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**CRAWFORD SCHOOLS PRELIMINARY EXAMINATION
SEPTEMBER 2020
MATHEMATICS: PAPER I**

Time: 3 hours

150 marks

NAME:

MARKER TO ENTER MARKS:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
16	11	9	11	18	7	10	10	15	10	9	15	9	150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 27 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete. Spare pages have been left for rough/extra working.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
8. It is in your own interest to write legibly and to present your work neatly.
9. Round off to two decimal places unless otherwise stated.

SECTION A

QUESTION 1

[16 Marks]

(a) Solve for x if:

(1) $8x^2 + 1 = 6x$ (3)

(2) $\sqrt{x^3} = 512$ (3)

(3) $7^{x+1} = 5,8$, correct to one decimal place. (3)

(4) $x^2(x+2) \leq 0$

(3)

(b) It is given that: $f(x) = x^3 - x^2 - 10x - 8$ (1) Prove that $(x+1)$ is a factor of $f(x)$.

(2)

(2) Hence, factorize $f(x)$ fully.

(2)

QUESTION 2

[11 Marks]

- (a) Calculate: $\sum_{n=3}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$ (4)

- (b) Matches are used to make the pattern below:

Diagram 1

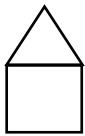


Diagram 2

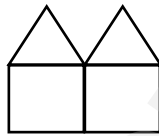
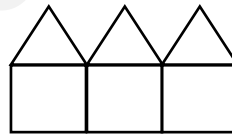


Diagram 3



- (1) If this pattern continues, calculate the maximum number of houses that can be formed in a row if a total of 196 matches are used. (5)

- (2) Explain why all the terms in this sequence will have a units digit of 1 or 6. (2)

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QUESTION 3

[9 Marks]

- (a) Determine the number of years it would take for money to treble in value if interest is 11, 2% per annum, compounded weekly.
(there are 52 weeks in a year)

(5)

- (b) Susan wishes to accumulate R500 000 by her 50th birthday.
She intends to pay equal payments into a savings account which pays an interest rate of 8% p.a. compounded monthly.
Payments start on her 25th birthday and end on her 50th birthday.
Determine how much she will need to pay into her account each month.

(4)

QUESTION 4

[11 Marks]

(a) If $f(x) = 3x - x^2$, determine $f'(x)$ from first principles.

(5)

(b) Determine $\frac{dy}{dx}$ in each of the following:

(1) $y = \frac{25x^2 - 9}{5x + 3}$ (3)

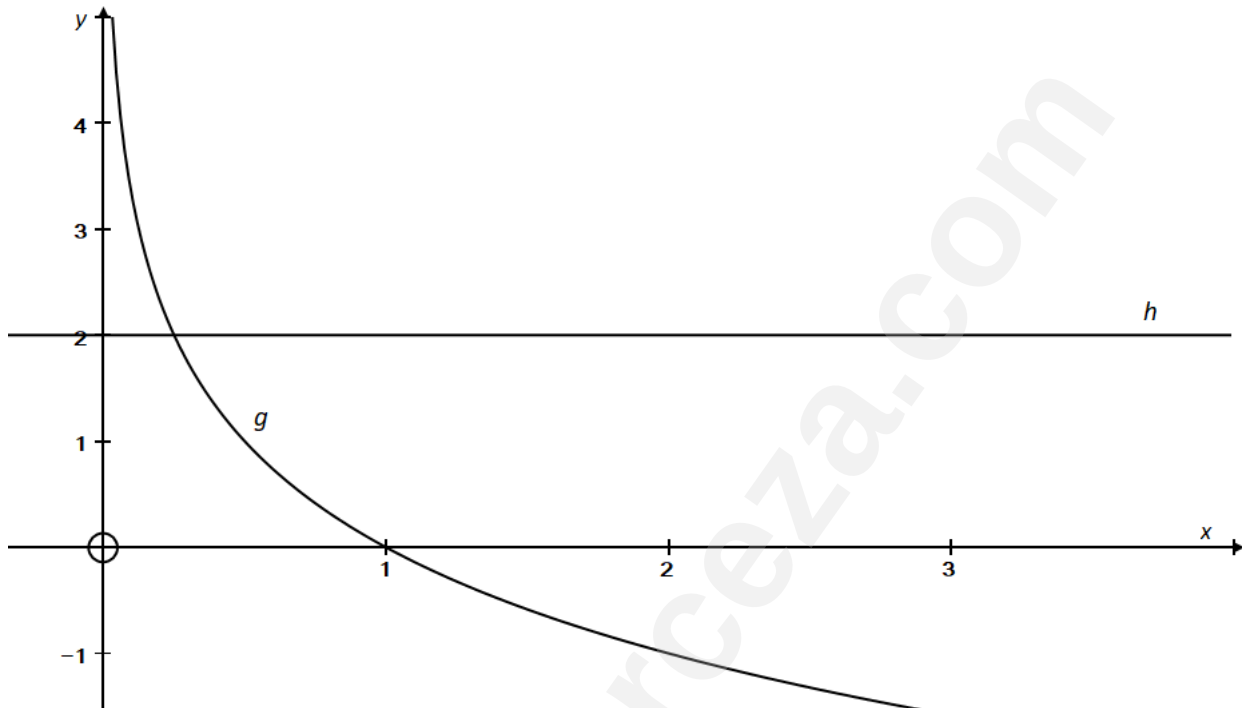
(2) $y = \frac{-8}{x} - 6\sqrt{x}$ (3)

QUESTION 5

[18 Marks]

- (a) The graphs of functions g and h with equations $g(x) = \log_{\frac{1}{2}} x$ and $h(x) = 2$

are sketched on the Cartesian plane as shown below.



- (1) Write down the domain of g . (1)

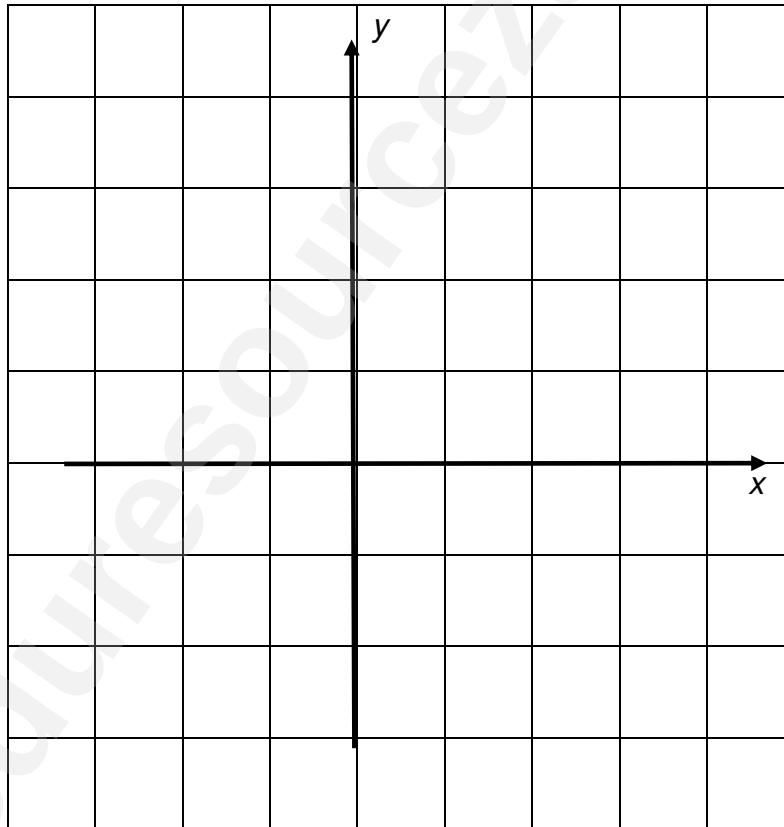
- (2) Calculate the value of x if $g(x) = h(x)$. (2)

- (3) The function f has equation $f(x) = \log_2 x$.

Describe the transformation that changes g to f .

(1)

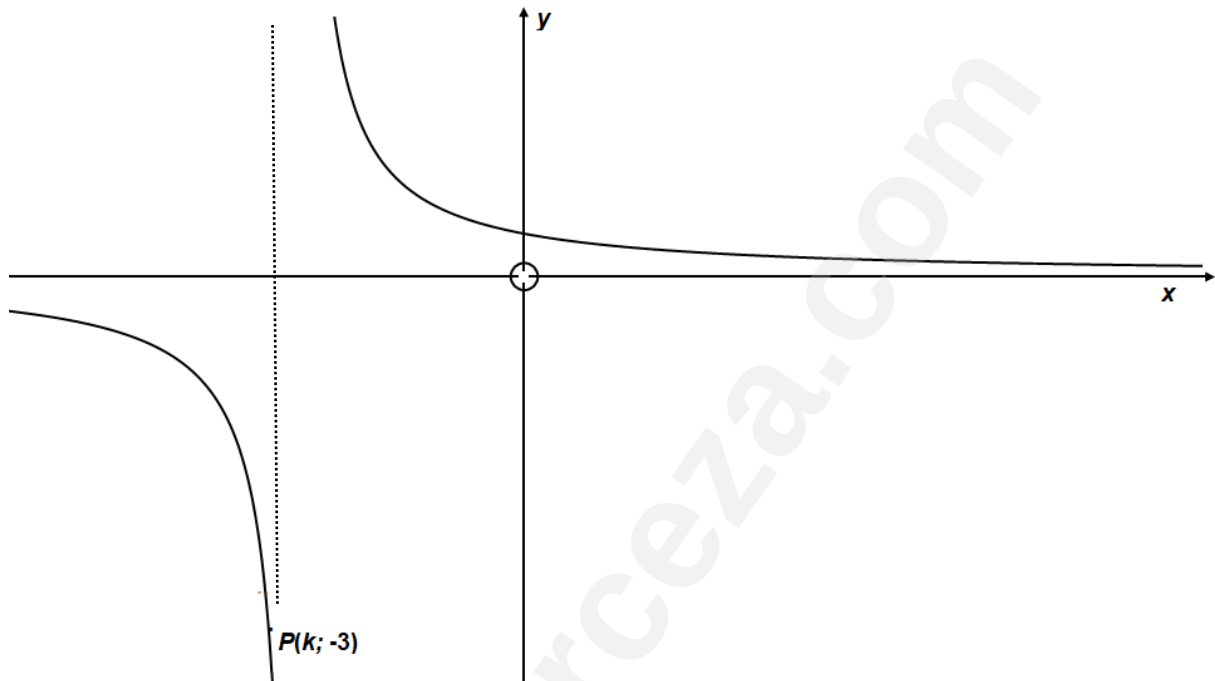
- (4) On the Cartesian plane below, sketch the graph of p if $p(x) = g^{-1}(x)$, the inverse of g . Clearly indicate the intercepts with the axes and one additional point.



(3)

- (b) The sketch represents the function of g with equation $g(x) = \frac{1}{x+1}$.

The point $P(k; -3)$ lies on the graph of g .



Determine:

- (1) the value of k , the x -coordinate of P . (3)

- (2) the equation of the axis of symmetry of g which has a negative gradient. (3)

- (3) the values of x for which $g(x) < 1$. (3)

- (4) determine the vertical and horizontal asymptotes of h ,
if $h(x) = g(-x) + 1$. (2)

QUESTION 6**[7 Marks]**

Each person from a sample of 140 males, chooses exactly one beverage from Pepsi (P), Coke (C) or Red Bull (R). The number of drinks chosen by two different age groups is shown below.

AGE DRINK	Number of males between ages 30 and 45 years (A)	Number of males between ages 45 and 60 (B)	TOTAL
PEPSI (P)	X	Y	84
COKE (C)		16	
RED BULL (R)	20		
	100	40	140

- (a) It is given that drinking Pepsi is independent of the two age groups.

Use this information to calculate the values of:

- (1) X (4)

(2) Y

(1)

- (b) Determine the probability that a male drinks Red Bull, given that he is aged between 45 and 60. (2)

SECTION B

QUESTION 7

[10 Marks]

(a) Given: $4^m = p(2^{2m-1}) + p$

(1) Show that for $p \neq 2$, the above equation can be written in the form:

$$2^{2m} = \frac{2p}{2-p} \quad (4)$$

(2) Use the answer in a (1) to calculate the following value(s):

$$m \text{ if } p = 1. \quad (2)$$

- (b) Calculate two values of k for which $x^3 - x^2 - x + k$ and $x^2 - 6x - 16$ will have a common factor.

(4)

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QUESTION 8

[10 Marks]

- (a) Calculate the value of n if $\sum_{k=0}^{19} 10 = \sum_{k=1}^n 2k - 10$ (5)

Source: www.source.com

- (b) Determine values of x for which the sequence below will converge:

$$2x-1; \quad \frac{-(2x-1)^2}{4}; \quad \frac{(2x-1)^3}{16}; \quad \dots\dots\dots \quad (5)$$

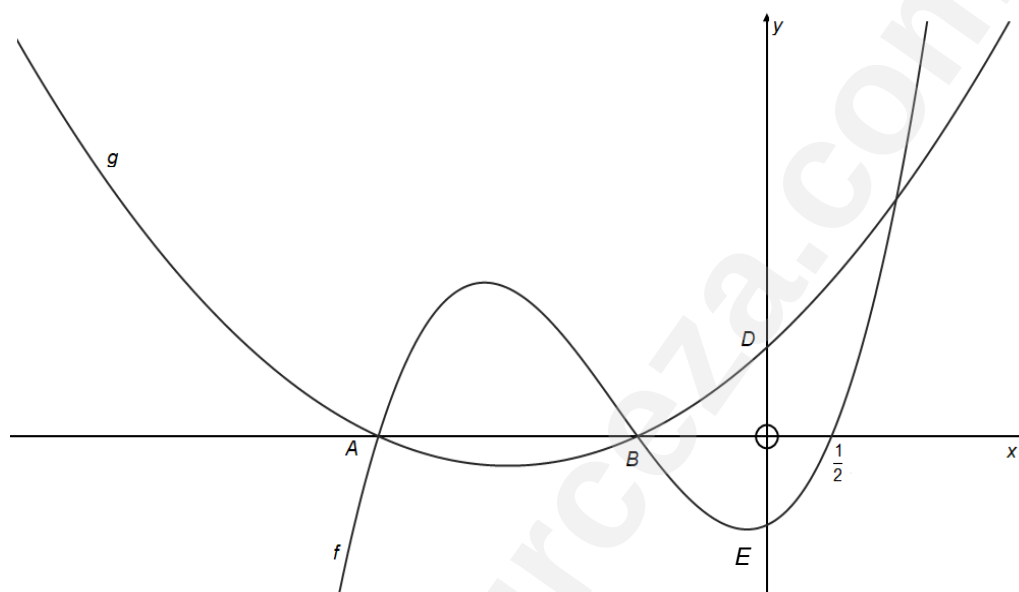
QUESTION 9

[15 Marks]

In the diagram below, the graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = x^2 + 4x + 3$ are given.

The two functions have x -intercepts at A and B and f has another x -intercept at $x = \frac{1}{2}$.

D and E are the y -intercepts and the length of $DE = 6$ units.



- (a) Show that $a = 2$, $b = 7$, $c = 2$ and $d = -3$. (5)

- (b) Determine the x -coordinate of the point of inflection of f . (3)

- (c) Find the gradient of the tangent to f at its point of inflection. (2)

- (d) Determine the value(s) of x , for which $f(x)$ and $g(x)$ will INCREASE, at the same rate. Leave your answer in simplest surd form. (3)

- (e) Determine the value(s) of t for which the equation
 $2(x+t)^3 + 7(x+t)^2 + 2(x+t) - 3 = (x+t)^2 + 4(x+t) + 3$ has three positive roots. (2)

QUESTION 10

[10 Marks]

A 30 year loan of R1 million is taken from a bank to buy a home. The interest charged on the loan is 8,4% p.a. compounded monthly. The repayments start one month after the loan was granted.

- (a) Calculate the monthly repayments. (4)

- (b) Calculate the balance outstanding on the loan after eight years. (3)

- (c) Calculate the amount of the loan that has been paid off **within** the 9th year? (4)

QUESTION 11

[9 Marks]

The letters of two words **SWEETS** and **SOUR** are used in a Scrabble game.

- (a) The letters of the word **SWEETS** are arranged to make 6-letter arrangements. Determine the possible number of arrangements that can be made so that the word starts with an E. (3)

- (b) Determine how many 10 letter arrangements can be created using the letters from both words. (3)

- (c) Hence, determine the probability of creating a 10-letter arrangement, using letters from both words, so that the three 'S'-letters and the two 'E'- letters are NOT next to each other. (3)

QUESTION 12

[15 Marks]

- (a) During a basketball match between the Los Angeles Lakers and the Toronto Raptors, Michael Jordan aimed at the hoop from a height of 2,2 meters.

The ball follows a parabolic route with equation, $h(t) = -t^2 + 2pt + p^2 + 1,2$ where $h(t)$ is the height (in meters) above the ground and t is the time (in seconds), where $p > 0$.



- (1) Show that $p = 1$.

(2)

(2) Determine the maximum height the ball reaches.

(4)

(3) Michael is successful in his attempt to shoot a goal. The ball, travelling in the given parabolic curve, reaches the hoop at the point where the parabola has a gradient of $-1,2$.

Calculate the height of the hoop at that point.

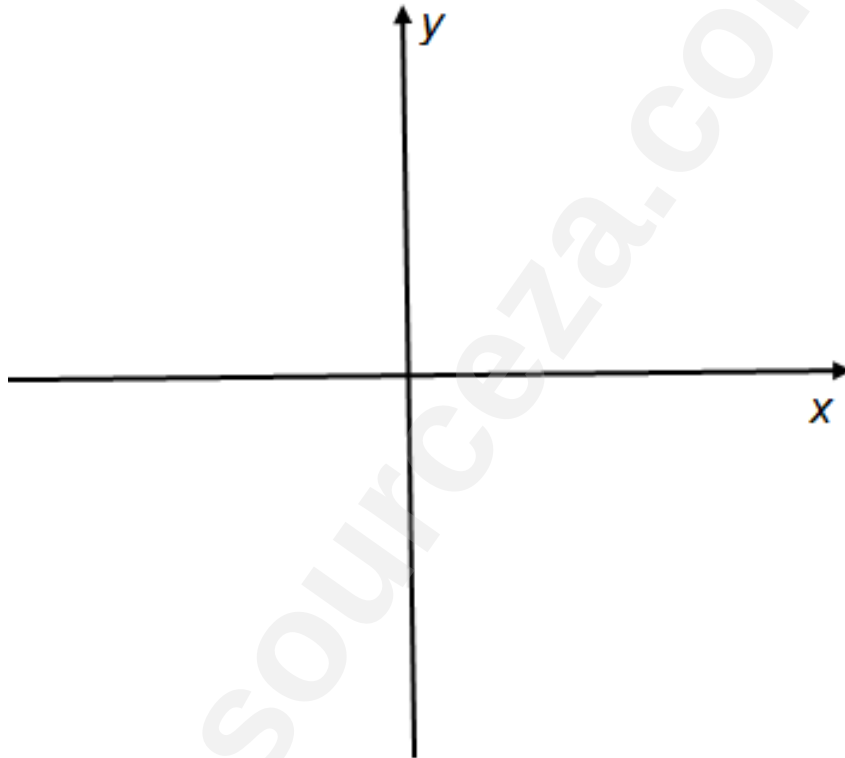
(5)

- (b) On the Cartesian plane below, draw a neat sketch of $f(x) = ax^2 + bx + c$ from the following information.

Show the x -intercepts and the turning point.

- The roots of f differ by 8 units
- $a = -b$
- The range of the function is $(-\infty; 4]$.

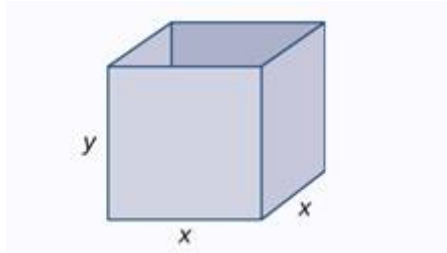
(4)



QUESTION 13

[9 Marks]

A rectangular box with a square base, an open top, and a volume of 216 cm^3 is to be constructed



The cost of the material for the base is R20 per cm^2 , and the cost of the material for the sides is R30 per cm^2

- (a) Show that the total cost of manufacturing the box, in terms of x , is

$$C(x) = 20x^2 + \frac{25920}{x} \quad (5)$$

- (b) Hence, determine the dimensions of the box so that the cost of manufacturing the box is a minimum. Round your answer off to TWO decimal places. (4)

EXTRA SPACE FOR WORKING

[illegible]

[illegible]