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VCAA MATHEMATICS**PAPER 1****2018****MEMORANDUM****QUESTION 1**

(a)

$$x(2x+5)=12$$

$$2x^2+5x-12=0 \quad \checkmark$$

$$(2x-3)(x+4)=0 \quad \checkmark$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-4 \quad \checkmark$$

(3)

(b)

$$81^{x-1} \cdot 27^x = 1$$

$$(3^4)^{x-1} \cdot 3^{3x} = 3^0 \quad \checkmark$$

$$3^{4x-4+3x} = 3^0 \quad \checkmark$$

$$7x-4 = 0$$

$$x = \frac{4}{7} \quad \checkmark$$

(3)

(c)

$$x^2+4x \geq -4$$

$$x^2+4x+4 \geq 0 \quad \checkmark$$

$$(x+2)^2 \geq 0 \quad \checkmark$$

$$\therefore x \in R \quad \checkmark$$

(3)

QUESTION 2

(a)

$$\text{common difference} = y - x$$

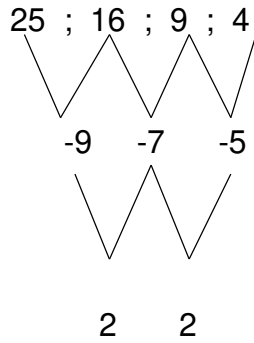
$$T_3 = T_2 + y - x$$

$$= y + y - x \quad \checkmark$$

$$= 2y - x \quad \checkmark$$

(2)

(b) (1)



$$\begin{array}{lll}
 2a = 2 & 3a + b = -9 & a + b + c = 25 \quad \checkmark \\
 a = 1 \quad \checkmark & 3 + b = -9 & 1 - 12 + c = 25 \\
 & b = -12 \quad \checkmark & c = 36 \quad \checkmark
 \end{array} \quad (4)$$

(b) (2)

$$D(t) = t^2 - 12t + 36$$

$$D(5) = (5)^2 - 12(5) + 36 = 1$$

$$D(6) = (6)^2 - 12(6) + 36 = 0$$

$$q = 1 \quad \checkmark \quad \text{and} \quad r = 0 \quad \checkmark$$

(2)

(b) (3)

As t approaches 6 - $D(t)$ is decreasing in value which show that the athlete is moving towards A. ✓ ie

$$D(1) = 25 \quad \checkmark$$

$$D(2) = 16$$

$$D(3) = 9 \text{ etc}$$

As t gets larger than 6 - $D(t)$ is increasing in value which shows that the athlete starts moving away from A. ie ✓

$$D(7) = 1 \quad \checkmark$$

$$D(8) = 4$$

(4)

$$D(9) = 9$$

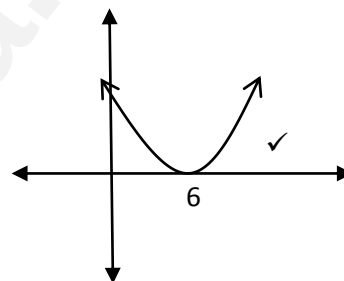
OR

$$T_n = (n-6)^2 \quad \checkmark$$

∴ minimum value at $n = 6$

for $0 < n < 6$; T_n is decreasing ✓

for $n > 6$; T_n is increasing ✓



QUESTION 3

(a) (1)

$$\text{domain: } x \in (-5 ; 4] \quad \checkmark \quad \checkmark$$

$$\text{range: } y \in \left(\frac{1}{2} ; 5 \right] \quad \checkmark \quad \checkmark$$

(4)

(2)

$$\text{maximum value is } -3 \quad \checkmark$$

(1)

(b) (1)

$$y = \log_5 x \quad \checkmark$$

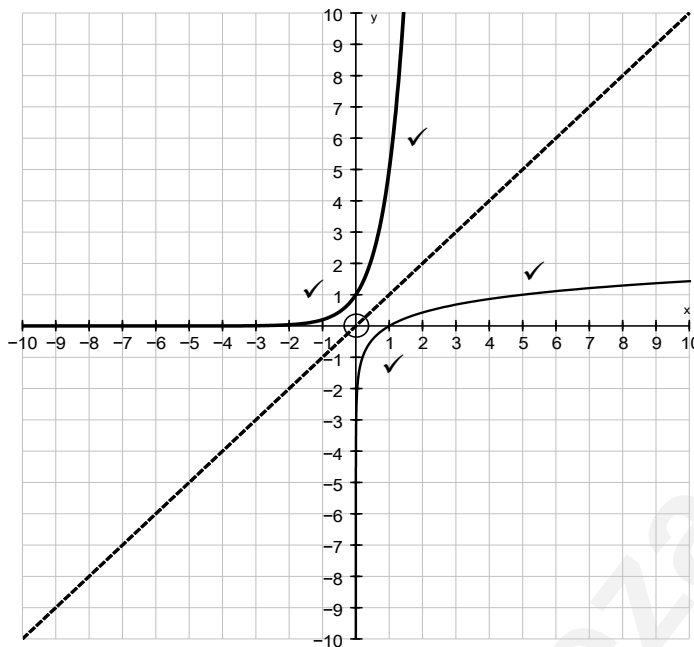
(1)

(2)

$$y \in \mathbb{R} \quad \checkmark \quad \checkmark \quad \text{or} \quad y \in (-\infty; \infty)$$

(2)

(3)



(4)

(4) $0 < x \leq 1$

(2)

(c)

$3x - 1 > 0$ ✓

$x > \frac{1}{3}$ ✓

(2)

QUESTION 4

(a) (1) $\frac{469280}{28000} = R16,76 \text{ per pound}$ ✓

(2)

(2) $14,86 \times 36500 = R542390$ ✓

Most expensive is in Europe. ✓

(2)

(b) (1)

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i} \quad \checkmark$$

$$1000000 = \frac{96250 \left[1 - \left(1 + \frac{12,5\%}{2} \right)^{-n} \right]}{\frac{12,5\%}{2}} \quad \checkmark \quad (3)$$

$$n = 17,2862$$

$$\therefore n = 17 \quad \checkmark$$

(2)

$$\text{Balance outstanding} = A - F \quad \checkmark$$

$$\begin{aligned} \text{Balance outstanding} &= 1000000 \left(1 + \frac{12,5\%}{2} \right)^{17} - \frac{96250 \left(\left(1 + \frac{12,5\%}{2} \right)^{17} - 1 \right)}{\frac{12,5\%}{2}} \\ &= R 26488,52 \quad \checkmark \end{aligned} \quad (4)$$

(3)

$$\text{loan amount} = P_v + x(1+i)^{-(n+1)} \quad \checkmark$$

$$1000000 = \frac{96250 \left(1 - \left(1 + \frac{12,5\%}{2} \right)^{-17} \right)}{\frac{12,5\%}{2}} + x \left(1 + \frac{12,5\%}{2} \right)^{-(18)} \quad \checkmark \quad (4)$$

$$x = R 28144,06 \quad \checkmark$$

QUESTION 5

(a)

$$b = 4 \quad \checkmark$$

$$c = -3 \quad \checkmark$$

$$\checkmark$$
$$-4 = \frac{a}{6-4} - 3 \quad \checkmark$$

$$-1 = \frac{a}{2}$$

$$\therefore a = -2 \quad \checkmark$$

(5)

(b)

$$y = x + c \quad \text{or}$$

$$y = -x + c$$

$$-3 = 4 + c \quad \checkmark$$

$$-3 = -4 + c \quad \checkmark$$

$$c = -7$$

$$c = 1$$

$$y = x - 7 \quad \checkmark$$

$$y = -x + 1 \quad \checkmark$$

(4)

(c)

$$g(x) = \frac{-2}{-x-4} - 3 \quad \checkmark \checkmark$$

(2)

QUESTION 6

(a) (1)

$$g(x) = 6x - 2x^2$$

$$\begin{aligned} g(x+h) &= 6(x+h) - 2(x+h)^2 \\ &= 6x + 6h - 2(x^2 + 2xh + h^2) \\ &= 6x + 6h - 2x^2 - 4xh - 2h^2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{6x + 6h - 2x^2 - 4xh - 2h^2 - (6x - 2x^2)}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} \frac{6h - 4xh - 2h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6 - 4x - 2h)}{h} \quad \checkmark \\ &= \lim_{h \rightarrow 0} 6 - 4x - 2h \\ &= 6 - 4x - 2(0) \\ &= 6 - 4x \quad \checkmark \end{aligned} \tag{5}$$

(2)

$$\begin{aligned} 6 - 4x &= 0 \quad \checkmark \\ x &= \frac{3}{2} \quad \checkmark \end{aligned} \tag{2}$$

(b) (1)

$$\begin{aligned} y &= \frac{x+1}{\sqrt{x}} \\ y &= \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \quad \checkmark \\ y &= x^{\frac{1}{2}} + x^{-\frac{1}{2}} \quad \checkmark \\ \frac{dy}{dx} &= \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} \end{aligned} \tag{4}$$

(2)

$$xy + 2y = x^2 + 5x + 6$$

$$y(x+2) = x^2 + 5x + 6 \quad \checkmark$$

$$y = \frac{x^2 + 5x + 6}{x+2} \quad \checkmark$$

$$y = \frac{(x+3)(x+2)}{x+2} \quad \checkmark$$

$$y = x + 3$$

$$\frac{dy}{dx} = 1 \quad \checkmark$$

(4)

QUESTION 7

(a)

$$x^2 + 5xy - 24y^2 = 0$$

$$(x+8y)(x-3y) = 0 \quad \checkmark$$

$$8y = -x \quad \text{or} \quad -3y = -x \quad \checkmark$$

$$\frac{y}{x} = \frac{-1}{8} \quad \checkmark \quad \text{or} \quad \frac{y}{x} = \frac{1}{3} \quad \checkmark$$

(4)

(b)

$$x + y = -7$$

$$-8y + y = -7 \quad \checkmark$$

$$-7y = -7$$

$$y = 1$$

$$\therefore x = -8 \quad \checkmark$$

or

$$x + y = -7$$

$$3y + y = -7 \quad \checkmark$$

$$4y = -7$$

$$y = \frac{-7}{4} \quad \checkmark$$

$$\therefore x = \frac{-21}{4}$$

(4)

QUESTION 8

(a)

$$T_3 = ar^2 = \frac{1}{3} \dots \text{equation 1}$$

$$T_4 = ar^3 = \frac{2}{9} \dots \text{equation 2}$$

$$\checkmark \frac{ar^3}{ar^2} = \frac{\frac{2}{9}}{\frac{1}{3}} \quad \checkmark$$

$$r = \frac{2}{3} \quad \checkmark$$

$$a \left(\frac{2}{3} \right)^2 = \frac{1}{3}$$

$$a = \frac{3}{4} \quad \checkmark$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{\frac{3}{4}}{1 - \frac{2}{3}}$$

$$= \frac{9}{4} \quad \checkmark$$

(5)

(b)

$$\sum_{n=1}^x 4 \cdot 3^n < 4400$$

$$12 + 36 + 108 + 324 \dots$$

$$r = 3 \quad \checkmark$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \checkmark$$

$$\frac{12(3^x - 1)}{3 - 1} < 4400 \quad \checkmark$$

$$3^x - 1 < \frac{2200}{3}$$

$$3^x < \frac{2203}{3}$$

$$x = \log_3 \frac{2203}{3} \quad \checkmark$$

$$x = 6$$

$$\therefore 0 < x \leq 6$$

$$x \in \mathbb{Z} \quad \checkmark$$

(5)

(c)

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad \checkmark$$

$$S_n = \frac{n}{2} [2a + (n-1)4] \quad \checkmark$$

$$2S_n = n[2a + 4n - 4]$$

$$2S_n = 2an + 4n^2 - 4n \quad \checkmark$$

$$S_{2n} = \frac{2n}{2} [2a + (2n-1)4] \quad \checkmark$$

$$2S_{2n} = 2n[2a + 8n - 4]$$

$$2S_{2n} = 4an + 16n^2 - 8n$$

$$S_{2n} = 2an + 8n^2 - 4n \quad \checkmark$$

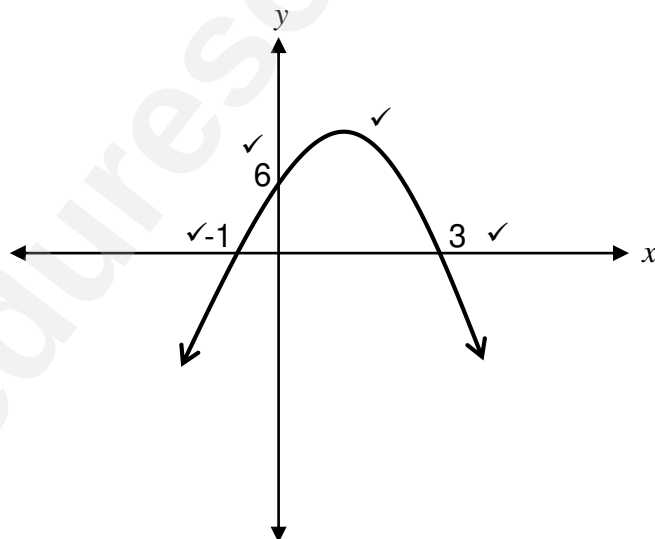
$$S_{2n} - 2S_n = 2an + 8n^2 - 4n - 2an - 4n^2 + 4n \quad \checkmark$$

$$S_{2n} - S_n = 4n^2 \quad \checkmark$$

$$\sqrt{S_{2n} - S_n} = 2n$$

(7)

QUESTION 9



(4)

QUESTION 10

(a) 18 meters ✓ (1)

(b)

$$\frac{-3}{800}(x-40)^2 + 18 = 12 \quad \checkmark$$

$$(x-40)^2 = (12-18) \div \frac{-3}{800}$$

$$(x-40)^2 = 1600$$

$$(x-40) = \pm 40 \quad \checkmark$$

$$\therefore x = 80\text{m} \quad \checkmark$$

(3)

(c)

$$\frac{-3}{800}(x-40)^2 + 18 = 16 \quad \checkmark$$

$$(x-40)^2 = (16-18) \div \frac{-3}{800}$$

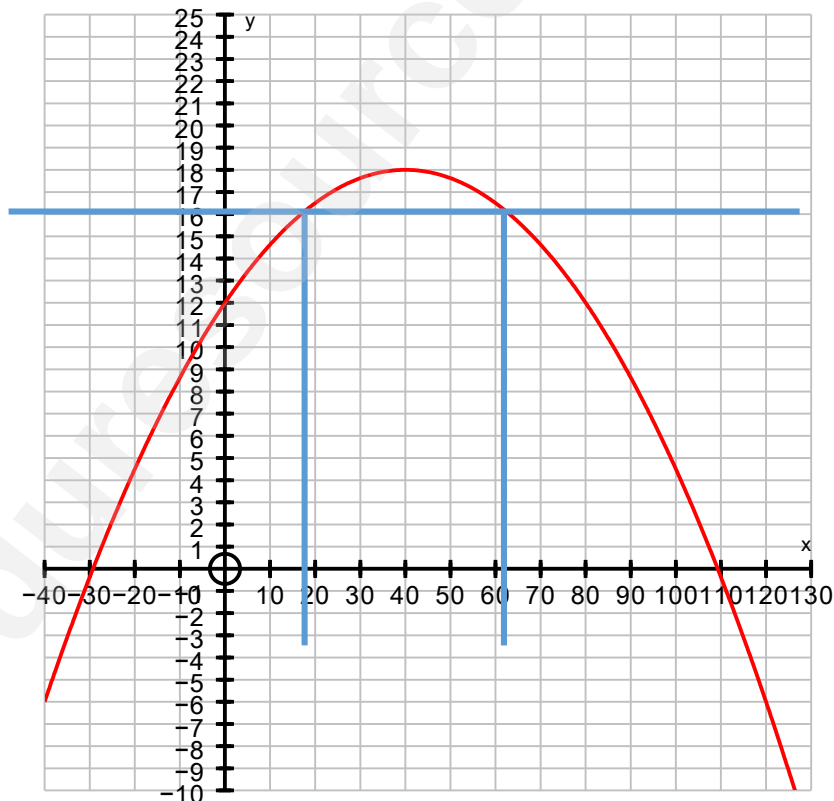
$$(x-40)^2 = \frac{1600}{3} \quad \checkmark$$

$$(x-40) = \pm \frac{40}{\sqrt{3}}$$

$$x = 63,09 \quad \text{or} \quad x = 16,906\text{m} \quad \checkmark$$

$\therefore$ horizontal distance covered will be $63,09 - 16,906 = 46,19\text{m}$ before landing.

(4)



QUESTION 11

(a)

$$y = \frac{1}{3}(45)^3 - 25(45)^2 + 600(45) \checkmark$$

$$y = 6750$$

$$y \in [0 ; 6750]$$

(3)

(b)

$$f'(x) = x^2 - 50x + 600$$

$$x^2 - 50x + 600 = 0 \checkmark$$

$$(x - 30)(x - 20) = 0 \checkmark$$

$$x = 30$$

or

$$x = 20 \quad \checkmark \text{ Both the values of } x$$

(5)

$$y = \frac{1}{3}(30)^3 - 25(30)^2 + 600(30)$$

$$y = 4500$$

$$A(30 ; 4500)$$

$$y = \frac{1}{3}(20)^3 - 25(20)^2 + 600(20)$$

$$y = 4666,7 \quad \checkmark \text{ Both the values of } y$$

$$B(20 ; 4666,7)$$

$\checkmark$ Co-ordinate form

(c)

turning points will be:

$$(31 ; 9000) \quad \checkmark \quad \text{and} \quad (21 ; 9333,3) \quad \checkmark$$

(4)

(d)

$$4500 < \frac{k}{3} < 4666,7 \quad \checkmark$$

$$13500 < k < 14000,1 \quad \checkmark$$

(2)

QUESTION 12

(a)

$$V = l \times b \times h \quad \checkmark$$

$$171 \frac{1}{2} = x \cdot x \cdot h$$

$$h = \frac{171,5}{x^2} \quad \checkmark$$

(4)

$$\text{surface area} = x^2 + 4(b \times h) \quad \checkmark$$

$$= x^2 + 4 \left(x \times \frac{171,5}{x^2} \right) \quad \checkmark$$

$$= x^2 + \frac{686}{x}$$

(b)

$$A(x) = x^2 + \frac{686}{x}$$

$$A'(x) = 2x - \frac{686}{x^2} = 0 \quad \checkmark$$

$$2x^3 - 686 = 0 \quad \checkmark$$

$$x^3 = 343$$

$$x = 7$$

dimension of the base is 7×7 $\checkmark$

(3)

QUESTION 13

(a)

$$\frac{42}{175} \checkmark \text{ or } 0,24 \text{ or } 24\% \quad (2)$$

(b)

$$\frac{37}{102} \checkmark \text{ or } 0,36 \text{ or } 36,3\% \quad (2)$$

(c)

$$P(< 40) = \frac{107}{175} \checkmark \quad P(\text{liking the movie}) = \frac{102}{175} \checkmark$$

$$P(< 40 \text{ and liking the movie}) = \frac{65}{175} = 0,371 \checkmark$$

$$P(< 40) \times P(\text{liking the movie}) = \frac{107}{175} \times \frac{102}{175} = 0,356 \checkmark \quad (4)$$

$$P(< 40 \text{ and liking the movie}) \neq P(< 40) \times P(\text{liking the movie})$$

$\therefore$ NOT INDEPENDENT

QUESTION 14

(a) $6 \times 5 = 30$ ways or Bruce/ Peter = 1 so $1 \overset{\checkmark} \times 5 \overset{\checkmark} \times 6 \overset{\checkmark} = 30$ (3)

(b) In Taxi P there are 5 ways to fill 4th seat but Bruce/Peter can switch therefore $5 \overset{\checkmark} \times 2 = 10$
 In Taxi Q there are $4 \overset{\checkmark} \times 3 \overset{\checkmark} \times 2 \overset{\checkmark} \times 1 \overset{\checkmark} = 24$ ways but the passengers can be swap therefore
 $5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 2 = 480$ ways ✓ (4)