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PRELIMINARY EXAMINATION

MATHEMATICS PAPER 2

SEPTEMBER 2020

MARKS : 150

DURATION : 3 HOURS

Name of Candidate	MEMO
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Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
7	17	22	6	9	8	8	7	13	6	11	13	10	13	150

INSTRUCTIONS

1. This question paper consists of 30 pages and an Information Sheet of 2 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all questions on the question paper.
4. Extra space is provided at the end of the paper, should this be necessary.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator.
7. Round off to one decimal place, unless specified otherwise.
8. Ensure that your calculator is in DEGREE mode.
9. All necessary working details must be clearly shown.
10. Give reasons for statements, unless specified otherwise.

SECTION A

QUESTION 1

The owner of *Pizza Perfect* uses the following set of data to illustrate the relationship between the number of people who visited two different restaurants over a period of 6 days.

Dino's Pizzeria (x)	12	14	17	21	26	30
Papachinos (y)	60	70	70	100	100	120

Round your answers off to two decimal places.

- (1) Determine the equation of the least squares line for the given data. (2)

$$y = 3,2x + 22,43$$

- (2) Calculate the correlation coefficient. (1)

$$0,96 \approx 1$$

- (3) Describe the strength of the relationship between the number of visits to Dino's and Papachinos. (2)

Very strong positive almost perfect

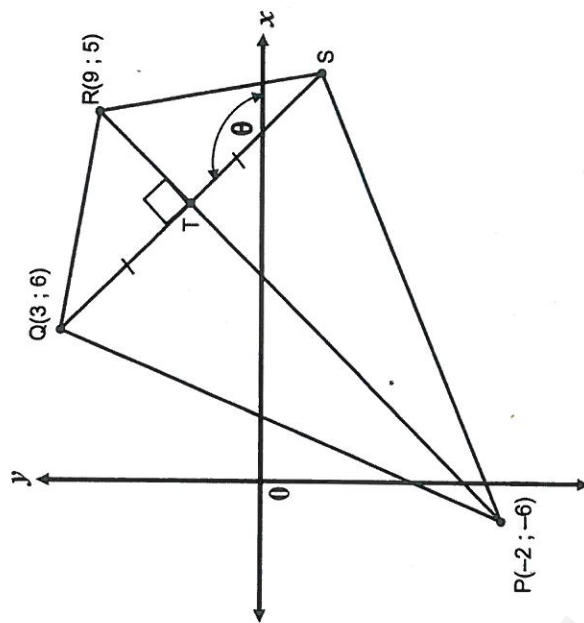
- (4) Predict the number of visits to Papachinos when there are 25 visits to Dino's. (2)

$$y = 102,7 \text{ (calc)} \quad \text{or} \quad y = 3,2(25) + 22,43 \\ \approx 103 \text{ visits} \quad \approx 102,4 \quad \approx 103 \text{ visits}$$

7 MARKS

QUESTION 2

Quadrilateral PQRS, a kite, is given with $QT = TS$ and $QS \perp PR$. $P(-2; -6)$, $Q(3; 6)$, $R(9; 5)$ and S are the vertices of the quadrilateral.



- (a) Calculate the length of PR and give your answer in simplest surd form. (2)

$$PR = \sqrt{(9+2)^2 + (5+6)^2} \\ = \sqrt{242} \\ = 11\sqrt{2}$$

(b) Determine the equation of PR.

(4)

$$\text{grad PR} = \frac{11}{11} \quad \checkmark$$

$$= 1 \quad \checkmark$$

$$\text{eqn PR: } y - 5 = (x - 9) \quad \checkmark$$

$$y = x - 4 \quad \checkmark$$

(c) Determine the equation of QS.

(3)

$$\text{grad QS} = -1 \quad (\text{perp lines}) \quad \checkmark$$

$$\text{eqn QS: } y - 6 = -(x - 3) \quad \checkmark$$

$$y = -x + 9 \quad \checkmark$$

(d) Determine the coordinates of T.

(4)

$$y = x - 4 \quad \dots \quad ①$$

$$y = -x + 9 \quad \dots \quad ②$$

$$① - ② \quad x - 4 = -x + 9 \quad \checkmark$$

$$2x = 13$$

$$x = \frac{13}{2} \quad \checkmark$$

$$y = \frac{5}{2} \quad \checkmark$$

$$T \left(\frac{13}{2}, \frac{5}{2} \right)$$

(e) Determine the co-ordinates of S.

(2)

$$\text{midpt QS} = \left(\frac{3+x_s}{2}, \frac{6+y_s}{2} \right)$$

$$\left(\frac{13}{2}, \frac{5}{2} \right) = \left(\frac{3+x_s}{2}, \frac{6+y_s}{2} \right)$$

$$\frac{13}{2} = \frac{3+x_s}{2} \quad \text{and} \quad \frac{5}{2} = \frac{6+y_s}{2}$$

$$x_s = 10 \quad \checkmark \quad y_s = -1$$

$$S(10, -1)$$

(f) Determine θ , the angle of inclination of line QS.

(2)

$$\tan \theta = -1 \quad \checkmark$$

$$\therefore \theta = 135^\circ \quad \checkmark$$

17 MARKS

QUESTION 3

Do NOT use a calculator to answer this question.

(a) Evaluate, without a calculator:

$$(1) \cos 69^\circ \cos 9^\circ + \sin 9^\circ \sin 69^\circ \quad (4)$$

$$= \cos 69^\circ \cos 9^\circ + \sin 9^\circ \sin 69^\circ \quad \checkmark$$

$$= \cos (69^\circ - 9^\circ) \quad \checkmark$$

$$= \cos 60^\circ \quad \checkmark$$

$$= \frac{1}{2} \quad \checkmark$$

$$(2) \frac{\sin 290^\circ \cdot \tan(-45^\circ) \cdot \sin 160^\circ \cdot \sin(90^\circ + x)}{\cos(-x - 360^\circ) \cdot \cos 300^\circ \cdot \sin 140^\circ} \quad (9)$$

$$= \frac{(-\sin 70^\circ) (-\tan 45^\circ) (\sin 20^\circ) (\cos x)}{\cos [-(360^\circ + x)] (\cos 60^\circ) (\sin 40^\circ)} \quad \checkmark$$

$$= \frac{-(\cos 20^\circ) (-1) (\sin 20^\circ) \cos x}{(\cos x) (\frac{1}{2}) \sin 40^\circ} \quad \checkmark \text{ special ratios}$$

$$= \frac{\cos 20^\circ \sin 20^\circ}{\frac{1}{2} (2 \sin 20^\circ \cos 20^\circ)} \quad \checkmark$$

$$= 1 \quad \checkmark \checkmark$$

(b) If $\sin 25^\circ = p$, express each of the following in terms of p . $\checkmark$



$$(1) \cos 205^\circ \quad (3)$$

$$= -\cos 25^\circ \quad \checkmark$$

$$= -\sqrt{1-p^2} \quad \checkmark$$

$$(2) \cos 50^\circ \quad (2)$$

$$= 1 - 2\sin^2 25^\circ \quad \checkmark$$

$$= 1 - 2p^2 \quad \checkmark$$

$$(3) \sin 55^\circ \quad (4)$$

$$= \sin (25^\circ + 30^\circ) \quad \checkmark$$

$$= \sin 25^\circ \cos 30^\circ + \sin 30^\circ \cos 25^\circ \quad \checkmark$$

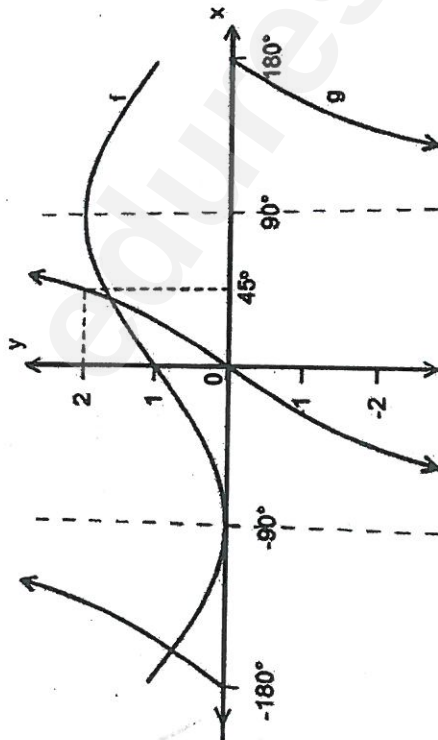
$$= p \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \sqrt{1-p^2} \quad \checkmark$$

$$= \frac{\sqrt{3}p + \sqrt{1-p^2}}{2} \quad \checkmark$$

22 MARKS

QUESTION 4

The graphs of $f(x) = a \sin x + q$ and $g(x) = b \tan x$ for the domain $x \in [-180^\circ; 180^\circ]$ are drawn below.



a) Give the values of a , q and b .

$$a = 1$$

$$q = 1$$

$$b = 2$$

b) State the period of g .

$$180^\circ$$

c) For which values of x will $f'(x) > 0$, given $x \in [-180^\circ; 180^\circ]$.

$$x \in (-90^\circ; 90^\circ)$$

interval

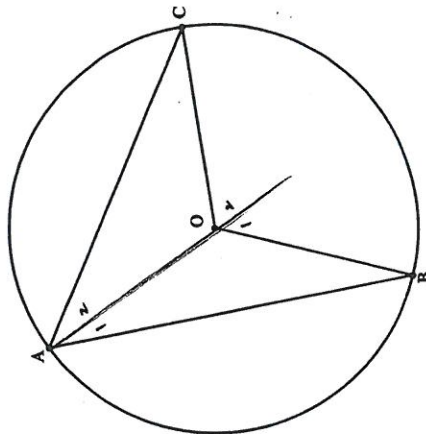
bracket

6 MARKS

QUESTION 5

(a) Using the diagram below, prove this theorem:

"The angle at the centre of circle O is double the angle at the circumference". (5)



Draw AO and extend ✓

$$\hat{O}_1 = \hat{B} + \hat{A}_1 \quad (\text{ext } \angle \text{ of } \triangle AOB) \quad \checkmark$$

But $OA = OB$ (radii)

$$\therefore \hat{A}_1 = \hat{B} \quad \checkmark \quad (\text{Isos } \triangle)$$

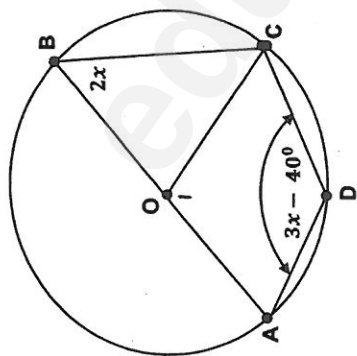
$$\therefore \hat{O}_1 = 2\hat{A}_1$$

$$\text{Similarly } \hat{O}_2 = 2\hat{A}_2 \quad \checkmark$$

$$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{A}_1 + \hat{A}_2) \quad \checkmark$$

$$\therefore \hat{BOC} = 2(\hat{A})$$

- (b) In the diagram below, O is the centre of the circle with diameter AB.



- (1) Find the measurement of reflex AOC in terms of x , without a reason. (1)

$$\text{Reflex } \angle AOC = 2(3x - 40^\circ) = 6x - 80^\circ \quad \checkmark$$

- (2) Find the numerical value of x . (3)

$$3x - 40^\circ + 2x = 180^\circ \quad \checkmark \quad (\text{opp } \angle \text{ s of cycl quad})$$

$$5x = 220^\circ$$

$$x = 44^\circ \quad \checkmark$$

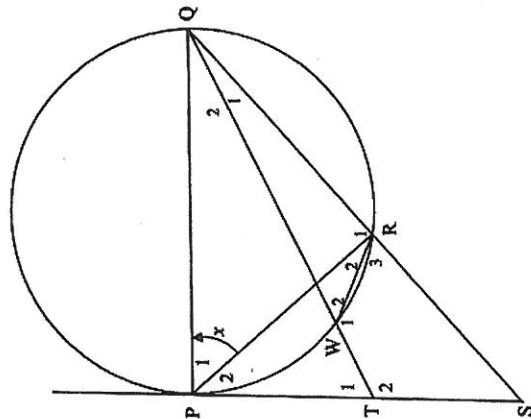
OR $\hat{O}_1 = 4x \quad \checkmark \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circ})$
 $4x + 6x - 80^\circ = 360^\circ \quad \checkmark \quad (\angle \text{ s round a pt})$
 $\therefore x = 44^\circ \quad \checkmark$

9 MARKS

QUESTION 6

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

Let $\hat{P}_1 = x$



- a) Why is $\hat{PRQ} = 90^\circ$ (1)

$\hat{R}_1 = 90^\circ$ ($\angle$ in a semi-circle) $\checkmark$

- b) Prove that $\hat{P}_1 = \hat{S}$ (2)

$$\hat{P}_1 + \hat{P}_2 = 90^\circ \quad (\text{diam } \perp \text{ tang})$$

$$\therefore \hat{P}_2 = 90^\circ - x$$

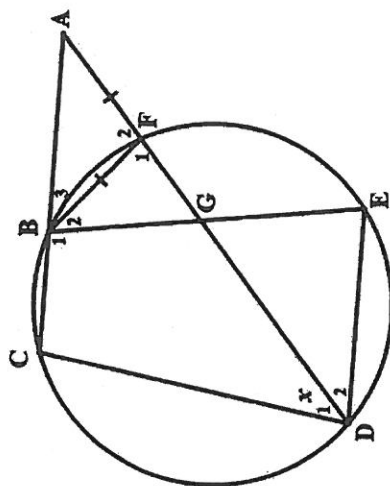
$$\hat{R}_2 + \hat{R}_3 = 90^\circ \quad (\text{adj } \angle \text{ s on str line})$$

$$\therefore \hat{S} = x \quad \checkmark \quad (\text{sum of } \angle \text{ s of } \triangle PSR)$$

$$\therefore \hat{P}_1 = \hat{S} = x$$

QUESTION 7

In the diagram B, C, D, E and F lie on the circle. AD bisects $\hat{B}$ and $AF = BF$. CB and DG produced meet at A. Let $\hat{D}_1 = x$.



Prove that:

(a) $\hat{B}_2 = \hat{B}_3$

(3)

$$\begin{aligned} \hat{D}_2 &= \hat{D}_1 = x && \text{(given)} \\ \hat{B}_2 &= \hat{D}_2 = x && (\angle s \text{ in same seg}) \\ \hat{B}_2 + \hat{B}_3 &= \hat{D} = 2x && (\text{ext } \angle \text{ of cyclic quad}) \\ \therefore \hat{B}_3 &= x && \\ \therefore \hat{B}_2 &= \hat{B}_3 && \end{aligned}$$

c) Prove that SRWT is a cyclic quadrilateral. (2)

$$\begin{aligned} \hat{W}_2 &= \hat{P}_1 = x && \checkmark (\angle s \text{ in same seg}) \\ \therefore \hat{W}_2 &= \hat{S} && (\text{both} = x) \\ \therefore \text{SRWT is a cyclic quad (ext } \angle = \text{int opp } \angle) &&& \checkmark \end{aligned}$$

d) Prove that $\Delta QWR \parallel \Delta QST$ (3)

In ΔQWR and ΔQST

- $\hat{Q}_1$ is common $\checkmark$
- $\hat{W}_2 = \hat{S}$ (proved) $\checkmark$
- $\hat{R}_1 + \hat{R}_2 = \hat{T}_2$ (3rd $\angle$ of Δ) $\checkmark$

$\therefore \Delta QWR \parallel \Delta QST (A, A, A)$

8 MARKS

(b) $DC = AC$ (2)

$$\hat{A} = \hat{B}_3 = x \quad (\text{Isos } \hat{A}) \quad \checkmark$$

$$\therefore \hat{A} = \hat{D}_1 \quad (\text{Both } = x) \quad \checkmark$$

$$\therefore DC = AC \quad (\text{sides opp eq } \angle s) \quad \checkmark$$

(c) $CDE = E$ (3)

$$\hat{E} = \hat{F}_1 \quad (\angle s \text{ in same seg}) \quad \checkmark$$

$$\hat{F}_1 = \hat{B}_3 + \hat{A} \quad (\text{ext } \angle \text{ of } \triangle BFA) \quad \checkmark$$

$$\therefore \hat{F}_1 = 2x$$

$$\therefore \hat{E} = 2x \quad \checkmark$$

$$CDE = 2x \quad (\text{given})$$

$$\therefore CDE = \hat{E}$$

8 MARKS

TOTAL SECTION A : 77 MARKS

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SECTION B

QUESTION 8

- (a) Which one of the following values is most influenced by extreme values :

mean ; median ; mode

(1)

Mean

- (b) A school announces in its media release:

'The Grade 12 pass rate has improved by a 100% of the pass rate from last year.'
Explain why this statement is misleading.

(2)

Pass rate from last year is unknown and
could have been very low. ✓

- (c) Data was collected to compare the length of time (x, in months) couples have been dating to the amount of money (y, in rands), spent when going out on a date.
The equation of regression was determined to be: $y = 165 - 6,3x$

- (1) Explain what the slope of the line tells us about the cost of an average date as the duration of the relationship increases.

(2)

As the duration of the relationship increases,
the cost of the date decreases. ✓

There is a negative gradient / neg relationship
between the two.

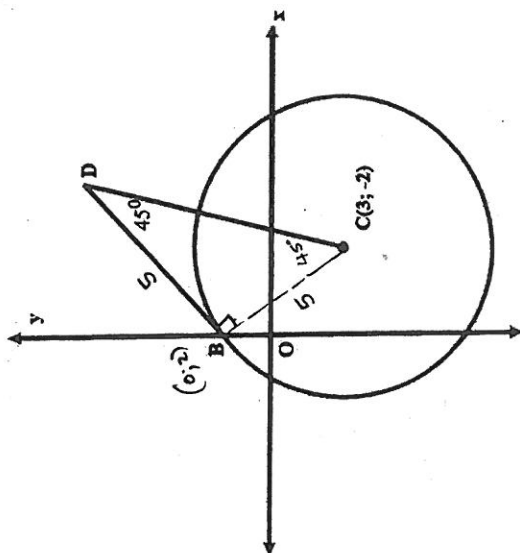
- (2) Can we use this line to predict the cost of a date in a 3 year long relationship?
Motivate your answer. (2)

3 years = 36 months
 $y = 165 - 6.3(36)$
 < 0
 No, as amount spent is negative

7 MARKS

QUESTION 9

In the diagram below, BD is a tangent to a circle at point B, which lies on the y -axis. The centre is $C(3; -2)$. The equation of the tangent BD is given by $3x - 4y + 8 = 0$. $\widehat{BDC} = 45^\circ$



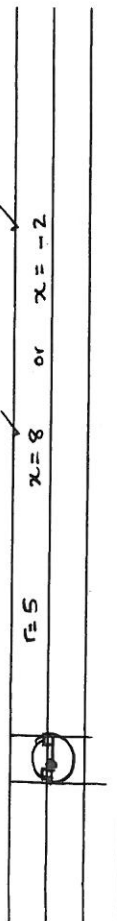
- (a) Determine the coordinates of B. (1)

$x_{int} : 3(0) - 4y + 8 = 0$
 $-y = 2 \quad B(0; -2) \quad \checkmark$

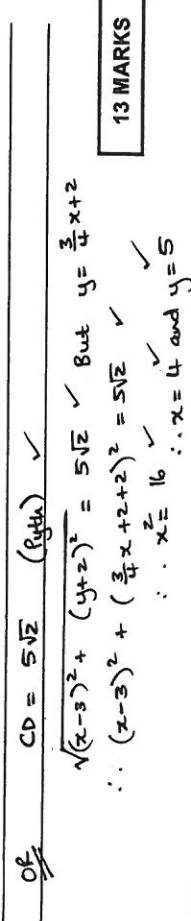
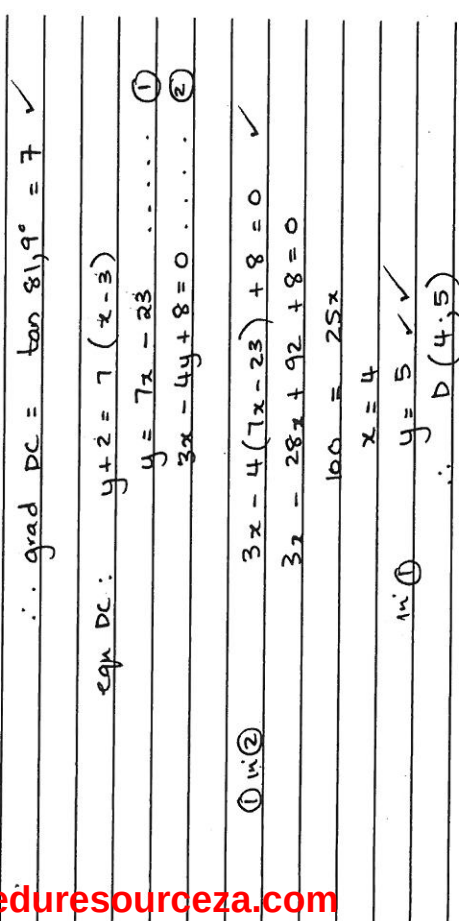
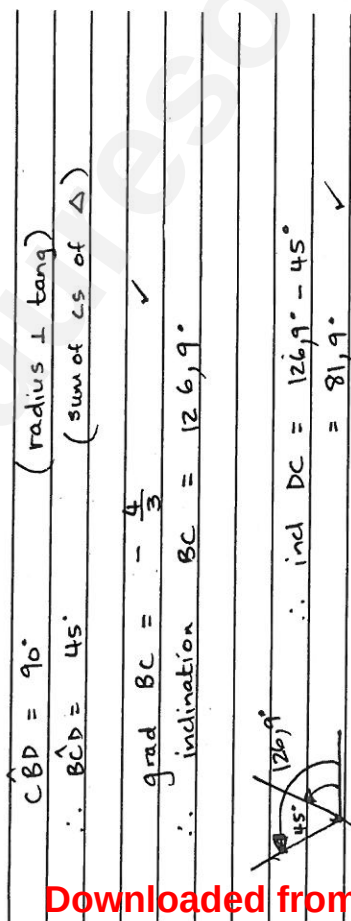
- (b) Show that the equation of the circle is $x^2 - 6x + y^2 + 4y - 12 = 0$ (4)

$(x-3)^2 + (y+2)^2 = r^2 \quad \checkmark$
 Subst $(0; -2) \quad \checkmark$
 $(0-3)^2 + (-2+2)^2 = r^2$
 $25 = r^2 \quad \checkmark$
 $\therefore (x-3)^2 + (y+2)^2 = 25 \quad \checkmark$
 $x^2 - 6x + y^2 + 4y - 12 = 0$

- (c) Find the equations of two tangents to the circle which are parallel to the y -axis. (2)



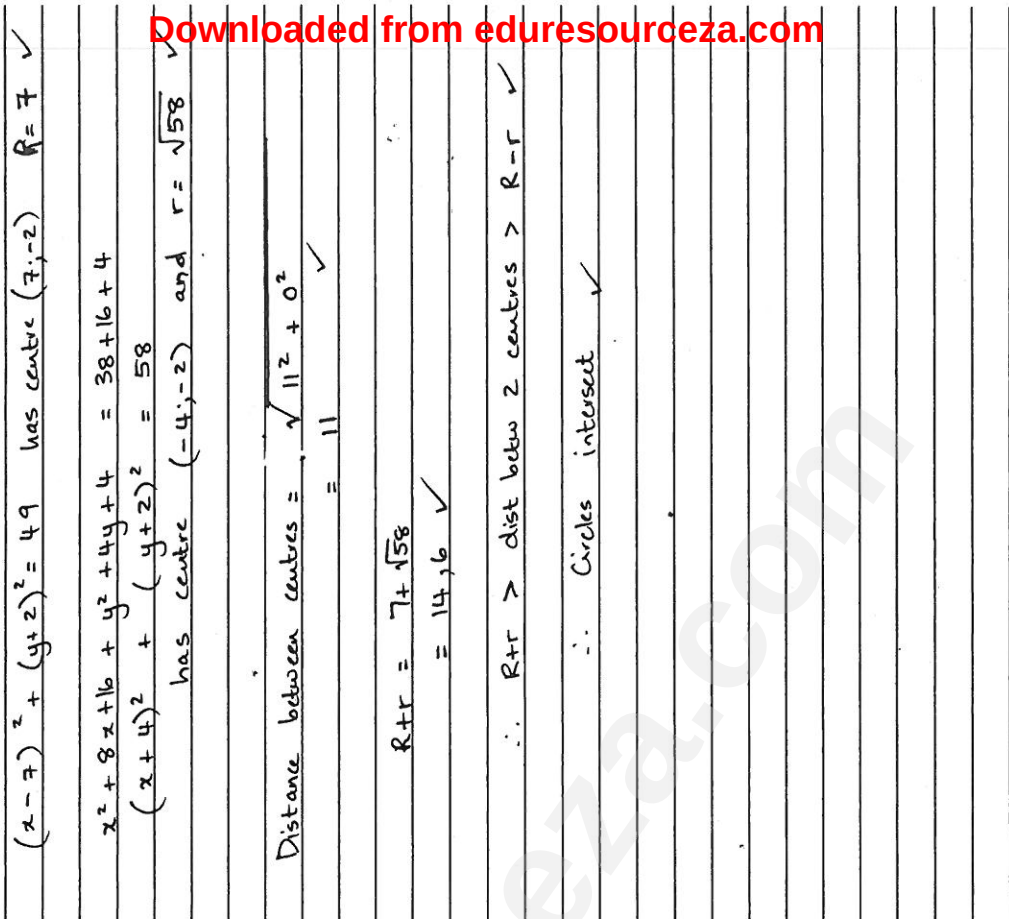
- (d) Determine the coordinates of D. (6)



QUESTION 10

Given: $(x-7)^2 + (y+2)^2 = 49$ and $x^2 + y^2 + 8x + 4y - 38 = 0$.

Using appropriate calculations, determine if the circles intersect.



6 MARKS

QUESTION 11

(a) Determine the general solution of the following equation:

$$\cos 6x = \sin 5x \cdot \cos x - \cos 5x \cdot \sin x \quad (6)$$

$$\begin{aligned} \cos 6x &= \sin(5x - x) \\ \cos 6x &= \sin 4x \quad \checkmark \\ \cos 6x &= \cos(90^\circ - 4x) \quad \checkmark \\ \therefore 6x &= 90^\circ - 4x + 360^\circ k \quad \checkmark \text{ or } 6x = -90^\circ + 4x + 360^\circ k \quad \checkmark \\ 10x &= 90^\circ + 360^\circ k \quad \checkmark \quad 2x = -90^\circ + 360^\circ k \quad \checkmark \\ x &= 9^\circ + 36^\circ k \quad \checkmark \quad x = -45^\circ + 180^\circ k \quad \checkmark \end{aligned}$$

($k \in \mathbb{Z}$)

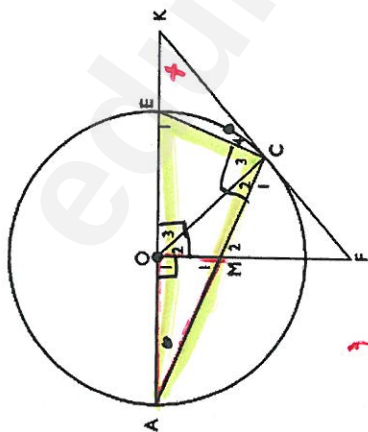
(b) Prove the following identity:

$$\frac{2 - \cos^2 \beta - 2 \sin \beta}{\cos^2 \beta} = \frac{1 - \sin \beta}{1 + \sin \beta} \quad (5)$$

$$\begin{aligned} \text{LHS} &= \frac{2 - (1 - \sin^2 \beta) - 2 \sin \beta}{1 - \sin^2 \beta} \quad \checkmark \\ &= \frac{\sin^2 \beta - 2 \sin \beta + 1}{1 - \sin^2 \beta} \quad \checkmark \\ &= \frac{(\sin \beta - 1)^2}{(1 - \sin \beta)(1 + \sin \beta)} \quad \checkmark \\ &= \frac{(1 - \sin \beta)^2}{(1 - \sin \beta)(1 + \sin \beta)} \\ &= \frac{1 - \sin \beta}{1 + \sin \beta} \\ &= \text{RHS} \end{aligned}$$

11 MARKS

FK is a tangent to the circle centre O.

 $FO \perp AK$ 

Prove that:

(a) $\Delta AOM \text{ /// } \Delta ACE$

$\hat{C}_2 + \hat{C}_3 = 90^\circ$ ✓ ($\angle$ in a semi-circle)
 $\hat{O}_1 = 90^\circ$ (given)

In ΔS POM and ACE

$$i) \hat{O}_1 = \hat{C}_2 + \hat{C}_3 = 90^\circ \quad (\text{proved})$$

ii) $\hat{A}$ is common ✓

$$\hat{M}_1 = \hat{E}_1 \quad \checkmark \quad (3^{\text{rd}} \text{ cof } \Delta)$$
$$\therefore \Delta AOM \parallel \Delta ACE$$

(b) $2AO^2 = AM \cdot AC$

(3)

$$\frac{AO}{AC} = \frac{OM}{CE} = \frac{AM}{AE} \quad \checkmark \text{ (equiangular } \Delta S)$$

$$AO \cdot AE = AM \cdot AC \quad \checkmark$$

But $AE = 2AO$ ✓

$$\therefore PO \cdot 2AO = AM \cdot AC$$

$$2AO^2 = AM \cdot AC$$

(c) $KC^2 = AK \cdot KE$

(9)

$$\hat{C}_4 = A \quad \checkmark \quad (\tan - \text{chord thm})$$

In ΔS KCE and KAC

k is common ✓

iii) $C_4 = A$ (proved)

$$\hat{E}_2 = \hat{C}_2 + \hat{C}_3 + \hat{C}_4 \quad (3^{\text{rd}} \text{ lot } \Delta) \quad \checkmark$$
$$\therefore \Delta KCE \equiv \Delta KAC \quad (A, A, A)$$

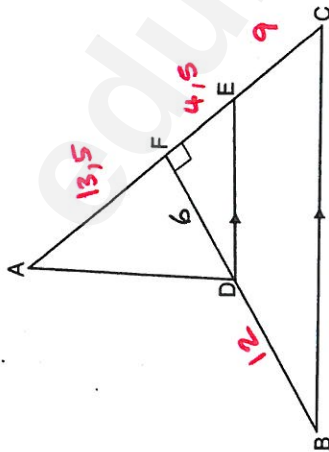
$$\therefore \underline{KC} = \underline{CE} = \underline{KE} \quad (\text{equilateral } \Delta S) \quad \checkmark$$

$$KC^2 = AK \cdot KE$$

13 MARKS

QUESTION 13

In the diagram below, F is the midpoint on AC with $BF \perp AC$. $DE \parallel BC$ with B on FD produced. $DF = 6$ cm with $DF:BF = 1:3$ and $EC = \frac{3}{2} DF$.



(a) Calculate the length of FE.

(5)

$$\frac{BF}{DF} = \frac{1}{3} \quad (\text{given})$$

$$\frac{6}{BF} = \frac{1}{3}$$

$$\therefore BF = 18 \quad \therefore BD = 12 \text{ cm}$$

$$\frac{FE}{EC} = \frac{FD}{DB} = \frac{1}{2} \quad \checkmark \quad (\text{line \parallel one side of } \triangle)$$

$$\text{But } \frac{EC}{6} = \frac{3}{2} \quad \checkmark$$

$$\therefore EC = 9 \text{ cm} \quad \checkmark$$

$$\therefore \frac{FE}{9} = \frac{1}{2}$$

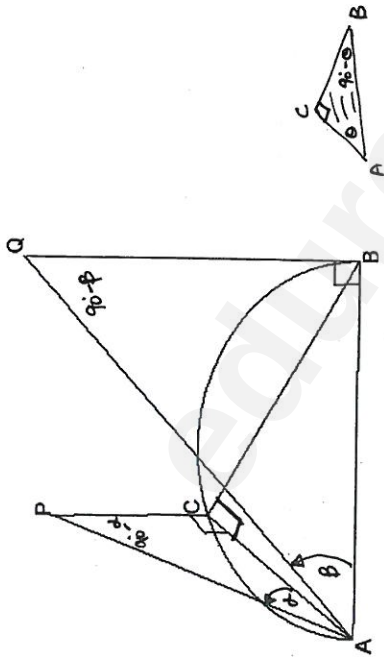
$$\therefore FE = 4,5 \text{ cm} \quad \checkmark$$

(b) Calculate $\frac{\text{Area of } \triangle EDF}{\text{Area of } \triangle ADE}$ (5)

$$\frac{\text{Area } \triangle EDF}{\text{Area } \triangle ADE} = \frac{\frac{1}{2} (4,5) (6)}{\frac{1}{2} (18) (6)} \quad \checkmark \checkmark$$

$$= \frac{1}{4} \quad \checkmark$$

QUESTION 14



AB is the diameter of a semi-circle lake.

Two perpendicular towers have been built on the banks (CP and BQ).

From A the angles of elevation of P and Q are α and β respectively. $\angle CAB = \theta$.

(a) Determine CA in terms of CP and α .

(2)

$$\frac{CP}{CA} = \tan \alpha \quad \checkmark \quad \text{OR} \quad \frac{CA}{\cos \alpha} = \frac{CP}{\sin \alpha}$$

$$\therefore CA = \frac{CP}{\tan \alpha} \quad \checkmark \quad \therefore CA = \frac{CP \cos \alpha}{\sin \alpha}$$

$$= \frac{CP}{\tan \alpha}$$

(b) Determine BQ in terms of AB and β .

(1)

$$\frac{BQ}{AB} = \tan \beta \quad \checkmark$$

$$\therefore BQ = AB \tan \beta \quad \checkmark$$

(c) Prove that $BQ = \frac{CP \tan \beta}{\tan \alpha \cos \theta}$. (5)

$$BQ = AB \tan \beta$$

$$\text{In } \triangle ACB, \quad \frac{AB}{\sin 90^\circ} = \frac{AC}{\sin(90^\circ - \theta)} \quad \checkmark$$

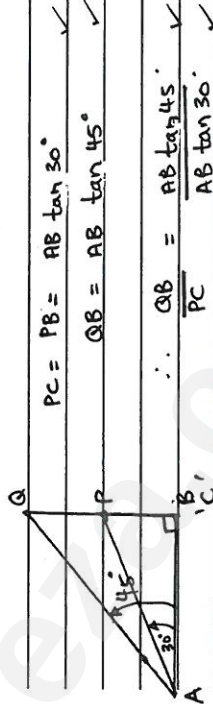
$$\therefore AB = \frac{AC}{\cos \theta} \quad \checkmark$$

$$\therefore BQ = \frac{AC}{\cos \theta} \cdot \tan \beta \quad \checkmark$$

$$= \frac{CP}{\cos \theta \tan \alpha} \cdot \tan \beta \quad \text{from (a)} \quad \checkmark$$

(d) If the point C moves along the semi-circle such that it co-incides with the point B

and $\beta = 45^\circ$ and $\alpha = 30^\circ$ determine the value of $\frac{QB}{PC}$. (5)



$$PC = PB = AB \tan 30^\circ \quad \checkmark$$

$$QB = AB \tan 45^\circ \quad \checkmark$$

$$\therefore \frac{QB}{PC} = \frac{AB \tan 45^\circ}{AB \tan 30^\circ} \quad \checkmark$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3} \quad \checkmark$$

13 MARKS

TOTAL SECTION B : 73 MARKS