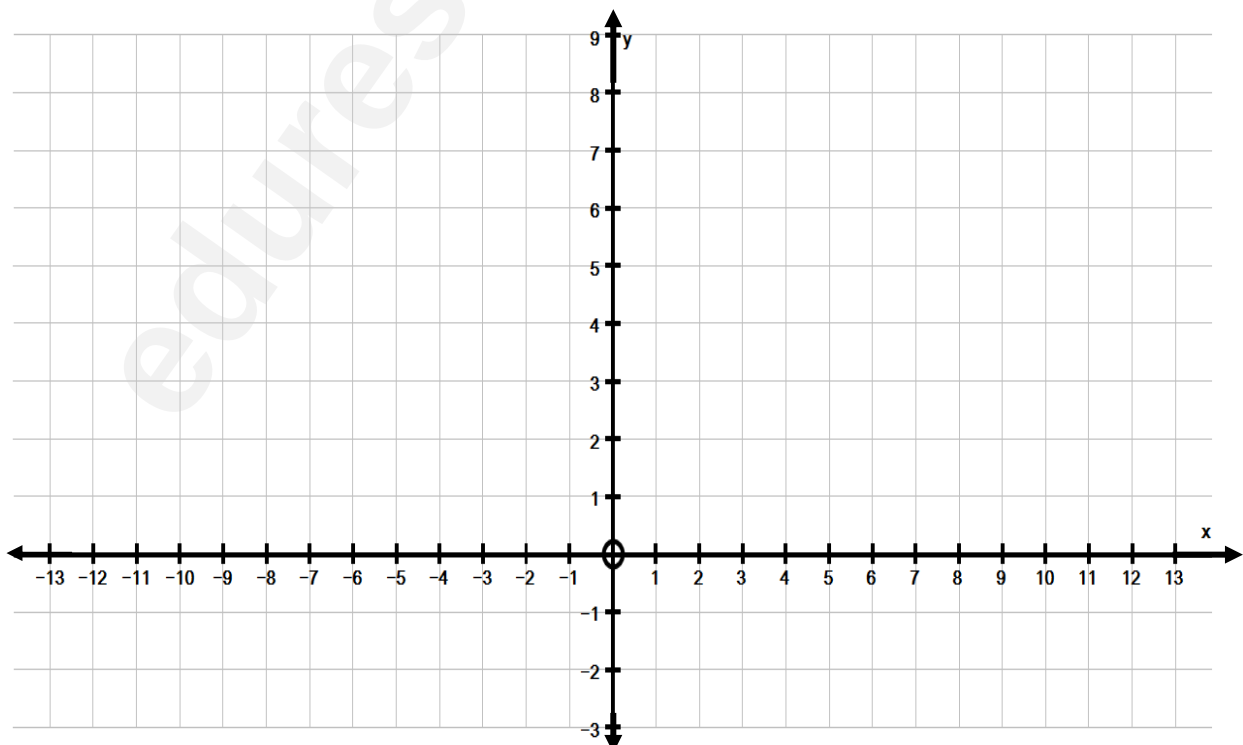
MATHEMATICS PAPER 1

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QUESTION 6



QUESTION 8



**MATHEMATICS**  
**INFORMATION SHEET**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}; \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 - i)^n$$

$$F = x \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$P = x \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum f x}{n} \quad \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$