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MATHEMATICS PAPER 1
VCAA 2016

150 marks

Time: 3 hours

PLEASE READ THESE INSTRUCTIONS CAREFULLY:

1. This question paper consists of 8 pages and an Information Sheet of 2 pages.
Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
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SECTION A

QUESTION 1

(a) Given: $x(x + 3) = p$

Solve for x if:

(1) $p = 0$ (2)

(2) $p = x + 3$ (4)

(b) Solve for x , showing all working out:

(1) $3\log_2(x - 2) = 9$ (3)

(2) $x^2 - 6x < 16$ (4)

(c) Given: $x^2 - 6ax + 5 = 0$, where a is a real constant.

(1) Determine the roots in terms of a . (2)

(2) Show that the product of the roots is independent of a . (3)

[18]

QUESTION 2

(a) Given: $1 - 2 + 4 - 8 + 16 \dots$

(1) State whether this is an arithmetic or geometric series. (1)

(2) Determine the sum of the first 30 terms. (3)

(b) The 4th term of an arithmetic sequence is -3 and the 20th is -35 . Determine the sum of the first 20 terms. (6)

(c) Given: $4k - 2 ; k + 1 ; k - 3$ are the first three terms of a geometric sequence. Determine the value of k if $k \in \mathbb{Z}$. (5)

[15]

QUESTION 3

(a) Determine $f'(x)$ from first principles if $f(x) = 4x^2 - 4$. (6)

(b) Find, leaving your answer with positive exponents:

(1) $f'(x)$ given $f(x) = \frac{x^2-4}{2-x}$ (3)

(2) $\frac{d}{dt} \left[\sqrt{t} - \frac{1}{t^2} \right]$ (4)

(c) Given $f(x) = x^3 - 20x + 30$.

Determine the equation of the tangent to $f(x)$ at the point where $x = 3$. (5)

[18]

QUESTION 4

(a) Mr King trades in his car for a new one every 5 years. He has just bought a new car for R 800 000 and has already decided that he will replace it with the same model in 5 years' time, when its trade-in value will be R 200 000. The replacement cost of a new car is expected to increase by 7,85% per annum effective.

(1) Using the car he has just bought as a trade-in, Mr King wants to pay cash for the new car in 5 years' time. Calculate how much extra cash he will need. (3)

(2) Mr King starts a sinking fund to make provision for the shortfall. He deposits x Rands into an account one month after the purchase and continues to deposit the same amount at the end of every month for five years. Calculate the amount that he needs to deposit each month if the interest rate is 9,75% per annum compounded monthly. (4)

(b) Ben borrows R 750 000 from his father to buy a holiday home. His father agrees that he can pay R 45 000 starting immediately and then at the end of every three months thereafter. An interest rate of 10,3% per annum compounded quarterly will be charged.

(1) How many payments of R 45 000 will Ben make? (4)

(2) What will Ben's final payment be? (3)

[14]

QUESTION 5

Mr Christians instructed his learners to sketch a graph for a function he had written on the board.
This is what they read:

$$y = 2^x + 1$$

Nontobeko read this as $y = 2x + 1$ while Amogelang read it as $y = 2^x + 1$.

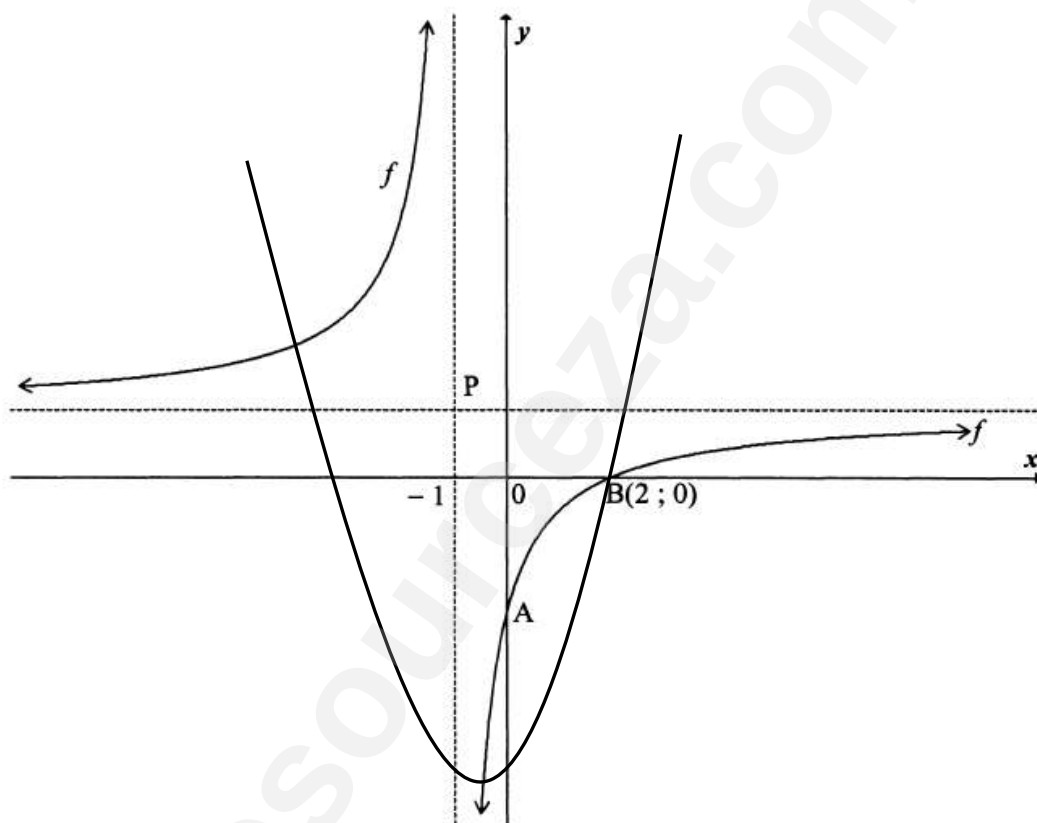
- (a) Write down two values of x which would generate the same y values for both options. (2)
- (b) Sketch the graphs of $y = 2x + 1$ and $y = 2^x + 1$ on the same set of axes. Indicate on the graphs your results from (a) as well as any intercepts on the axes. (5)
- (c) Draw the graph of the inverse of $y = 2^x + 1$ on the same axes as the graphs in (b). (3)

[10]

75 marks

SECTION B**QUESTION 6**

- (a) Sketches of a hyperbola $f(x)$ and a parabola $g(x)$ are given below. $f(x) = \frac{x-d}{x-p}$ where d and p are constants, and $g(x) = x^2 + x - 6$. The dotted lines are the asymptotes of the hyperbola. The asymptotes intersect at P and $B(2; 0)$ is a point on f .



- (1) Determine the values of d and p . (3)
- (2) Substitute $d = 2$ and $p = -1$ in the equation of f and rewrite it in the form $y = \frac{K}{x-p} + q$. (4)
- (3) Determine $h(x)$ if $h(x) = g(x - 2) - 2$ and describe the transformation from g to h . (4)
- (4) Determine the values of x for which $f(x) \cdot g'(x) < 0$ (4)

- (b) Consider a linear function with equation $f(x) = ax + b$, where $a > 0$ and $b > 0$, and a function with equation $g(x) = dx^2 + c$, where $d < 0$ and $c < 0$.

(1) Draw a sketch of possible graphs of f and g on the same set of axes. (5)

(2) If the vertical distance between the functions is defined by the equation $y = x^2 + \frac{2}{3}x + 7$, determine the minimum distance between f and g . (5)

[25]

QUESTION 7

- (a) The first term of a geometric series with n terms is x , and the sum of the series is $\frac{x(x^{10}-1)}{x^2-1}$.

(1) Determine a possible value of n . (2)

(2) If the common ratio is x^2 , show that $T_n = x^{2n-1}$. (3)

- (b) (1) For which values of k does the series below converge?

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n$$

(4)

- (2) If the series does converge, prove that:

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n = \frac{2k-3}{5-2k}$$

(4)

- (3) Hence, if possible, evaluate:

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n \text{ if } k = 1$$

(2)

[15]

QUESTION 8

(a) Given $f(x) = (x + 2)(x^2 + 4)$

(1) Show that the graph of $y = f(x)$ has no stationary point. (4)

(2) Calculate the values of x for which $y = f(x)$ is concave up. (4)

(3) Sketch the graph of $y = f(x)$, indicating all important points. (4)

(b) Consider the function $g(x) = x^3 - 3x^2 + kx + 8$.

Calculate the values of k for which $g(x)$ will be an increasing function for $x \in R$. (5)

[17]

QUESTION 9

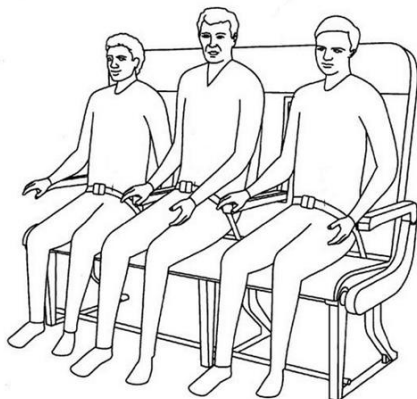
(a) A South African computer company, Cool-digital, has been having problems with one of their laptop models. They previously outsourced the assembly of some of these units to a company in Bangladesh, but are now assembling the units in South Africa. A sample of 483 laptops has revealed the following:

	Assembled in South Africa	Assembled in Bangladesh	Totals
Faulty	14	x	98
Non Faulty	y	330	385
Totals	69	414	483

(1) Write down the values of x and y . (2)

(2) By testing for independence, decide whether Cool-digital should continue with the assembly of units in South Africa or outsource to Bangladesh. (6)

- (b) A bus has seats for three passengers as shown in the diagram:



https://www.google.co.za/search?q=bus+seats+for+3+line+drawing&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjag7ics7DLAhVKAxoKHQXZAhYQ_AUIBygB&biw=1600&bih=731&safe=active&ssui=on#imgsrc=YR75zmJNouVZeM%3A

- (1) In how many different ways can Liam, John and Thando be seated? (1)
- (2) Liam and John want to sit next to each other; how many different options are there for the three friends to be seated? (2)
- (3) The name of the bus is HEADHAMMER. In how many different ways can the letters from the word be arranged? (3)
- (c) Three cards are drawn from a deck of 52 cards, one after the other without replacement. What is the probability of obtaining at least one club? (4)

[18]

75 marks