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MATHEMATICS PAPER 1 - MARKING GUIDELINE

VCAA 2016

150 marks

Time: 3 hours

PLEASE READ THESE INSTRUCTIONS CAREFULLY:

1. This question paper consists of 8 pages and an Information Sheet of 2 pages. Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
-

SECTION A

QUESTION 1

(a) Given: $x(x + 3) = p$

Solve for x if:

(1) $x(x + 3) = 0$

$x = 0 \text{ or } x = -3$ (2)

(2) $x(x + 3) = x + 3$

$x^2 + 3x - x - 3 = 0$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3 \text{ or } x = 1$ (4)

(b) Solve for x :

(1) $3 \log_2(x - 2) = 9$

$\log_2(x - 2) = 3$

$2^3 = (x - 2)$

$x - 2 = 8$

$x = 10$

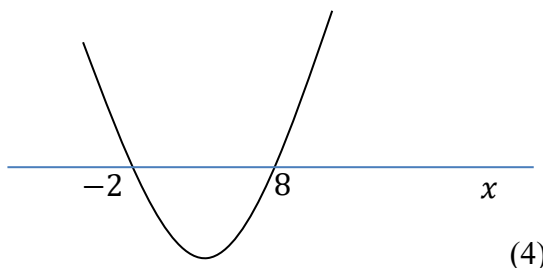
$\therefore x = 10$ (3)

(2) $x^2 - 6x < 16$

$x^2 - 6x - 16 < 0$

$(x - 8)(x + 2) < 0$

$-2 < x < 8$



(4)

(c) Given: $x^2 - 6ax + 5 = 0$, where a is a real constant.

$$\begin{aligned}
 x &= \frac{6a \pm \sqrt{36a^2 - 4(1)(5)}}{2} \\
 (1) \quad &= \frac{6a \pm \sqrt{36a^2 - 20}}{2} & (2) \\
 &= \frac{6a \pm 2\sqrt{9a^2 - 5}}{2} \\
 &= 3a \pm \sqrt{9a^2 - 5}
 \end{aligned}$$

$$(2) \quad (3a + \sqrt{9a^2 - 5})(3a - \sqrt{9a^2 - 5}) = 9a^2 - (9a^2 - 5) = 5 \quad (3)$$

[18]

QUESTION 2

(a) Given: $1 - 2 + 4 - 8 + 16 \dots$

(1) Geometric (1)

$$(2) \quad s_n = \frac{a(r^n - 1)}{r - 1}$$

$$r = -2$$

$$s_n = \frac{1((-2)^{30} - 1)}{-2 - 1}$$

$$= -357913941 \quad (3)$$

(b) $a + 3d = -3 \quad \dots \dots (1)$ $a + 19d = -35. \quad \dots \dots (2)$ *subtract (2) from (1)*

$$-16d = 32$$

$$d = -2$$

$$\therefore a = 3$$

$$S_{20} = 10[2(3) + 19(-2)]$$

$$= -320 \quad (6)$$

(c)

$$\frac{k+1}{4k-2} = \frac{k-3}{k+1}$$

$$4k^2 - 14k + 6 = k^2 + 2k + 1$$

$$3k^2 - 16K + 5 = 0$$

$$(3k - 1)(k - 5) = 0$$

$$\therefore k = \frac{1}{3} \text{ or } k = 5$$

$$N/A \quad k = 5 \quad (5)$$

[15]

QUESTION 3

- (a) If
- $f(x) = 4x^2 - 4$
- ; determine
- $f'(x)$
- from first principles.

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 4 - (4x^2 - 4)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4(x^2 + 2xh + h^2) - 4 - (4x^2 - 4)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4x^2 + 8xh + 4h^2 - 4 - 4x^2 + 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{8xh + 4h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(8x + 4h)}{h} \\
 &= \lim_{h \rightarrow 0} 8x + 4h \\
 &= 8x
 \end{aligned} \tag{6}$$

- (b)

$$\begin{aligned}
 (1) \quad f(x) &= \frac{x^2 - 4}{2 - x} \\
 f(x) &= \frac{-(x-2)(x+2)}{x-2} \\
 f(x) &= -x - 2 \\
 f'(x) &= -1
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 (2) \quad \frac{d}{dt} \left[\sqrt{t} - \frac{1}{t^2} \right] \\
 &= \frac{d}{dt} \left[t^{\frac{1}{2}} - t^{-2} \right] \\
 &= \frac{1}{2} t^{-\frac{1}{2}} + 2t^{-3} \\
 &= \frac{1}{2\sqrt{t}} + \frac{2}{t^3}
 \end{aligned} \tag{4}$$

(c) Given $f(x) = x^3 - 20x + 30$

$$f'(x) = 3x^2 - 20$$

$$f'(3) = 3(3)^2 - 20$$

$$= 7$$

$$f(x) = x^3 - 20x + 30$$

$$f(3) = (3)^3 - 20(3) + 30$$

$$= -3$$

$$(3; -3)$$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = 7(x - 3)$$

$$y = 7x - 24$$

(5)

[18]

QUESTION 4

(a)

$$(1) \quad A = P(1 + i)^n$$

$$A = 800\,000(1 + 0,0785)^5$$

$$= 1\,167\,322,17$$

Mr King will therefore need:

$$1\,167\,322,17 - 200\,000$$

$$= 967\,322,17$$

(3)

$$(2) \quad 967\,322,17 = \frac{x \left[\left(1 + \frac{0,0975}{12} \right)^{60} - 1 \right]}{\frac{0,0975}{12}}$$

$$\therefore x = 12\,574,46$$

(4)

(b)

(2)

(4)

(3)

[14]

QUESTION 5

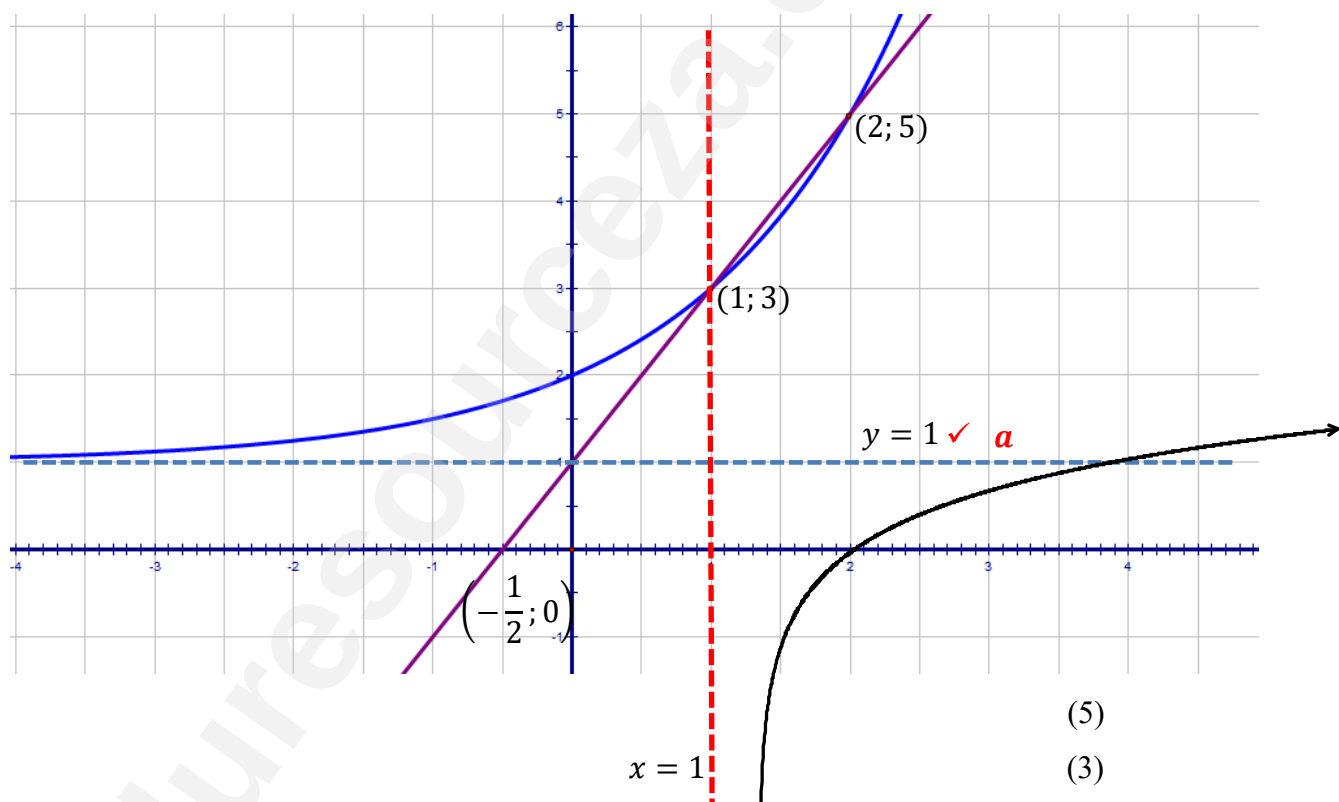
Mr Christians instructed his students to sketch a graph for a function he wrote on the board. This is what they read:

$$y = 2^x + 1$$

Nontobeko read this as $y = 2x + 1$ while Amogelang read it as $y = 2^x + 1$

(a) $x = 1$ and $x = 2$

(2)



(b)

(5)

(c)

(3)

[10]

75 marks

SECTION B**QUESTION 6**

(a)

$$(1) \quad p = -1$$

$$\therefore f(x) = \frac{x-d}{x+1}.$$

$$\text{subst } (2; 0)$$

$$0 = \frac{2-d}{2+1}.$$

$$2 - d = 0$$

$$d = 2$$

(3)

$$(2) \quad f(x) = \frac{x-2}{x+1}.$$

$$f(x) = \frac{x+1}{x+1} + \frac{-3}{x+1}$$

$$f(x) = \frac{-3}{x+1} + 1$$

(4)

$$(3) \quad h(x) = g(x-2) - 2$$

$$= (x-2)^2 + (x-2) - 6 - 2$$

$$= x^2 - 4x + 4 + x - 2 - 6 - 2$$

$$= x^2 - 3x - 6$$

(4)

2 units right, 2 units down

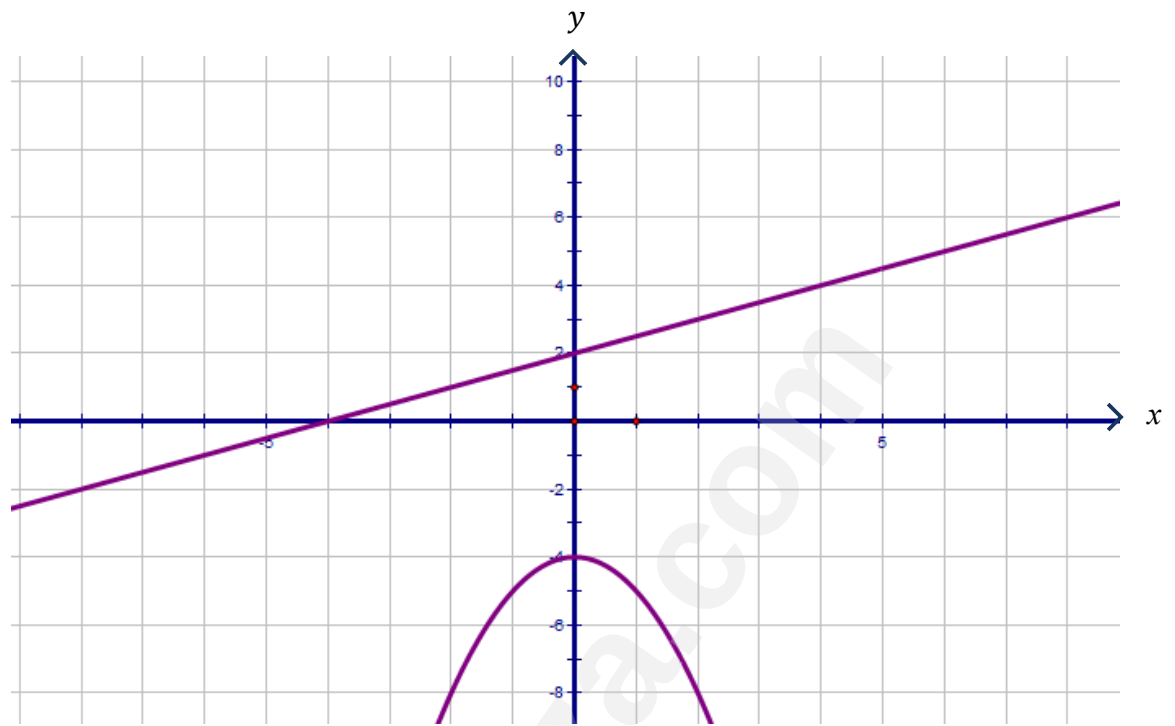
$$(4) \quad g'(x) = 2x + 1$$

$$\therefore x = -\frac{1}{2}$$

$$\therefore (-\infty; -1) \text{ or } (-\frac{1}{2}; 2)$$

(4)

(b)



(1)

(5)

$$(2) \quad y = x^2 + \frac{2}{3}x + 7$$

$$\frac{dy}{dx} = 2x + \frac{2}{3}$$

$$2x + \frac{2}{3} = 0$$

$$\therefore x = -\frac{1}{3}$$

$$y = x^2 + \frac{2}{3}x + 7$$

$$y = \left(-\frac{1}{3}\right)^2 + \frac{2}{3}\left(-\frac{1}{3}\right) + 7$$

$$= \frac{62}{9}$$

(5)

[25]

QUESTION 7

- (a) The first term of a geometric series with n terms is x , and the sum of the series is $\frac{x(x^{10}-1)}{x^2-1}$.

$$r = x^2$$

$$(1) \quad S_n = \frac{x(x^{10}-1)}{x^2-1}$$

$$x^{2n} = x^{10}$$

$$\therefore 2n = 10$$

$$n = 5$$

(2)

$$(2) \quad T_n = ar^{n-1}$$

$$T_n = x \cdot x^{2(n-1)}$$

$$T_n = x^{2n-2+1}$$

$$T_n = x^{2n-1}$$

(3)

- (b) (1)

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n$$

$$\left(k - \frac{3}{2}\right); \left(k - \frac{3}{2}\right)^2; \left(k - \frac{3}{2}\right)^3 \dots$$

$$r = k - \frac{3}{2}$$

$$-1 < r < 1$$

$$-1 < k - \frac{3}{2} < 1$$

$$\frac{1}{2} < k < \frac{5}{2}$$

(4)

- (2)

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n = \frac{2k-3}{5-2k}$$

$$S_{\infty} = \frac{k - \frac{3}{2}}{1 - (k - \frac{3}{2})}$$

$$= \frac{2k-3}{2} \times \frac{2}{-2k+5}$$

$$= \frac{2k-3}{5-2k}$$

(4)

(3)

$$\frac{2k - 3}{5 - 2k}$$

$$= \frac{2(1) - 3}{5 - 2(1)}$$

$$= \frac{-1}{3}$$

(2)

[15]

QUESTION 8(a) Given $f(x) = (x + 2)(x^2 + 4)$

$$(1) \quad f(x) = x^3 + 2x^2 + 4x + 8$$

$$f'(x) = 3x^2 + 4x + 4$$

Stationary points

$$3x^2 + 4x + 4 = 0$$

non – real roots

$$b^2 - 4ac$$

$$= (4)^2 - 4(3)(4)$$

$$= -32$$

$$\Delta < 0$$

 $\therefore$ no stationary points

(4)

(2) Concave up

$$f''(x) > 0$$

$$f'(x) = 3x^2 + 4x + 4$$

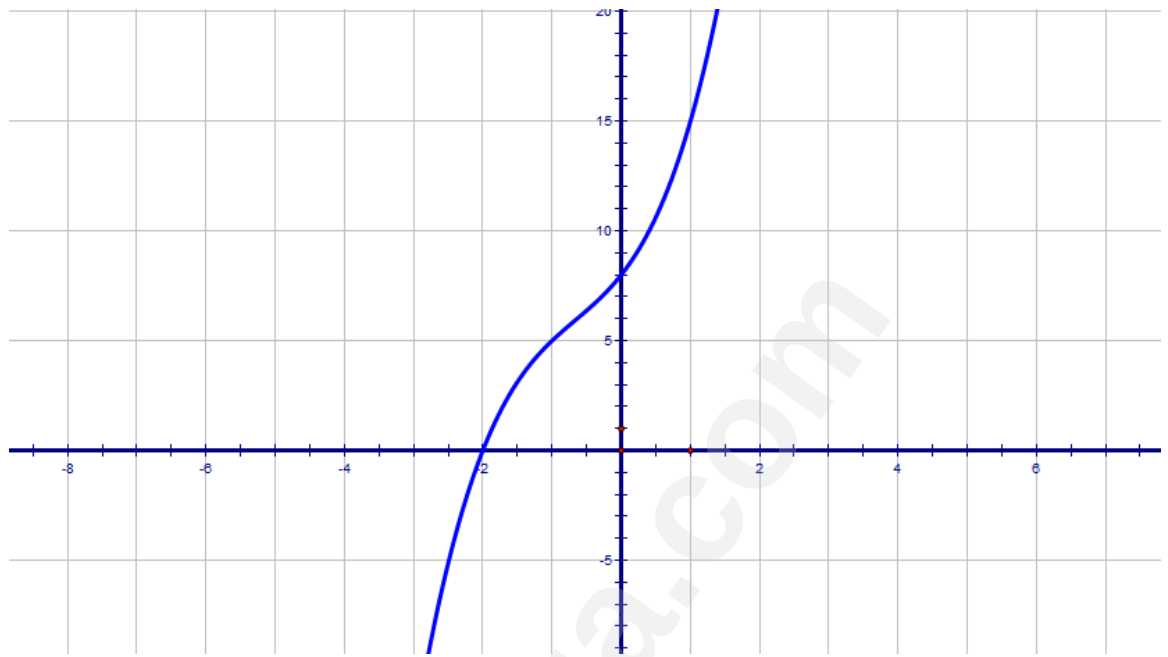
$$f''(x) = 6x + 4$$

$$6x + 4 > 0$$

$$x > -\frac{2}{3}$$

(4)

(3)



(4)

(b) Consider the function $g(x) = x^3 - 3x^2 + kx + 8$

$$g'(x) = 3x^2 - 6x + k$$

$$b^2 - 4ac < 0$$

$$36 - 12k < 0$$

$$k > 3$$

(5)

[17]

QUESTION 9

(a)

$$(1) \quad x = 84 \text{ and } y = 55 \quad (2)$$

$$(2) \quad P(\text{faulty} \cap \text{South Africa}) = P(\text{faulty}) \times P(\text{South Africa})$$

LHS

$$P(\text{faulty} \cap \text{South Africa})$$

$$= \frac{14}{483}$$

$$= 0,029$$

RHS

$$P(\text{faulty}) \times P(\text{South Africa})$$

$$= \frac{98}{483} \times \frac{69}{483}$$

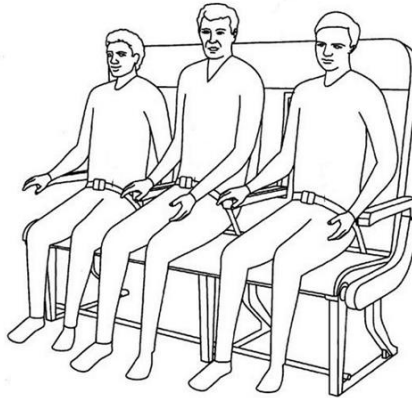
$$= 0,029$$

$$\therefore P(\text{faulty} \cap \text{South Africa}) = P(\text{faulty}) \times P(\text{South Africa})$$

These events are independent,

and Cool – digital should not continue outsourcing their work. (6)

- (b) A bus has seats that can sit three people on it as shown in the diagram below.



https://www.google.co.za/search?q=bus+seats+for+3+line+drawing&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjag7ics7DLAhVKAxoKHQXZAhYQ_AUIBygB&biw=1600&bih=731&safe=active&ssui=on#imgsrc=YR75zmJNouVZeM%3A

$$(1) \quad 3 \ 2 \ 1 \rightarrow 3! = 6 \text{ ways} \quad (1)$$

$$(2) \quad 2! \ 2! \rightarrow 4 \text{ ways} \quad (2)$$

$$(3) \quad \frac{10!}{2! \times 2! \times 2! \times 2!} = 226 \ 800 \text{ ways} \quad (3)$$

$$(c) \quad (No \ Clubs) = \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} \approx 0,41$$

$$P(At \text{ least one Club}) = 1 - 0,41 \approx 0,59$$

(4)

[18]

75 marks