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BALLITO

MEMO

Mathematics	
Paper 1	
FORM 5	
25 August 2021	
TIME: 3 hours	TOTAL: 150 marks
Examiner: Miss Eastes	Moderated: Mrs Smith and Mrs Gunning
Name and Surname:	

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

- This question paper consists of 10 questions and 21 pages (pages 19 to 21 are blank for you to use if necessary) and an information sheet is given.
- Read the questions carefully.
- **ALL** questions must be answered on the question paper.
- Diagrams are **NOT** necessarily drawn to scale.
- All answers must be given correct to **one decimal place** where necessary, unless stated otherwise.
- Approved calculators may be used, unless stated otherwise.
- All the necessary working details must be clearly shown.

Question	1	2	3	4	5	6	7	8	9	10	Total	%
	21	13	6	18	21	17	14	23	13	6	150	

allison

1

malih

QUESTION 1

a) Solve for x :

[Where necessary solve for x in terms of p in simplest form, show all working details]

1) $(2^x - 1)(2^x + 4)(2^x - 6) = 0$

✓m

(4)

$$\begin{array}{lll} 2^x = 1 & 2^x = -4 & 2^x = 6 \\ \checkmark a & \text{no solution} & \checkmark a \\ x = 0 & & \log_2 6 = x \end{array}$$

2) $(x - 3)(x + 4) = p$

(3)

$$\begin{aligned} x^2 + x - 12 &= p \\ x^2 + x - 12 - p &= 0 \quad \checkmark a \\ x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-12-p)}}{2(1)} \quad \checkmark a \\ x &= \frac{-1 \pm \sqrt{1+48+4p}}{2} = \frac{-1 \pm \sqrt{49+4p}}{2} \quad \checkmark a \end{aligned}$$

3) $\log_x(p - 4) = 2$

(2)

$$\begin{aligned} x^2 &= p - 4 \quad \checkmark a \\ x &= \pm \sqrt{p - 4} \quad \checkmark a \end{aligned}$$

4) $\sqrt{5-x} + 1 + x = 0$

(5)

$$\begin{aligned}\sqrt{5-x} &= -1-x \quad \checkmark^m \\ 5-x &= (-1-x)^2 \\ 5-x &= 1+2x+x^2 \quad \checkmark^a \\ 0 &= x^2+3x-4 \quad \checkmark^{ca} \\ 0 &= (x+4)(x-1) \\ x &= -4 \quad \checkmark^{ca} \quad \text{or} \quad x = 1 \quad \checkmark^{\text{test}}\end{aligned}$$

b) Solve for x and y:

$$4 - 2y = x$$

$$2^{x(x-y)} = 4$$

(6)

$$\begin{aligned}2^{x^2-xy} &= 2^2 \quad \checkmark^a \\ x^2 - xy &= 2 \quad \checkmark^m \\ (4-2y)^2 - (4-2y)y &= 2 \quad \checkmark^{ca} \\ 16 - 16y + 4y^2 - 4y + 2y^2 &= 2 \quad \checkmark^{ca} \\ 6y^2 - 20y + 14 &= 0 \\ (y-1)(3y-7) &= 0 \quad \checkmark^{ca} \\ y = 1 \quad \text{or} \quad y = \frac{7}{3} \quad \checkmark^{ca} \\ x = 4 - 2(1) \quad \text{or} \quad x = 4 - 2\left(\frac{7}{3}\right) \quad \checkmark^{ca}\end{aligned}$$

RP

[21]

QUESTION 2

- a) 1) Determine the derivative from first principles of $f(x) = -x + 3x^2$. $\checkmark 9$ (4)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{-(x+h) + 3(x+h)^2 - (-x + 3x^2)}{h} \quad \checkmark 9 \\
 &= \lim_{h \rightarrow 0} \frac{-x-h + 3(x^2 + 2xh + h^2) + x - 3x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-x-h + 3x^2 + 6xh + 3h^2 + x - 3x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(-1 + 6x + 3h)}{h} \quad \checkmark m \\
 &= -1 + 6x + 3(0) \quad \checkmark 9 \\
 &= -1 + 6x \quad \checkmark 9
 \end{aligned}$$

- 2) Hence, determine the gradient of $f(x)$ at $x = 2$. (2)

$$\begin{aligned}
 f'(x) &= -1 + 6(2) \quad \checkmark \\
 &= 11 \quad \checkmark 9
 \end{aligned}$$

- b) 1) Determine: $D_x \left[\frac{2x^3+3}{x} - 3\sqrt[3]{x^2} \right], x \neq 0$. (4)

$$\begin{aligned}
 D_x \left[\frac{2x^3+3}{x} - 3x^{\frac{2}{3}} \right] &\quad \checkmark m \quad \checkmark a \\
 D_x \left[2x^2 + 3x^{-1} - 3x^{\frac{2}{3}} \right] &\quad \checkmark 9 \\
 &= 4x - 3x^{-2} - 2x^{-\frac{1}{3}} \quad \checkmark m
 \end{aligned}$$

- 2) Determine: $f'(-1)$ if $f(x) = \frac{x^2+2x-8}{x-2}$

$$\begin{aligned}
 f(x) &= \frac{3x^2 - 7x - 6}{3-x} \quad \checkmark 9 \\
 &= \frac{(3x+2)(x-3)}{-(x-3)} \quad \checkmark 9
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \frac{(x+4)(x-2)}{(x-2)} \quad \checkmark 9 \\
 &= x+4
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= -3x - 2 \quad [13] \\
 &= -3 \quad \checkmark m
 \end{aligned}$$

$$f'(x) = 1 \quad \checkmark$$

QUESTION 3

Find the equation of the tangent to the curve $y = x - \frac{1}{x^2}$ at the point where the curve crosses the x-axis.

(6)

$$y = x - x^{-2}$$

$$\frac{dy}{dx} = 1 + 2x^{-3} \checkmark$$

$$(x, 0)$$

~~$$\frac{dy}{dx} = 1 + 2x^{-3}$$~~

$$0 = x - x^{-2} \checkmark$$

$$0 = x^3 - 1$$

$$1 = x^3$$

$$1 = x \checkmark$$

$$\frac{dy}{dx} = 1 + 2(1)^{-3}$$

$$= 3 \checkmark$$

$$y = 3x + C$$

$$0 = 3(1) + C \checkmark$$

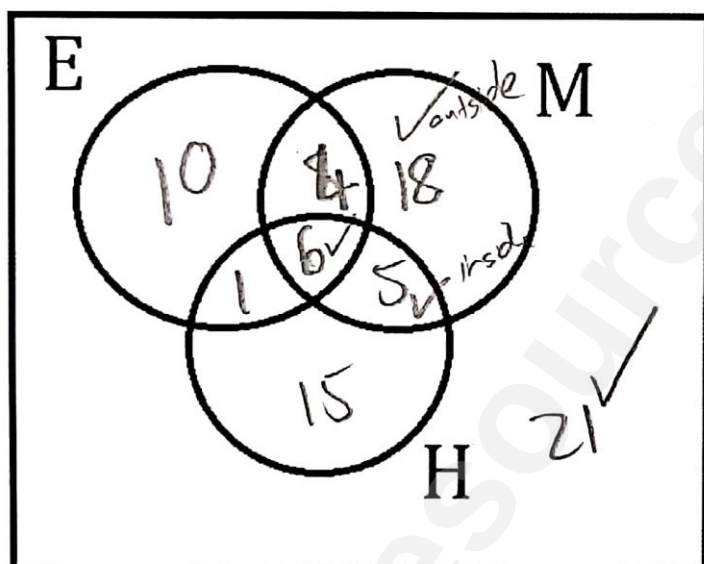
$$-3 = C$$

$$\boxed{y = 3x - 3} \checkmark$$

QUESTION 4

- a) A group of 80 students were asked which of the 3 subjects – Mathematics, History and/ or Economics, they intend taking in Grade 10 in 2022.
- 21 Students will not be taking any of the 3 subjects
 - 6 Students will be taking all 3 subjects
 - 10 Students will be taking Mathematics and Economics
 - 11 Students will be taking Mathematics and History
 - Of the 21 students who are taking Economics, 10 students are taking Economics only.
 - 27 Students are taking History.

- 1) Complete the Venn diagram below to represent the above information. (4)



E = Economics
M = Mathematics
H = History

- 2) Are the events taking Mathematics and taking Economics independent? (4)
Prove your answer.

$$P(M) = \frac{33}{80} \checkmark$$

$$P(E) = \frac{21}{80} \checkmark$$

$$P(M \cap E) = \frac{10}{80} \checkmark$$

$$P(M) \times P(E)$$

$$= \frac{33}{80} \times \frac{21}{80}$$

$$= \frac{693}{6400} \checkmark$$

$$P(M \cap E) \neq P(M) \times P(E)$$

not independent $\checkmark$

- 3) Out of the group of 80 students how many arrangements can be made for the top 3 academic positions in the group. (2)

$$80 \times 79 \times 78 \checkmark_9 \\ = 492960 \checkmark_{ca}$$

- b) Consider the letters of the word: **M A T H E M A T I C S**

- 1) How many unique arrangements can be made with the word **M A T H E M A T I C S**. (Leave your answer in factorial form) (4)

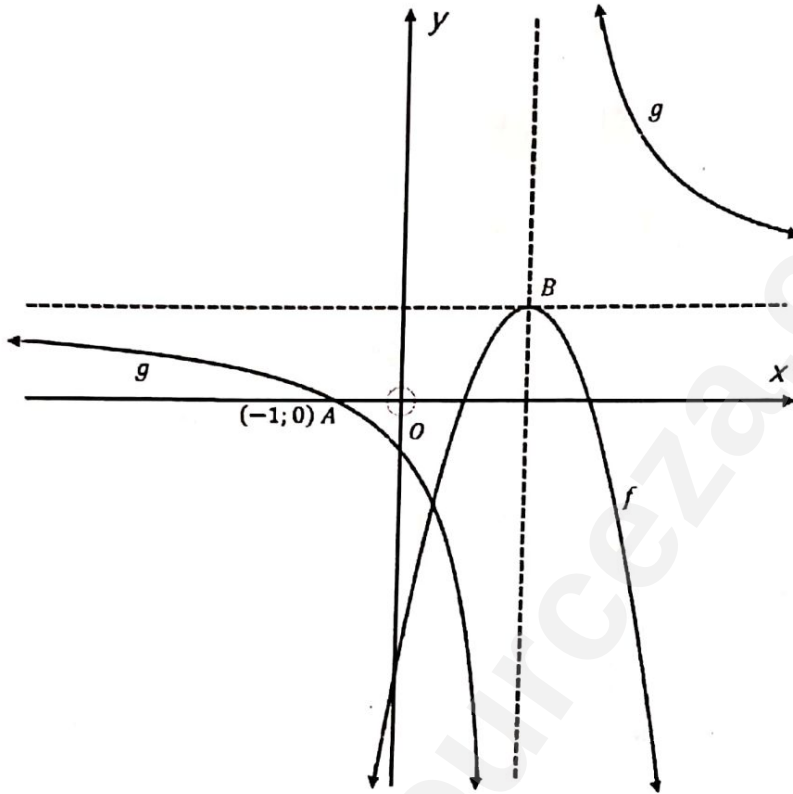
$$\frac{11! \checkmark_9}{2! 2! 2! \checkmark_9 \checkmark_9 \checkmark_9}$$

- 2) How many unique arrangements can be made with the word **M A T H E M A T I C S** if all the M's and all the A's have to be grouped together. (4)

$$\begin{array}{cccccccc} & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ m & m & A & A & - & - & - & - & - & - \\ \hline & & 4! \checkmark & & & & & & & \\ & & \underline{2! \times 2!} & & & & & & & \\ & & \checkmark & & & & & & & \end{array} \times 8! \checkmark$$

QUESTION 5

- a) The sketch below is a parabola f , with the equation $f(x) = -x^2 + 4x - 3$ and hyperbola g with equation $g(x) = \frac{3}{x+a} + p$.
 B, the turning point of f , lies at the point of intersection of the asymptotes of g .
 A(-1; 0) is the x-intersect of g .



- 1) Show that the co-ordinates of B are (2; 1). (2)

$$\begin{aligned}
 f'(x) &= -2x + 4 \\
 0 &= -2x + 4 \quad \checkmark a \\
 2x &= 4 \quad \checkmark 19 \\
 x &= 2 \\
 -4 + 8 - 3 &= 1 \\
 B(2; 1)
 \end{aligned}$$

- 2) Write down the range of f . (1)

$$y \in (-\infty, 1] \quad \checkmark a$$

- 3) Determine the values of a and p .

(2)

$$y = \frac{3}{x-2} + 1\sqrt{a}$$

- 4) For which values of x will $g(x) \geq 0$.

(2)

$$x \leq -1 \quad \text{or} \quad x > 2$$

$\sqrt{a}$ $\sqrt{a}$

- 5) Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x+4)$.

(2)

$$y = \frac{3}{x+4-2} + 1$$

$$= \frac{3}{x+2} + 1$$

$$x = -2$$

- 6) For which values of x will $f(x) \cdot f'(x) \geq 0$.

(4)

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

3 or 1

$$x \in [1, 3]$$

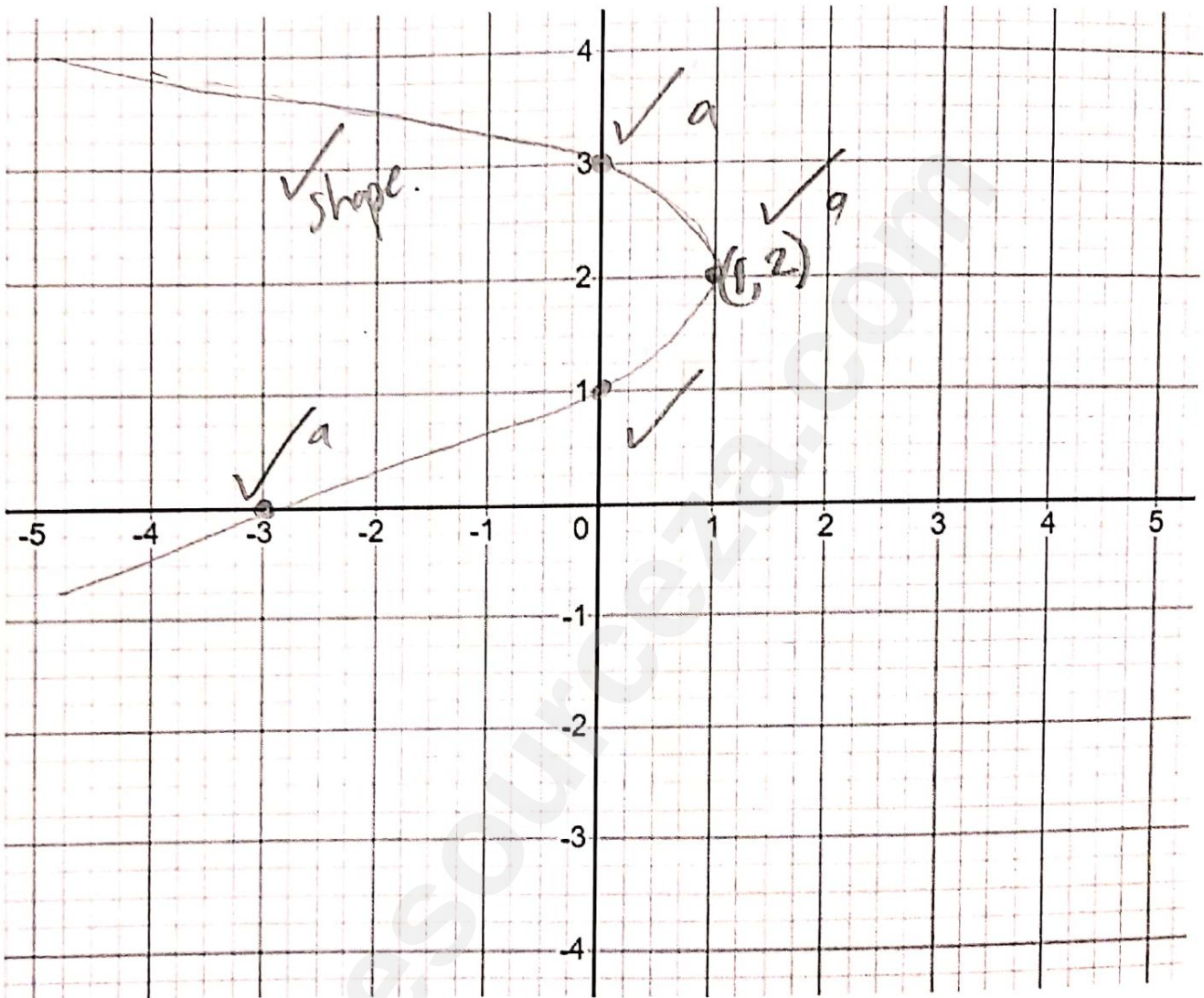
$$x \in [1, 2)$$

$$x \in (3, \infty)$$

7) Draw the graph of $f^{-1}(x)$ on the axes below.

Indicate the turning point and any intercepts with the axes.

(4)



8) How can we restrict the domain of $f(x)$ such that $f^{-1}(x)$ is a function.

(2)

$$x > 1 \quad \checkmark a$$

or

$$x < 1 \quad \checkmark a$$

b) The function $f(x) = \left(\frac{1}{2}\right)^x + 1$ is given.

1) Write down the range of $f(x)$.

(1)

$$y \in (1, \infty) \checkmark$$

2) Write down $f^{-1}(x) = \dots\dots\dots$

(3)

$$x = \left(\frac{1}{2}\right)^y + 1 \checkmark$$

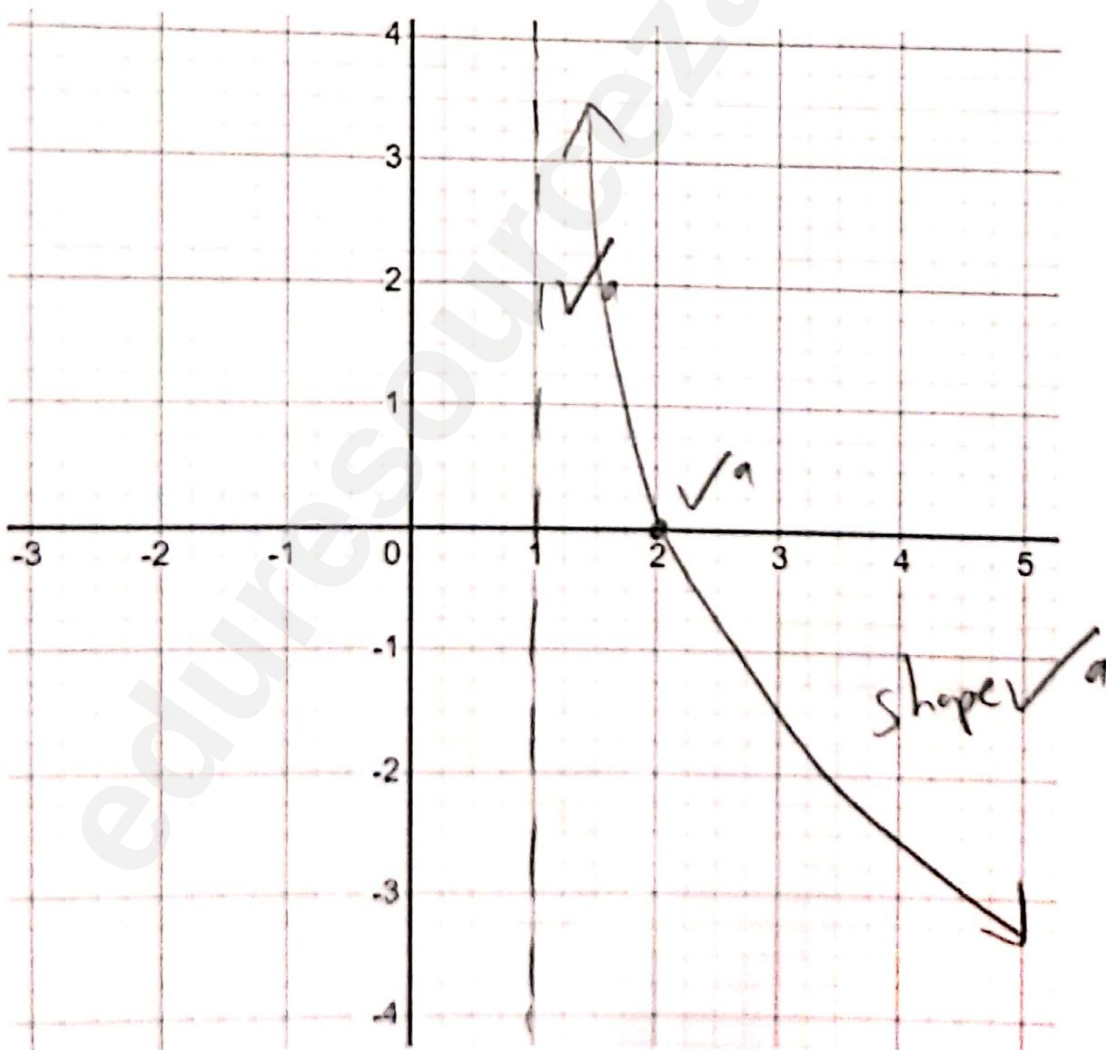
$$x - 1 = \left(\frac{1}{2}\right)^y \checkmark$$

$$\log_{\frac{1}{2}}(x-1) = y \checkmark$$

more space

3) Sketch $f^{-1}(x)$. Show asymptote and x-intersect.

(3)



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QUESTION 6

- a) Mrs Ndlovu invests R15 500 for k years. She earns interest at a rate of 9% per annum, compounded quarterly. At the end of k years her investment is worth R33 400.

1) Calculate the effective annual rate of Mrs Ndlovu's investment. (2)

$$1 + i_e = \left(1 + \frac{9\%}{4}\right)^4 \checkmark$$

$$i_e = 9.31\% \checkmark$$

2) Determine the value of k . (4)

$$33400 = 15500 \left(1 + \frac{9\%}{4}\right)^{4 \times k} \checkmark$$

$$\frac{334}{155} = \left(\frac{409}{400}\right)^{4k} \checkmark$$

$$\log \frac{409}{400} \frac{334}{155} = 4k \checkmark$$

$$8.62 = k \checkmark$$

- b) Khwezi is planning to buy her first home. The bank will allow her to use a maximum of 30% of her monthly salary to repay the bond. She earns R18 480 per month. Suppose, at the end of each month, Khwezi repays the maximum amount allowed by the bank. The first payment is made one month after the loan is granted.

1) How much money does Khwezi borrow if she takes 25 years to repay the loan at a rate of 8% p.a. compounded monthly? (4)

$$\frac{30}{100} \times 18480 = 5544 \checkmark$$

$$P = \frac{5544 \left(1 - \left(1 + \frac{8\%}{12}\right)^{-25 \times 12}\right)}{\frac{8\%}{12}} \checkmark$$

$$= 718305.71 \checkmark$$

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2) Calculate the outstanding balance after 20 years of paying back the loan.

(3)

$$B = \frac{5544 \left(1 - \left(1 + \frac{8\%}{12}\right)^{-5 \times 12}\right)}{\frac{8\%}{12}} \quad \checkmark_{eq}$$

$$= 273421,40 \quad \checkmark_{eq}$$

3) How much interest was paid after 20 years?

(4)

$$718305,71 - 273421,40 \quad \checkmark_1$$
$$= 444884,31 \quad m \checkmark$$

$$5544 \times 12 \times 20 = 1330560 \quad \checkmark_{eq}$$

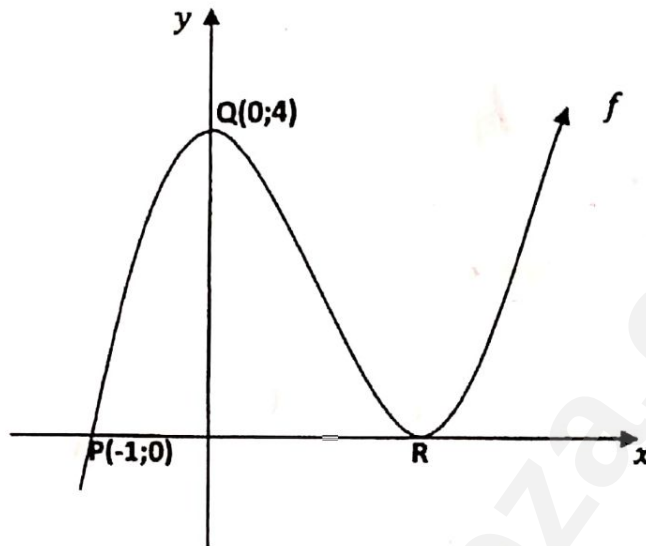
$$\therefore 1330560 - 444884,31$$

[17]

$$= 885675,69 \quad \checkmark_{eq}$$

QUESTION 7

- a) The sketch below represents the curve of $f(x) = x^3 + ax^2 + bx + c$.
 $Q(0; 4)$ is a turning point and a local minimum occurs at R.
 Study the graph and then the questions that follow.



- 1) Write down the value of c .

(1)

$$4 \checkmark$$

- 2) Hence, show that $a = -3$ and $b = 0$.

(5)

$$0 = 3x^2 + 2ax + b \quad \checkmark \quad \checkmark$$

$$0 = 3(0) + 2a(0) + b$$

$$b = 0 \quad \checkmark$$

$$0 = (-1)^3 + a(-1)^2 + 4 \quad \checkmark$$

$$0 = -1 + a + 4 \quad \checkmark$$

- 3) If $f(x) - k = 0$ has only one real root, use your graph to find the values of k .

$$k > 4 \quad \checkmark \text{ or } k < 0 \quad \checkmark$$

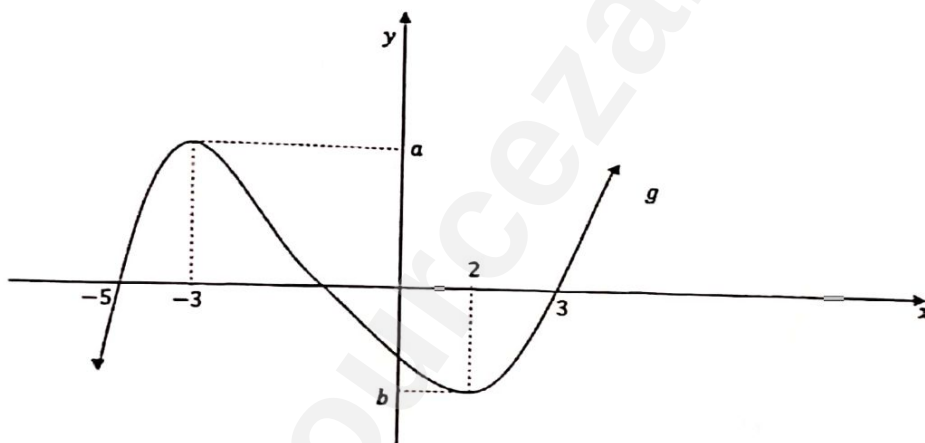
(2)

- 4) If the equation f was changed to $y = (x + 3)^3 - 3(x + 3)^2 + 4$, what would the new co-ordinates of point Q be?

$$Q(-3; 4) \quad \checkmark$$

(1)

- b) The graph of a cubic polynomial g is drawn below. Study the graph carefully and then answer the questions that follow.



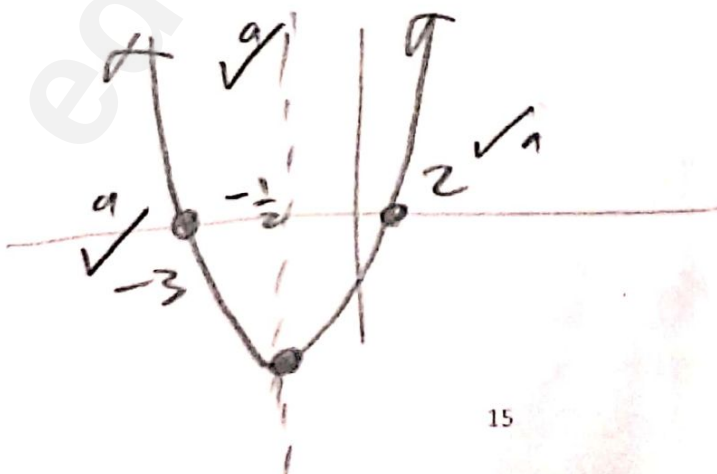
- 1) For which values of x is $g'(x) > 0$?

(2)

$$x < -3 \quad \checkmark \text{ or } x > 2 \quad \checkmark$$

- 2) Draw a rough sketch graph of the derivative $g'(x)$ indicating the x-intercepts on the graph and the axis of symmetry.

(3)



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QUESTION 8

- a) Determine the sum of the following series:

$$3 - 6 + 12 - \dots - 1536$$

(5)

$$-1536 = 3(-2)^{n-1} \checkmark$$

$$(-2)^9 = (-2)^{n-1} \checkmark$$

$$10 = n \checkmark$$

$$S_n = \frac{3((-2)^{10} - 1)}{-2 - 1} \checkmark$$

$$= -1023 \checkmark$$

- b) The seventh, eighth and thirteenth terms of an arithmetic sequence are
- $3x - 1$
- ;
- x
- and
- $-8x + 20$
- respectively. Determine the numerical value of
- x
- .

(6)

$$x - (3x - 1)$$

$$d = -2x + 1 \checkmark$$

$$a + 12d = -8x + 20 \checkmark$$

$$a + 12(-2x + 1) = -8x + 20 \checkmark$$

$$a - 24x + 12 = -8x + 20$$

$$a = 16x + 8 \checkmark$$

$$x = a + 7d$$

$$x = 16x + 8 + 7(-2x + 1) \checkmark$$

$$x = -15 \checkmark$$

- c) Determine the value of x if:

$$\sum_{k=1}^{\infty} 9x^k = \sum_{t=1}^{14} (24 - 3t)$$

(6)

$$9x, 9x^2, 9x^3 \checkmark$$

$$24 - 3 \quad \checkmark_m$$

$$21 \quad ; \quad 18 \quad ; \quad 15$$

$$\underbrace{\quad} \quad \underbrace{\quad}$$

$$-3 \quad -3$$

$$S_{\infty} = \frac{9x}{1-x} \checkmark$$

$$\frac{9x}{1-x} = 21 \checkmark$$

$$9x = 21 - 21x$$

$$x = \frac{21}{30} \checkmark_{eq}$$

[23]

$$S_{14} = \frac{14}{4} [2(21) + 13(-3)]$$

$$= 21 \quad \checkmark_{eq}$$

QUESTION 9

a) If $y = \log(-x^2 + 2x + 99)$ and it is given that $y = \log_a b$ where $b > 0$

- 1) Determine the values of x for which y is defined. Show all working. (5)

$$-x^2 + 2x + 99 > 0 \quad \checkmark$$

$$x^2 - 2x - 99 < 0 \quad \checkmark$$

$$(x-11)(x+9) < 0 \quad \checkmark$$

$$-9 < x < 11 \quad \checkmark$$

- 2) Showing all your workings, determine the maximum value of y . (4)

$$\frac{dy}{dx} = -2x + 2 \quad \checkmark$$

$$0 = -2x + 2$$

$$1 = x \quad \checkmark$$

$$y = \log(-1^2 + 2 + 99)$$

$$y = -2 \quad \checkmark$$

- b) Given that $x, y, z, \dots$ form a geometric sequence.

Show that $\log x, \log y, \log z, \dots$ form an arithmetic sequence. (4)

$$\frac{y}{x} = \frac{z}{y} \quad \checkmark$$

$$y^2 = x \cdot z$$

$$\log y^2 = \log x + \log z \quad \checkmark$$

$$\log y + \log y = \log x + \log z \quad \checkmark$$

$$\log y - \log x = \log z - \log y \quad \checkmark$$

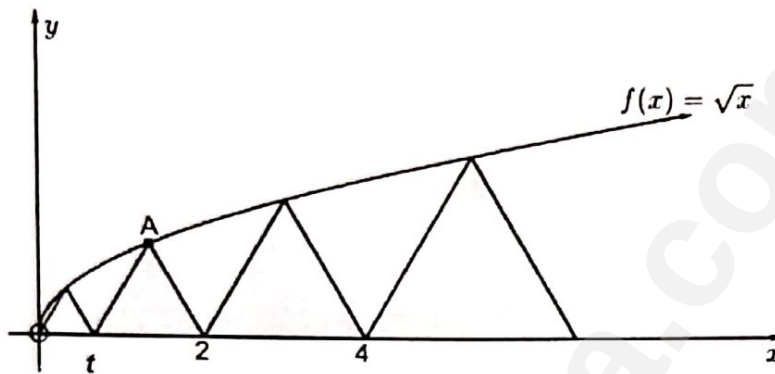
[13]

QUESTION 10

The graph below shows the graph of the function $f(x) = \sqrt{x}$ and a sequence of points on the x-axis are $\{0, t, 2, 4\}$

The points are drawn such that a sequence of equilateral triangles result.

The points on the x-axis form a quadratic sequence.



- a) Determine the value of t .

(3)

$$0; x; 2-x; 2\sqrt{9}$$

$$2-2x-x = x-(2-2x)\sqrt{9}$$

$$x = \frac{2}{3}\sqrt{9}$$

- b) Hence, or otherwise, the co-ordinates of the point A.

(3)

$$2 - \frac{2}{3}\sqrt{9} \quad \sqrt{\frac{4}{3}}\sqrt{9}$$

$$y = \frac{2\sqrt{3}}{3}\sqrt{9}$$

$$\left(\frac{4}{3}; \frac{2\sqrt{3}}{3}\right)$$

[6]