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CRAWFORD SCHOOLS PRELIMINARY EXAMINATIONS

SEPTEMBER 2022

MATHEMATICS : PAPER ONE

Time: 3 hours

150 marks

SUGGESTED ANSWERS

FOR OFFICE USE ONLY : MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
18	25	13	14	4	19	21	14	14	8	150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of **24** pages and an Information Sheet of 2 pages (i–ii).
Please check that your question paper is complete.
Spare pages have been left for rough/extra working.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your name in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
8. It is in your own interest to write legibly and to present your work neatly.
9. Round off to one decimal place unless otherwise stated.

SECTION A

QUESTION 1

[18 marks]

(a) Solve for x correct to ONE decimal place where necessary.

$$(1) \quad x(x-2) = 8 \quad (3)$$

$$x^2 - 2x - 8 = 0 \checkmark$$

$$(x+2)(x-4) = 0 \checkmark$$

$$x = -2 \quad \text{or} \quad x = 8 \checkmark$$

$$(2) \quad x^2 - px = -2 \quad (3)$$

$$x^2 - px = -2$$

$$x^2 - 6x + 2 = 0$$

$$x = \frac{p \pm \sqrt{(-p)^2 - 4(1)(2)}}{2(1)} \checkmark \checkmark$$

$$x = \frac{p \pm \sqrt{p^2 - 8}}{2} \checkmark$$

$$(3) \quad 2\sqrt{x-1} + 1 = x \quad (4)$$

$$2\sqrt{x-1} = x - 1$$

$$4(x-1) = (x-1)^2 \checkmark$$

$$4x - 4 = x^2 - 2x + 1$$

$$x^2 - 6x + 5 = 0 \checkmark$$

$$(x-5)(x-1) = 0$$

$$x = 5 \quad \checkmark \quad \text{or} \quad x = 1 \checkmark$$

(4) $3^{x+3} - 3^{x+2} + 3^0 = 487$ (4)

$$3^{x+3} - 3^{x+2} = 486$$

$$3^x(3^3 - 3^2) = 486 \checkmark$$

$$3^x(18) = 486 \checkmark$$

$$3^x = 27$$

$$3^x = 3^3 \checkmark$$

$$x = 3 \checkmark$$

(b) Solve for x and y if $(2x-1)^2 + (2y+3x)^2 = 0$ (4)

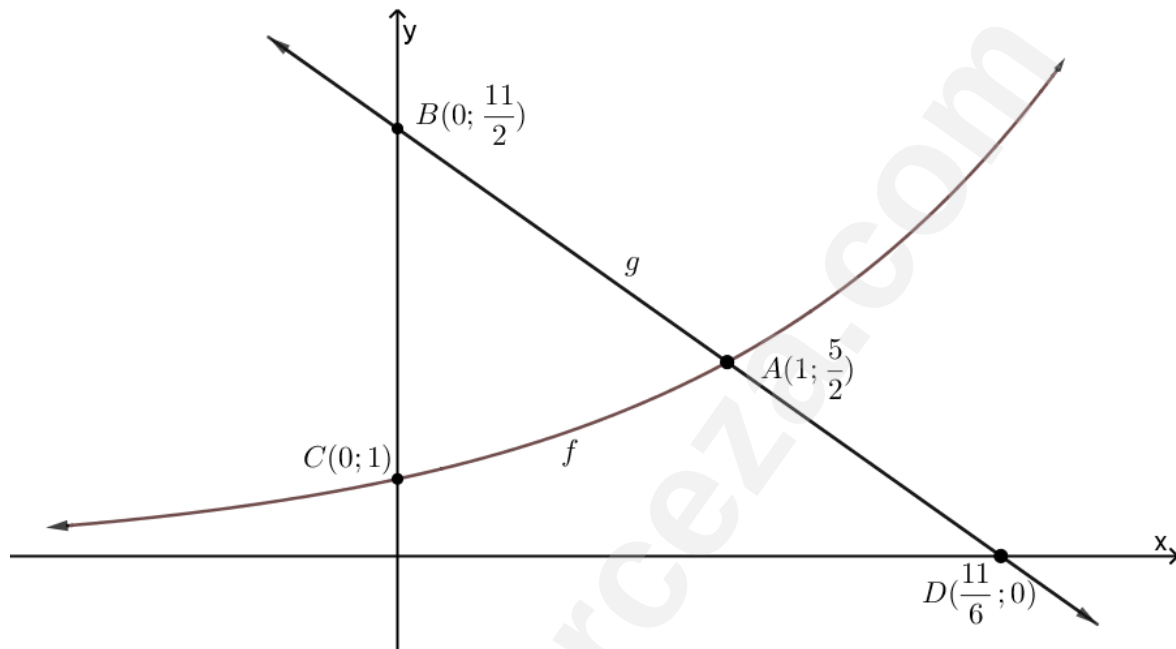
$$2x - 1 = 0 \checkmark$$

$$x = \frac{1}{2} \checkmark$$

$$2y + 3\left(\frac{1}{2}\right) = 0 \checkmark$$

$$y = -\frac{3}{4} \checkmark$$

- (a) The graphs of $f(x) = \left(\frac{5}{2}\right)^x$ and $g(x) = mx + c$ are drawn. $A\left(1; \frac{5}{2}\right)$ is the point of intersection of the two graphs. $B\left(0; \frac{11}{2}\right)$ is the y-intercept of g , and $C(0; 1)$ is the y-intercept of f .



- (1) Determine the values of m and c . (2)

$$m = -3 \checkmark \text{ a}$$

$$c = \frac{11}{2} \checkmark \text{ a}$$

- (2) Determine f^{-1} , the inverse of f in the form $y = \dots$ (2)

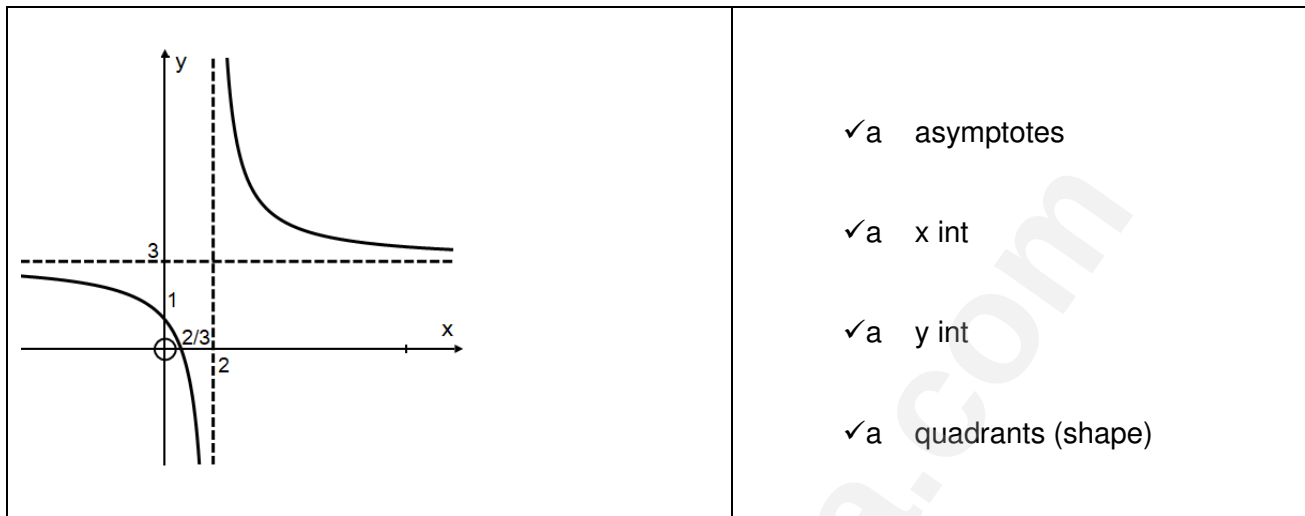
$$y = \log_{\frac{5}{2}} x \checkmark \checkmark$$

- (3) Hence, write down the domain of $f^{-1}(x-2)$. (2)

$$x > 2 \checkmark \checkmark$$

(b) On the grid below: **Downloaded from eduresourceza.com**

- (1) Sketch the graph of $f(x) = \frac{4}{x-2} + 3$ showing all intercepts and asymptotes clearly. (4)



- (2) The function f has an axes of symmetry with a negative gradient. Determine the equation of this line of symmetry. (3)

$y = -x + c$ $subst(2;3)$ $3 = -2 + c$ $5 = c$ $y = -x + 5$	✓a neg gradient ✓m subst ✓a c value in equation
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- (c) Given: $p(x) = \frac{3x+1}{x-2}$

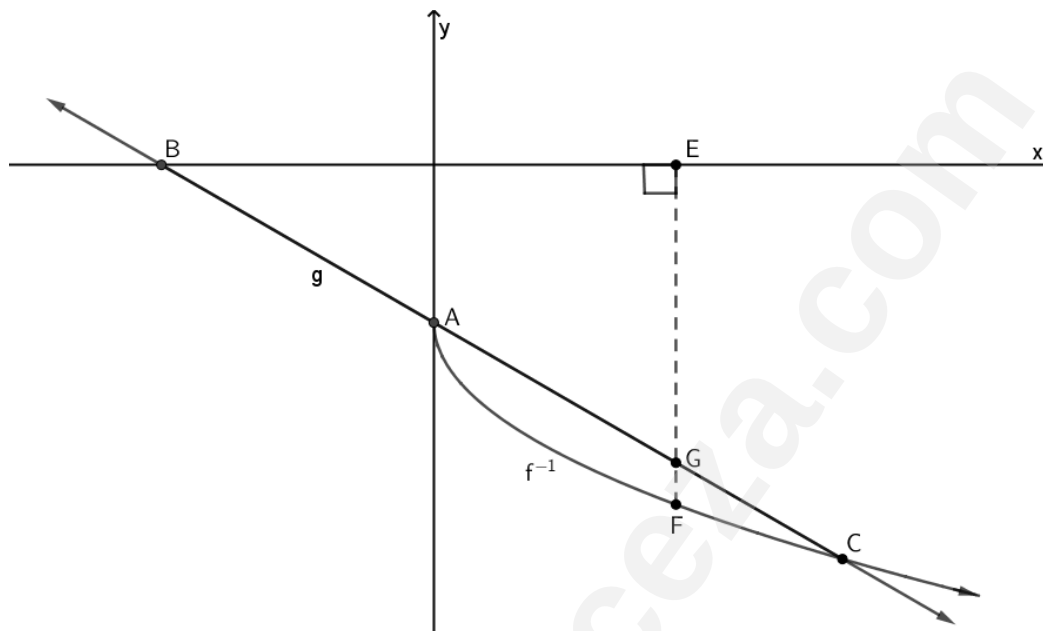
Rewrite p in the form $p(x) = \frac{a}{x-2} + n$ where a and n are integers. (4)

$$\frac{3x+1}{x-2} = \frac{3(x-2)+7}{x-2} = \frac{7}{x-2} + 3$$

✓✓ ✓✓

- (d) Refer to the figure below, showing the graphs of $f^{-1}(x) = -\sqrt{x} - 2$ and $g(x) = -\frac{1}{3}x - 2$.

A and B are the intercepts on the coordinate axes. A and C are the points of intersection of f and g .



- (1) Is f^{-1} a function? Justify your answer. (2)

Yes ✓ a, it passes the "vertical line test" (1:1 correspondence) ✓ a

- (2) Determine the equation of the function f , in the form $f(x) = \dots$ (4)

$$y = -\sqrt{x} - 2$$

$$\therefore x = -\sqrt{y} - 2 \quad \checkmark \text{ m}$$

$$\sqrt{y} = -2 - x \quad \checkmark \text{ m (squaring)}$$

$$y = (-2 - x)^2$$

$$\therefore f(x) = (-2 - x)^2 = y \quad \checkmark \text{ a}; x \leq -2 \quad \checkmark \text{ a}$$

- (3) The position of the vertical line GF is variable such that $x_A < x_E < x_C$.

Determine the length of the vertical line segment GF, in terms of x . (2)

$$\begin{aligned} GF &= y_G - y_F \\ &= -\frac{1}{3}x - 2 - (-\sqrt{x} - 2) \quad \checkmark \text{ m} \checkmark \text{ a} \\ &= -\frac{1}{3}x + \sqrt{x} \end{aligned}$$

QUESTION 3

[13 marks]

- (a) If 2 ; x ; 50 form a geometric sequence, determine the value of x , if $x < 0$. (2)

$$\frac{x}{2} = \frac{50}{x} \quad \checkmark$$

$$x^2 = 100$$

$$(x = 10) \text{ n/a} \quad \text{or} \quad x = -10 \quad \checkmark$$

- (b) Calculate the value of n if $\sum_{k=1}^n (-5k + 21) = -630$ (6)

$$16; 11; 6; \quad \checkmark$$

$$= \frac{n}{2} [2(16) + (n-1)(-5)] = -630; \quad \checkmark$$

$$= \frac{n}{2} [32 - 5n + 5] = -630;$$

$$n[-5n + 37] = -1260$$

$$-5n^2 + 37n + 1260 = 0$$

$$5n^2 - 37n - 1260 = 0 \quad \checkmark \quad \text{CA}$$

$$(n-20)(5n + 63) = 0 \quad \checkmark \quad \text{CA}$$

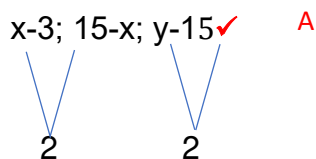
$$n=20 \quad \checkmark \quad \text{CA}$$

$$\text{or } n = -\frac{63}{5} \text{ (reject)} \quad \checkmark \quad \text{CA}$$

- (c) 3 ; x ; 15 ; y form a quadratic number pattern.
If the 2nd common difference is 2, determine the value of x and y.

(5)

3; x; 15; y



$$15 - x - (x - 3) = 2 ✓$$

CA

$$15 - x - x + 3 = 2$$

$$-2x = -16$$

$$x = 8; ✓$$

CA

$$y - 15 - (15 - 8) = 2; ✓$$

CA

$$y - 15 - 15 + 8 = 2;$$

$$y = 24 ✓$$

CA

QUESTION 4

[14 marks]

(a) Given: $m(x) = x^2 - 3x$

(1) Determine $m'(x)$ using first principles. (5)

$$m(x) = x^2 - 3x$$

Determine $m'(x)$ using first principles.

$$\begin{aligned} m(x+h) &= (x+h)^2 - 3(x+h) \\ &= x^2 + 2xh + h^2 - 3x - 3h \quad \checkmark \end{aligned}$$

$$\begin{aligned} m'(x) &= \lim_{h \rightarrow 0} \frac{m(x+h) - m(x)}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h} \quad \checkmark \text{sub} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h} \\ &= 2x + 0 - 3 \\ &= 2x - 3 \quad \checkmark \end{aligned}$$

(2) hence, determine the gradient of the tangent at $x = -2$. (1)

$$\begin{aligned} m'(-2) &= 2(-2) - 3 \quad \checkmark \\ &= -7 \end{aligned}$$

- (b) (1) Determine $f'(x)$ if $f(x) = \left(\frac{3}{\sqrt{x}} + \frac{\sqrt{x}}{3}\right)^2$ (4)

$$\begin{aligned} f(x) &= \left(\frac{3}{\sqrt{x}} + \frac{\sqrt{x}}{3}\right)^2 \\ &= \frac{9}{x} + 2 + \frac{x}{9} \quad \checkmark \checkmark \\ &= 9x^{-1} + 2 + \frac{1}{9}x \end{aligned}$$

$$f'(x) = -9x^{-2} + \frac{1}{9} \quad \checkmark \checkmark$$

- (2) Determine $\frac{dy}{dx}$ if $y + xy = x^2 - 1$ (4)

$$\begin{aligned} y + xy &= x^2 - 1 \\ y(1+x) &= x^2 - 1 \quad \checkmark \\ y &= \frac{x^2 - 1}{1+x} \quad \checkmark \\ y &= \frac{(x-1)(x+1)}{1+x} \\ y &= x-1 \quad \checkmark \end{aligned}$$

$$\frac{dy}{dx} = 1 \quad \checkmark$$

QUESTION 5

[4 marks]

Zinhle plans to save R350 000 towards buying a new car in 3 years' time. She makes 3 equal deposits at the end of each year into a savings account. The interest paid on the money in the savings account is 11% p.a. compounded quarterly.

Calculate how much money she will need to deposit on each occasion. (4)

$$\begin{aligned} x \left(1 + \frac{11\%}{4}\right)^8 + x \left(1 + \frac{11\%}{4}\right)^4 + x &= 350\,000 \\ x &= R104\,259,70 \quad \checkmark \end{aligned}$$

74 marks

QUESTION 6

[19 marks]

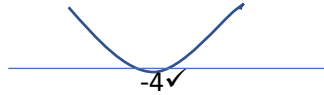
(a) Given $f(x) = x^2 + 8x + 16$ (1) Solve for x for which $f(x) > 0$

(3)

$$x^2 + 8x + 16 > 0$$

$$(x + 4)^2 > 0 \checkmark$$

$$x \in \mathbb{R} - \{-4\} \checkmark$$

(2) For which value(s) of p will $f(x) = p$ have two unequal negative roots?

(3)

$$0 < p < 16$$

✓

✓✓

(b) Consider the geometric sequence: $2(2x-1)$; $(2x-1)^2$; $\frac{1}{2}(2x-1)^3$ where $x \neq \frac{1}{2}$.(1) Calculate the value of x for which the sequence converges.

(4)

$$r = \frac{(2x-1)^2}{2(2x-1)} = \frac{1}{2}(2x-1) \checkmark$$

$$\text{FOR CONVERGENCE: } -1 < r < 1 \checkmark$$

$$-1 < \frac{1}{2}(2x-1) < 1 \checkmark$$

$$-2 < 2x-1 < 2$$

$$-1 < 2x < 3$$

$$-\frac{1}{2} < x < \frac{3}{2} \checkmark$$

- (2) Calculate the sum to infinity of the sequence if $x = \frac{1}{4}$. (3)

$$\begin{aligned}
 S_{\infty} &= \frac{a}{1-r} \checkmark & A \\
 &= \frac{-1}{1+\frac{1}{4}} \checkmark & CA \\
 &= -\frac{4}{5} \checkmark & CA
 \end{aligned}$$

- (c) Consider the following sequence of numbers:

2 ; 10 ; 2 ; 14 ; 2 ; 18 ; 2 ; 22 ;

- (1) Write down the next TWO terms of the sequence, given that the pattern continues. (2)

2; 26 ✓✓

- (2) Calculate the sum of the first 100 terms of the sequence. (4)

$$\begin{aligned}
 50 \times 2 &= 100 \checkmark & A \\
 S_n &= \frac{50}{2} [2(10) + 49(4)] \checkmark & A \\
 &= 5400 \checkmark & A \\
 \text{Sum} &= 100 + 5400 = 5\,500 \checkmark & CA
 \end{aligned}$$

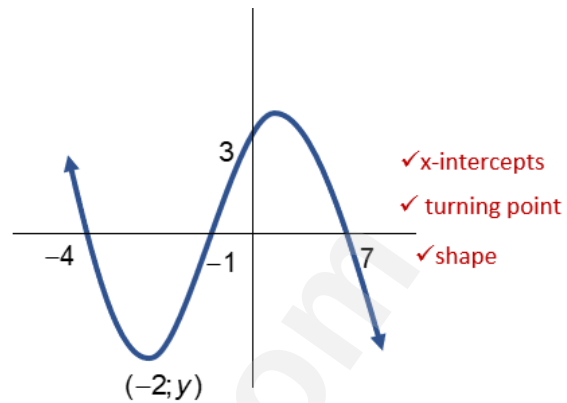
QUESTION 7

[22 marks]

- (a) Consider the function $f(x) = ax^3 + bx^2 + cx + d$.

Draw a rough sketch graph of f where

- $a < 0$
- $d = 3$
- $f(-4) = f(7) = f(-1) = 0$
- $f'(-2) = f'(4) = 0$



- (b) Calculate the values of t such that the line $y = 3x + t$ is a tangent to the curve $g(x) = -x^3 + x^2 + 8x - 12$.

$$-3x^2 + 2x + 8 = 3 \quad \checkmark \checkmark$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0 \quad \checkmark$$

$$x = \frac{5}{3} \quad \text{or} \quad x = -1 \quad \checkmark$$

$$y = -\frac{14}{27} \quad \text{or} \quad y = -18 \quad \checkmark$$

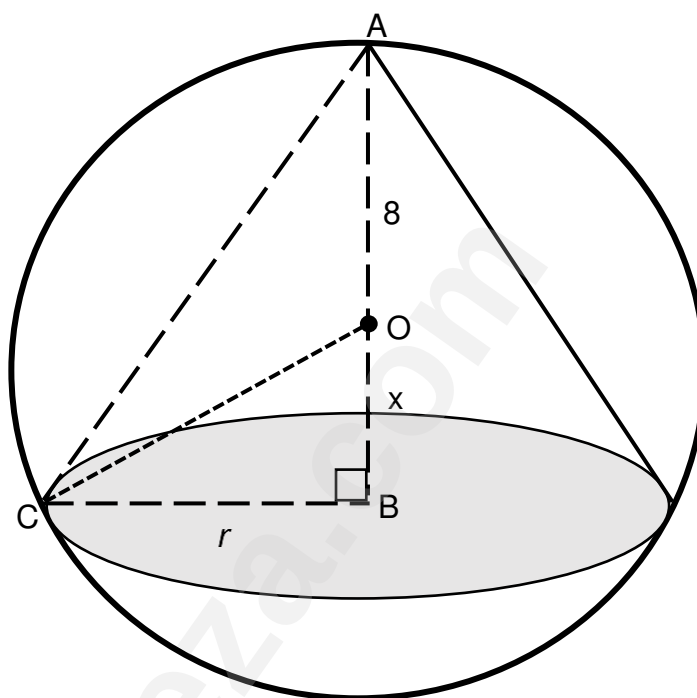
$$t = -\frac{14}{27} - 3\left(\frac{5}{3}\right) = -\frac{149}{27} \quad \checkmark$$

or $t = -18 - 3(-1) = -15 \quad \checkmark$

- (c) A cone with radius r cm and height AB is inscribed in a circle with centre O and a radius = 8cm. $OB = x$.

$$\text{volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{volume of a sphere} = \frac{4}{3}\pi r^3$$



- (1) Calculate the volume of the sphere. (1)

$$V = \frac{4}{3}\pi(8)^3 = \frac{2048}{3}\pi \quad \checkmark$$

- (2) Show that $r^2 = 64 - x^2$ (1)

$$OC = OA = 8\text{cm} \quad (\text{radii})$$

$$r^2 = 8^2 - x^2 \quad (\text{Thm of Pythag}) \quad \checkmark$$

$$r^2 = 64 - x^2$$

(3) Determine the value of x for which the volume of the cone is a maximum.

(9)

$$V = \frac{1}{3}\pi r^2 h$$

$$V(x) = \frac{1}{3}\pi(64 - x^2)(x + 8) \quad \checkmark \quad \checkmark$$

$$V(x) = \frac{1}{3}\pi(64x + 512 - x^3 - 8x^2) \quad \checkmark$$

$$V(x) = \frac{64}{3}\pi x + \frac{512}{3}\pi - \frac{1}{3}\pi x^3 - \frac{8}{3}\pi x^2$$

$$V'(x) = 0 \quad \checkmark$$

$$\frac{64}{3}\pi - \pi x^2 - \frac{16}{3}\pi x = 0 \quad \checkmark \quad \checkmark$$

$$\frac{64}{3} - x^2 - \frac{16}{3}x = 0$$

$$3x^2 + 16x - 64 = 0 \quad \checkmark$$

$$(3x - 8)(x + 8) = 0 \quad \checkmark$$

$$x = \frac{8}{3} \quad \checkmark \quad \text{or} \quad x \neq -8$$

- (a) Justin invests a certain amount of money (P) at an interest rate of $i\%$ p.a on a straight line basis. After two years, his investment grows to R17 024 and after six years, his investment grows to R25 472.

Determine P, the initial amount invested.

$$\frac{25\,472 - 17\,024}{4} = 2112 \quad \checkmark\checkmark$$

$$y = 2112x + P$$

$$17\,024 = 2112(2) + P \quad \checkmark$$

$$P = R12\,800 \quad \checkmark$$

- (b) Mr Govender and Mrs Williams have both taken loans from Absa Bank. Payments are made consistently after one month of granting of the loan. The details are tabulated below.

	Mr Govender	Mrs Williams
Loan Amount	R75 000	P
Monthly payment	R1 200	R 1 180,42
Interest rate	15% p.a compounded monthly	13% p.a compounded monthly
Balance outstanding after 62 months.	K	K

If the interest rates and monthly payments remain constant then determine:

- (1) K, the balance outstanding on Mr Govender's loan after 62 months. (5)

$\checkmark\checkmark$

$$\begin{aligned} \text{Balance outstanding} &= 75000 \left(1 + \frac{15\%}{12}\right)^{62} - \frac{1\,200 \left[\left(1 + \frac{15\%}{12}\right)^{62} - 1\right]}{\frac{15\%}{12}} \quad \checkmark\checkmark \\ &= R50\,636,01 \quad \checkmark \end{aligned}$$

- (2) Immediately after 62 months, Mrs Williams received the following summary from ABSA Bank.

Balance Outstanding	R50 636
Total interest paid	R44 764

Using this information, calculate P, the initial amount that Mrs Williams borrowed from ABSA Bank. (5)

✓✓

$$62 \times 1180,42 - (79057 - 50636) \quad \checkmark\checkmark$$

$$= R44\,764,10 \quad \checkmark$$

- (a) An electrical company manufactures capacitors. Three machines, Alpha, Beta and Gamma are used to manufacture the different components of the capacitor. They produce 20%, 30% and 50% respectively of the total production. 1%, 2% and 6% respectively of the capacitors produced by the three machines are defective.

- (1) At the quality control centre, a capacitor is picked up at random.

What is the probability that it was produced by the Beta machine and it is not defective? (2)

$$P(B,ND) = 0,3 \times 0,98 \checkmark$$

$$= 0,29 \checkmark$$

- (2) What is the probability that the capacitor is defective? (4)

$$\begin{aligned} P(D) &= P(A,D) \times P(B,D) \times P(G,D) \\ &= (0,2 \times 0,01) + (0,3 \times 0,02) + (0,5 \times 0,06) \\ &= 0,04 \checkmark \quad \text{CA} \end{aligned}$$

- (b) Consider the letters of the word “**EMBEZZLEMENTS**”

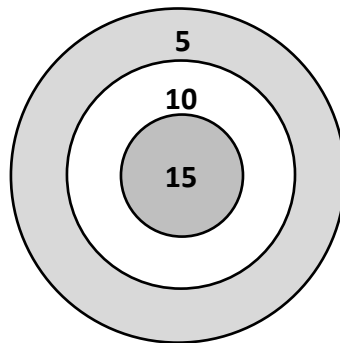
How many different 13 letter arrangements are possible if the letters **B**, **T** and **S** follow each other in any order. (4)

✓ ✓

$$\frac{3! \times 11!}{4! \times 2! \times 2!} = 2494800 \checkmark$$

✓

- (c) Dan is playing a game where a dart is thrown at a board with regions as shown. The outcomes of each throw are all independent of each other.



The probabilities of scoring in the different regions are indicated in the table:

15	10	5	0 (miss)
x	0,15	0,3	0,45

If 5 darts are thrown, what is the probability that **at least one** of the darts scores a “15”?

Give your answer to 3 decimal places.

(4)

$$x = 0,1 \quad \checkmark$$

$$\begin{aligned} P(\text{at least one of the darts scores a “15”}) &= 1 - (0,9)^5 \quad \checkmark\checkmark \\ &= 0,410 \quad \checkmark \end{aligned}$$

QUESTION 10

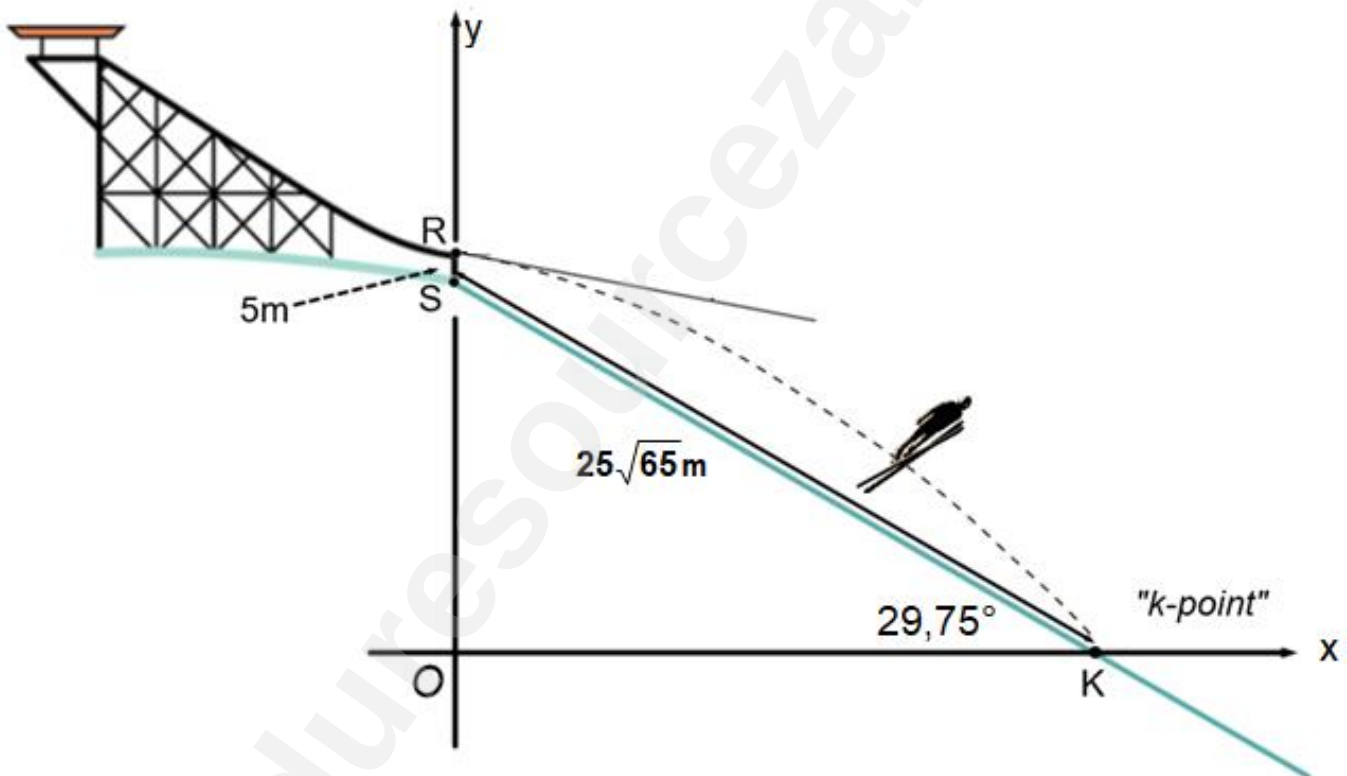
[8 marks]

Diego-the-Daredevil has spent his life pursuing his dream of participating in the Winter Olympics. His favourite event is the Ski-jump, so he does some research into the ramp design, as well as the mechanics and motion of the jump. During his research, Diego discovers that, contrary to popular belief, the ramp does not go up at the end.

The data suggests that:

- The trajectory of the skier, in flight, is parabolic in nature, $y = ax^2 + bx + c$; $x \geq 0$.
- Skiers land at the designated "k-point" $25\sqrt{65}$ metres down the slope.
- The slope is inclined at $29,75^\circ$ to the horizontal.
- There is a vertical drop of 5m from the end of the ramp to the top of the slope.
- The tangent to the curve at R is $y = -0,25x + k$ and $RS = 5m$.

Find an equation which models the skier's trajectory.



$$\sin 29,75^\circ = \frac{OS}{25\sqrt{65}} \checkmark \therefore OS = 100 \text{ m}$$

$$OR = 105 \text{ m} \checkmark$$

$$\cos 29,75^\circ = \frac{OK}{25\sqrt{65}} \checkmark \therefore OK = 175 \text{ m}$$

$$K(175 ; 0) \checkmark$$

$$y = ax^2 + bx + 105$$

$$2ax + b = -0,25 \quad \checkmark$$

$$2a(0) + b = -0,25$$

$$b = -0,25 \quad \checkmark$$

$$y = ax^2 - 0,25x + 105$$

$$0 = a(175)^2 - 0,25(175) + 105 \quad \checkmark$$

$$a = -0,002 \quad \checkmark$$

$$y = -0,002x^2 - 0,25x + 105$$

76 marks