





ST BENEDICT'S

SUBJECT	Mathematics	PAPER	Prelim Paper 2
GRADE	12	DATE	29 July 2024
EXAMINER NAME	Mr Hayward	MARKS	150
TEACHER	Memo	MODERATOR	Ms McConkey Mrs Meredith
	GH SM TM KM	DURATION	3 hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
SECTION A		75	
1	Analytical Geometry	6	
2	Analytical Geometry	15	
3	Statistics	10	
4	Statistics	9	
5	Trigonometry	25	
6	Euclidean Geometry	10	
SECTION B		75	
7	Trigonometry	9	
8	Analytical Geometry	18	
9	Trigonometry	6	
10	Trigonometry	14	
11	Euclidean Geometry	8	
12	Euclidean Geometry	7	
13	Euclidean Geometry	13	
TOTAL		150	

INSTRUCTIONS:

1. This paper consists of 14 questions and 24 pages. A page for extra working has been provided. A separate formula sheet has been included. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all questions on the question paper and hand it in at the end of the exam.
4. It is in your interest to write legibly and present your work neatly.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded marks.**
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Round your answers to **one decimal place** where necessary unless otherwise stated.
8. Diagrams are not necessarily drawn to scale.

SECTION A

QUESTION 1

6 MARKS

$A(6; 1)$, $B(8; 2)$, $C(3; a)$ and $D(b; c)$ are points in the cartesian plane.

Determine the value(s) of:

- a) a if A , B and C are collinear.

(3)

$$m_{AB} = m_{AC} \quad \checkmark A$$

$$\frac{2-1}{8-6} = \frac{a-1}{3-6} \quad \checkmark A$$

$$\frac{1}{2} = \frac{a-1}{-3}$$

$$-\frac{3}{2} = a-1$$

$$-\frac{1}{2} = a \quad \checkmark CA$$

—————→

- b) b and c if B is the midpoint of A and D .

(3)

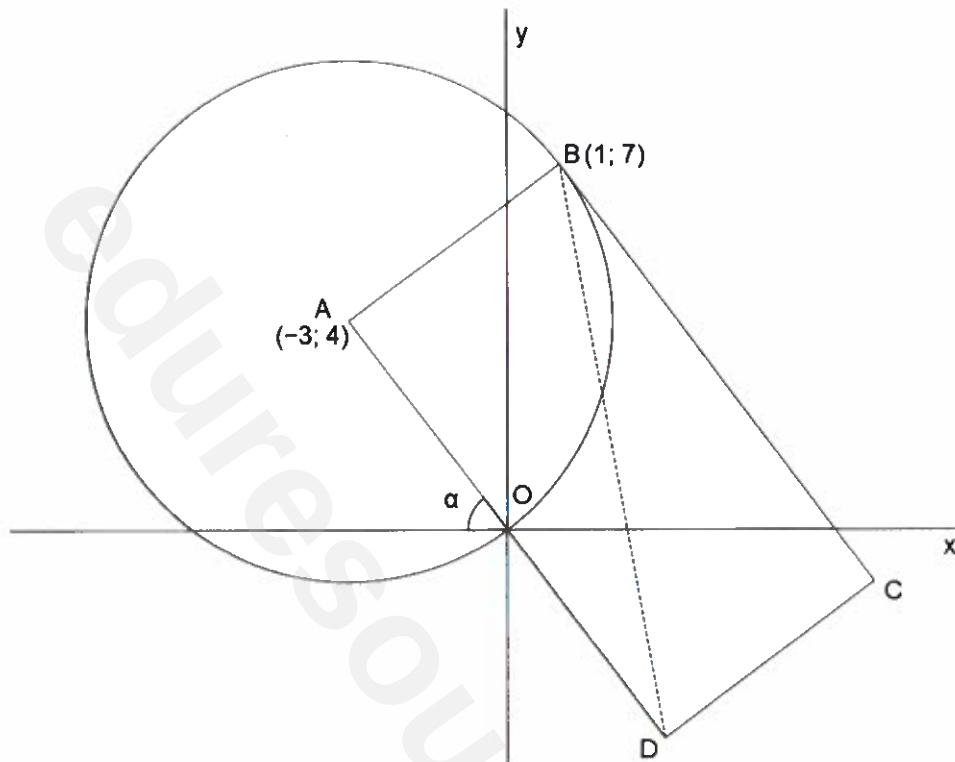
$$\left(\frac{6+b}{2}; \frac{1+c}{2} \right) = (8; 2) \quad \checkmark A$$

$$\therefore b = 10; \quad c = 3 \quad \checkmark CA$$

QUESTION 2

15 MARKS

$ABCD$ is a rectangle with $A(-3; 4)$ and $B(1; 7)$, with O at the origin.



- a) Determine the gradient of AB .

$$m = \frac{7-4}{1-(-3)} = \frac{3}{4}$$

- b) Determine the equation of line AD .

$$m_{AD} = -\frac{4}{3}$$

$$y = -\frac{4}{3}x$$

$$y-4 = -\frac{4}{3}(x-(-3))$$

$$y = -\frac{4}{3}x - 4 + 4$$

$$y = -\frac{4}{3}x$$

- c) Calculate the value of the angle marked α on the diagram.

(2)

$$\begin{aligned}\tan \theta &= -\frac{4}{3} \quad \checkmark \text{CA} \\ \therefore \theta &= -53,1^\circ + 180^\circ \\ &= 126,9^\circ \\ \therefore \alpha &= 53,1^\circ \quad \checkmark \text{CA}\end{aligned}$$

- d) If line BD is described by the equation $2y + 11x = 25$, determine the coordinates of point D .

(3)

$$\begin{aligned}2\left(-\frac{4}{3}x\right) + 11x &= 25 \quad \checkmark \text{CA} - \text{subs} \\ -\frac{8}{3}x + 11x &= 25 \\ -8x + 33x &= 75 \\ 25x &= 75 \quad \checkmark \text{CA} - \text{Alg} \\ x &= 3 \\ \therefore y &= -\frac{4}{3}(3) \quad \checkmark \text{CA} - \text{subs} \\ &= -4 \\ D(3; -4)\end{aligned}$$

- e) If D is the point $(3; -4)$, determine the area of rectangle $ABCD$.

(3)

$$\begin{aligned}d_{AB} &= \sqrt{(-3-1)^2 + (4-7)^2} = 5 \quad \checkmark \\ d_{AD} &= \sqrt{(-3-3)^2 + (4-(-4))^2} = 10 \quad \checkmark \\ A &= (5)(10) = 50 \quad \checkmark\end{aligned}$$

- f) If a second circle is drawn, with centre $D(3; -4)$ and radius DC , how many points of intersection will it have with the sketched circle, with centre A ? Explain by giving mathematical evidence.

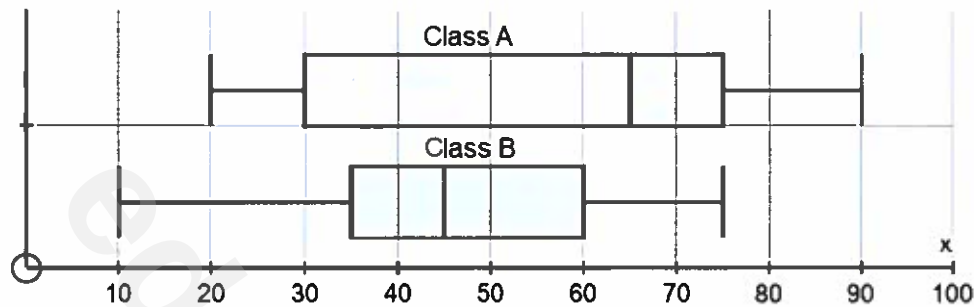
(3)

$$\begin{aligned}d_{AD} &= 10 \quad R + r = 5 + 5 = 10 \quad \checkmark \\ d_{AD} &= R + R \quad \checkmark \\ \therefore \text{The circles touch once, tangentially.} \quad \checkmark\end{aligned}$$

QUESTION 3

10 MARKS

- a) The box and whisker plots show the English results of class A and class B in a recent assessment. The assessment was out of 100 marks.



- 1) Which class would have a higher mean mark?

(1)

Class A ✓

- 2) Which class has a greater interquartile range?

(1)

Class A ✓

- 3) What percentage of class B achieved less than 60?

(1)

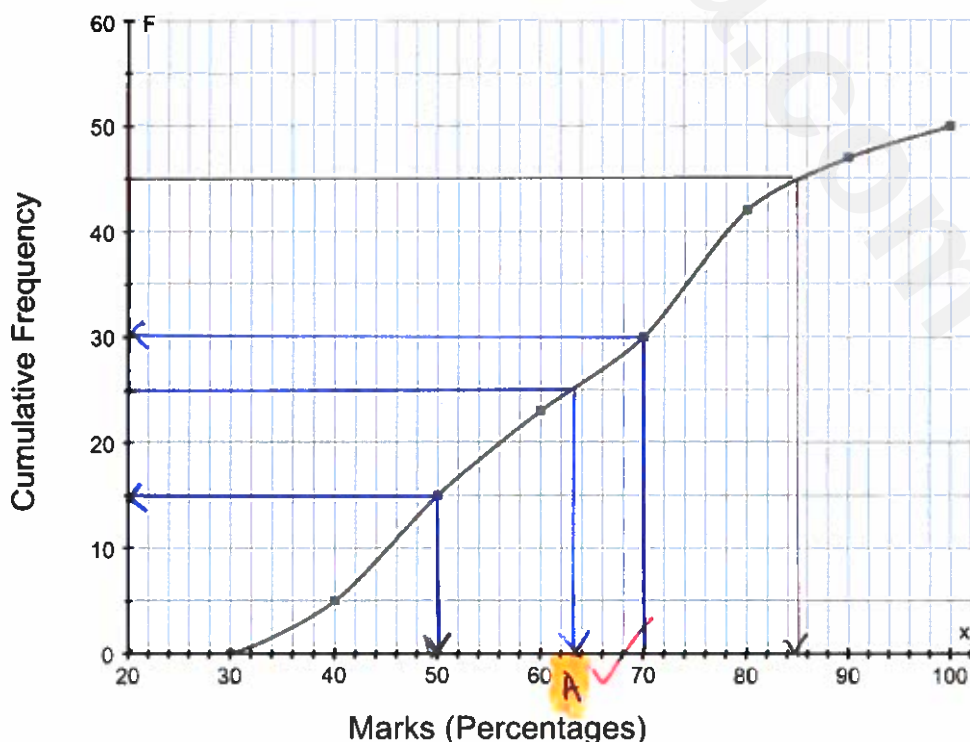
75% ✓

- 4) If all the students in class A were given an extra 5 marks, what would happen to the standard deviation of class A?

(1)

Remain the same. ✓

A class of 50 students wrote a Physical Science examination. The marks, in percentages, obtained by the students were tabulated and an ogive was drawn as shown below.



Study the graph and answer the following:

- 1) What is the median mark? Indicate with "A" where you took your reading on the graph. (2)

63 ✓

- 2) Above what mark did the top 10% of students score? (1)

85 ✓

- 3) How many students got between 50 and 70 percent for the examination? (1)

$$30 - 15 = 15 \text{ Students} \quad \checkmark$$

- 4) If 70% of the students passed the test, what was the passing mark? Show your calculations. (2)

$$50 - 70\%(50) = 15 \quad \checkmark$$

∴ 50 marks ✓

QUESTION 4

9 MARKS

The following table shows the term marks as a percentage for English and Maths for a random sample of 10 Matrics.

English (x)	54	54	64	62	70	70	72	80	78	83
Maths (y)	43	46	50	71	72	76	77	82	87	91

- a) Calculate the correlation coefficient, to 4 decimal places, and comment on what it tells us about the relationship between performance in English and Maths. (3)

$$r = 0,9289 \text{ Strong Positive linear correlation}$$

- b) Determine the equation of the line of best fit, to 4 decimal places. (3)

$$y = -38,4403 + 1,5712x$$

- c) Use your equation to predict the Maths mark, to the nearest whole number, of a student who scores 30% for English. (2)

$$\begin{aligned} y &= -38,4403 + 1,5712(30) \text{ ✓ subs} \\ &= 8,6952 \\ &= 9 \text{ ✓} \end{aligned}$$

- d) Comment, with an explanation, on the reliability of your prediction from c). (1)

Unreliable, Extrapolation ✓

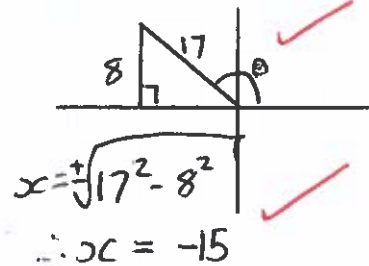
QUESTION 5

25 MARKS

- a) Given: $\sin \beta = \frac{8}{17}$ and $90^\circ \leq \beta \leq 270^\circ$. With the aid of a sketch and without the use of a calculator, determine the following:

1) $\tan \beta$

$$= \frac{8}{-15}$$



(3)

2) $\sin(90^\circ + \beta)$

$$= \cos \beta$$

$$= \frac{-15}{17}$$

(2)

3) $\cos 2\beta$

$$= 1 - 2\sin^2 \beta$$

$$= 1 - 2\left(\frac{8}{17}\right)^2$$

$$= \frac{161}{289} = 0,557$$

$$\approx 0,6$$

(Alt)

$$= 2\cos^2 \beta - 1$$

$$= 2\left(\frac{-15}{17}\right)^2 - 1$$

$$= \frac{161}{289}$$

(Alt)

$$= \cos^2 \beta - \sin^2 \beta$$

$$= \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$$

$$= \frac{161}{289}$$

(3)

- b) Simplify the following to a single trigonometric ratio, without the use of a calculator:

$$\begin{aligned} & \frac{\sin 190^\circ \times \tan 225^\circ}{\cos 100^\circ} - \sin(90^\circ - \alpha) \times \cos(-\alpha) \\ &= \frac{-\sin 10^\circ \cdot \tan 45^\circ}{-\cos 80^\circ} - \cos \alpha \times \cos \alpha \\ &= \frac{-\sin 10^\circ (1)}{-\sin 10^\circ} - \cos^2 \alpha \\ &= 1 - \cos^2 \alpha \\ &= \sin^2 \alpha \end{aligned}$$

(6)

c) Prove that:

(4)

$$\frac{\cos 2x - 1}{\sin 2x} = -\tan x$$

$$\begin{aligned}\text{LHS} &= \frac{\cos 2x - 1}{\sin 2x} \\ &= \frac{1 - 2\sin^2 x - 1}{2\sin x \cos x} \\ &= \frac{-2\sin^2 x}{2\sin x \cos x} \\ &= -\frac{\sin x}{\cos x} \\ &= -\tan x \\ &= \text{RHS}\end{aligned}$$

Given the equation: $\tan(5\theta) = \tan \theta$

1) Determine the general solution.

(3)

$$\begin{aligned}CA &= \theta \\ 5\theta &= \theta + k180^\circ \\ 4\theta &= k180^\circ \\ \theta &= k45^\circ \quad k \in \mathbb{Z}\end{aligned}$$

2) Write down the value(s) of $\theta \in [-90^\circ; 90^\circ]$ for which $\tan \theta$ is undefined.

(1)

$$\theta = -90^\circ \quad \text{OR} \quad \theta = 90^\circ$$

3) Hence, or otherwise, determine the values of $\theta \in [-90^\circ; 90^\circ]$ which satisfy the equation.

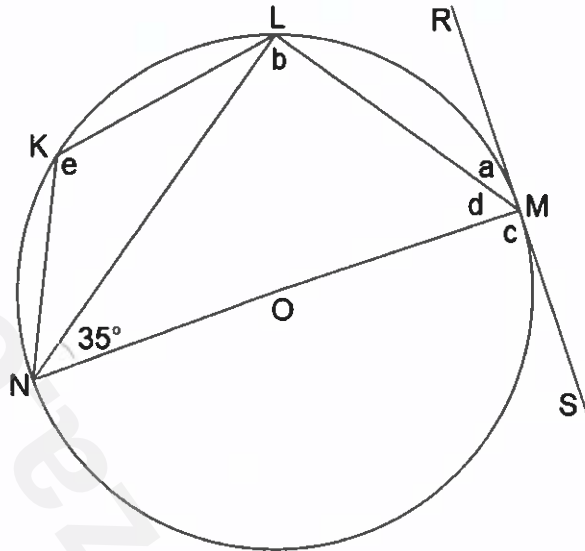
(3)

$$\theta \in \{-45^\circ; 0^\circ; 45^\circ\}$$

QUESTION 6

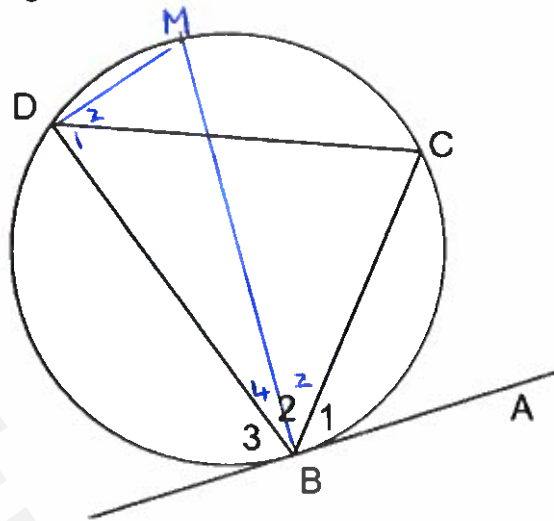
10 MARKS

- a) In the diagram, RMS is a tangent to the circle at M , and O is the centre of the circle. MON is a straight line. $\angle LNM = 35^\circ$. Determine the sizes of the angles marked a-e, giving reasons. (5)



- $a = 35^\circ$ ✓ tan chord
 $b = 90^\circ$ ✓ $\angle$ in semi circle
 $c = 90^\circ$ ✓ tan $\perp$ rad
 $d = 55^\circ$ ✓ $\angle$ s in a Δ
 $e = 125^\circ$ ✓ Int $\angle$'s of the cyclic quad.

- b) In the diagram below a tangent has been drawn to the circle at B . Prove that $\hat{B}_1 = \hat{D}$. (5)



Construct BM , diameter and chord DM

$$\hat{B}_1 + \hat{B}_2 = 90^\circ \quad \checkmark \quad \text{tan} \perp \text{rad}$$

$$\hat{D}_1 + \hat{D}_2 = 90^\circ \quad \checkmark \quad \angle \text{ in semicircle}$$

$$\hat{D}_2 = \hat{B}_2 \quad \checkmark \quad \angle \text{ same segment}$$

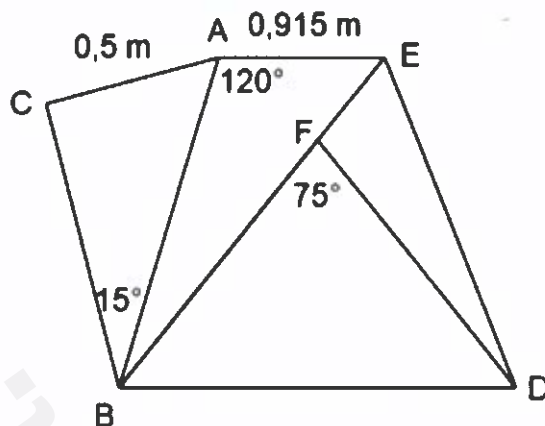
$$\therefore \hat{B}_1 = \hat{D}_1 \quad \checkmark$$

SECTION B

QUESTION 7

9 MARKS

The figure shows a collection of triangles. AC is 0,5 m, AE is 0,915 m, $\hat{ABC} = 15^\circ$, $\hat{BAE} = 120^\circ$ and $\hat{C} = 90^\circ$.



- a) Calculate the length of AB , to one decimal place.

$$\sin 15^\circ = \frac{0,5}{AB}$$

$$AB = 1,93 \text{ m}$$

$$= 1,9$$

- b) Calculate the length of BE .

$$BE^2 = AE^2 + AB^2 - 2(AE)(AB)\cos 120^\circ$$

$$= (0,915)^2 + (1,9)^2 - 2(0,915)(1,9)\cos 120^\circ$$

$$= 6,3$$

$$BE = 2,5099... = 2,5 \text{ m}$$

- c) Calculate the area of $\triangle BFD$ if $\hat{BFD} = 75^\circ$, $BF = FD$ and $BF = \frac{5}{7}BE$.

$$BF = \frac{5}{7}(2,5)$$

$$= \frac{25}{14}$$

$$\text{Area} = \frac{1}{2}(FB)(FD)\sin 75^\circ$$

$$= \frac{1}{2}\left(\frac{25}{14}\right)\left(\frac{25}{14}\right)\sin 75^\circ$$

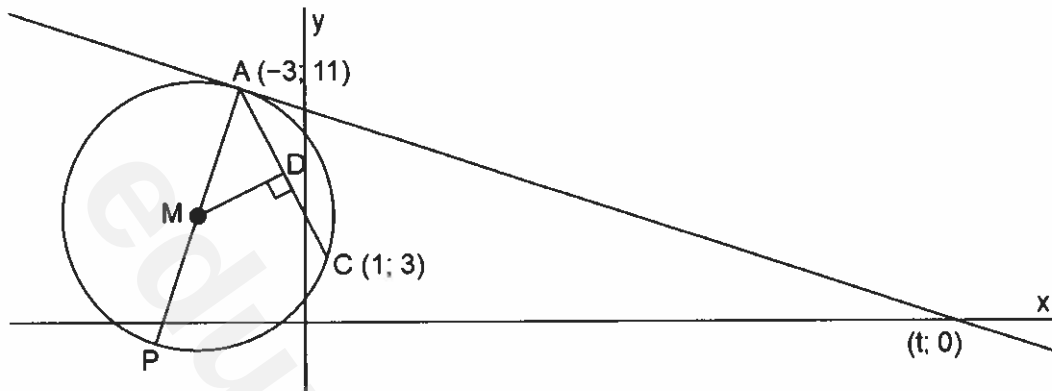
$$= 1,54006...$$

$$= 1,5 \text{ m}^2$$

QUESTION 8

18 MARKS

$A(-3; 11)$ and $C(1; 3)$ and P are points on the circle with centre M . $MD \perp AC$.
 AP the diameter of the circle, has equation $y = 3x + 20$.



- a) Determine the equation of DM

(5)

$$D = M_{AC} = \left(\frac{-3+1}{2}, \frac{11+3}{2} \right)$$

$$= (-1; 7)$$

$$m_{AC} = \frac{11-3}{-3-1}$$

$$= -2$$

$$m_{DM} = \frac{1}{2}$$

$$y-7 = \frac{1}{2}(x+1)$$

$$y = \frac{1}{2}x + \frac{15}{2}$$

b) Show that the equation of the circle is $(x + 5)^2 + (y - 5)^2 = 40$.

(6)

$$\frac{1}{2}x + \frac{15}{2} = 3x + 20$$

$$x + 15 = 6x + 40$$

$$-5x = 25$$

$$x = -5$$

$$\therefore y = 3(-5) + 20$$

$$= 5$$

$$M(-5; 5)$$

$$d_{MC} = \sqrt{(-5-1)^2 + (5-3)^2}$$

$$= \sqrt{40}$$

$$\therefore (x+5)^2 + (y-5)^2 = 40$$

c) Determine, correct to one decimal place, the size of $\hat{MAD}$.

(3)

$$m_{MA} = \frac{11-5}{-3-(-5)}$$

$$= \frac{6}{2}$$

$$= 3$$

$$m_{AD} = -2$$

$$\beta = \tan^{-1}(-2)$$

$$= 116,6^\circ$$

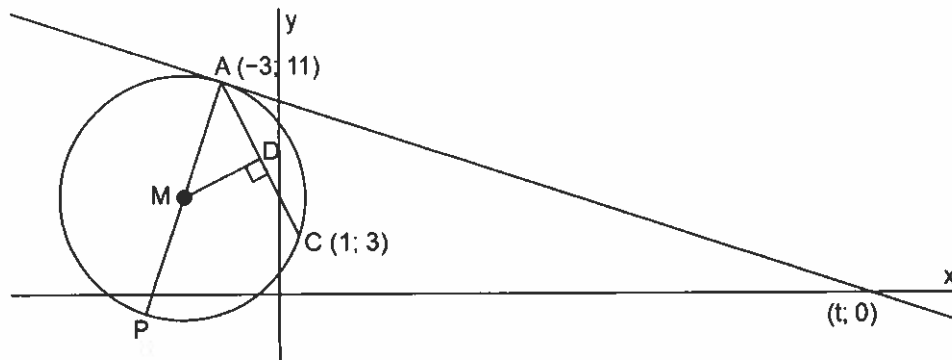
$$\theta = \tan^{-1}(3)$$

$$= 71,6^\circ$$

$$\hat{MAD} = 116,6^\circ - 71,6^\circ$$

$$= 45^\circ$$

Ext $\angle$ of Δ



- d) Find the numerical value of t if the tangent to the circle at A meets the x -axis at the point $(t; 0)$. (4)

$$m_t = -\frac{1}{3} \quad A(-3; 11)$$

$$y - 11 = -\frac{1}{3}(x + 3)$$

$$y = -\frac{1}{3}x + 10$$

$$\text{let } y = 0$$

$$0 = -\frac{1}{3}x + 10$$

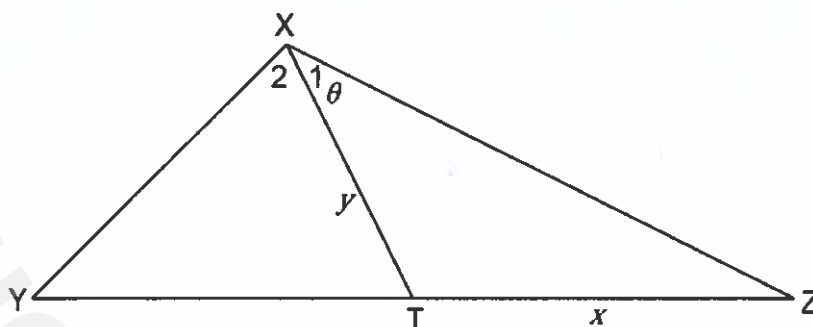
$$\frac{1}{3}x = 10$$

$$x = 30 = t$$

QUESTION 9

6 MARKS

In the triangle XYZ , T is the midpoint of YZ , $\hat{X}_2 = 2\hat{X}_1$ and $XT = y$.



- a) Write $\sin Y$ in terms of θ , x and y .

$$\frac{\sin Y}{y} = \frac{\sin 2\theta}{x}$$

$$\sin Y = \frac{y \sin 2\theta}{x}$$

- b) Write $\sin Z$ in terms of θ , x and y .

$$\frac{\sin Z}{y} = \frac{\sin \theta}{x}$$

$$\sin Z = \frac{y \sin \theta}{x}$$

- c) Hence, show that:

$$\frac{\sin Y}{\sin Z} = 2 \cos \theta$$

$$\frac{\sin Y}{\sin Z} = \frac{y \sin 2\theta}{x} \times \frac{x}{y \sin \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta}$$

$$= 2 \cos \theta$$

QUESTION 10

14 MARKS

a) Given the equation:

$$\frac{1}{2} \tan x = \sin 2x$$

1) Show that the equation can be simplified to:

$$\sin x (4 \cos^2 x - 1) = 0$$

(3)

$$\frac{1}{2} \left(\frac{\sin x}{\cos x} \right) = 2 \sin x \cos x$$

$$\sin x = 4 \sin x \cos^2 x$$

$$0 = 4 \sin x \cos^2 x - \sin x$$

$$0 = \sin x (4 \cos^2 x - 1)$$

2) Hence, determine the general solution to the equation.

(5)

$$\sin x = 0 \quad \text{OR} \quad \cos^2 x = \frac{1}{4}$$

$$\cos x = \frac{1}{2} \quad \text{OR} \quad \cos x = -\frac{1}{2}$$

$$\textcircled{1} \quad x = 0^\circ + k/180^\circ$$

$$\textcircled{2} \quad CA = 60^\circ$$

$$\text{OR} \quad x = 60^\circ + k360^\circ$$

$$x = -60^\circ + k360^\circ$$

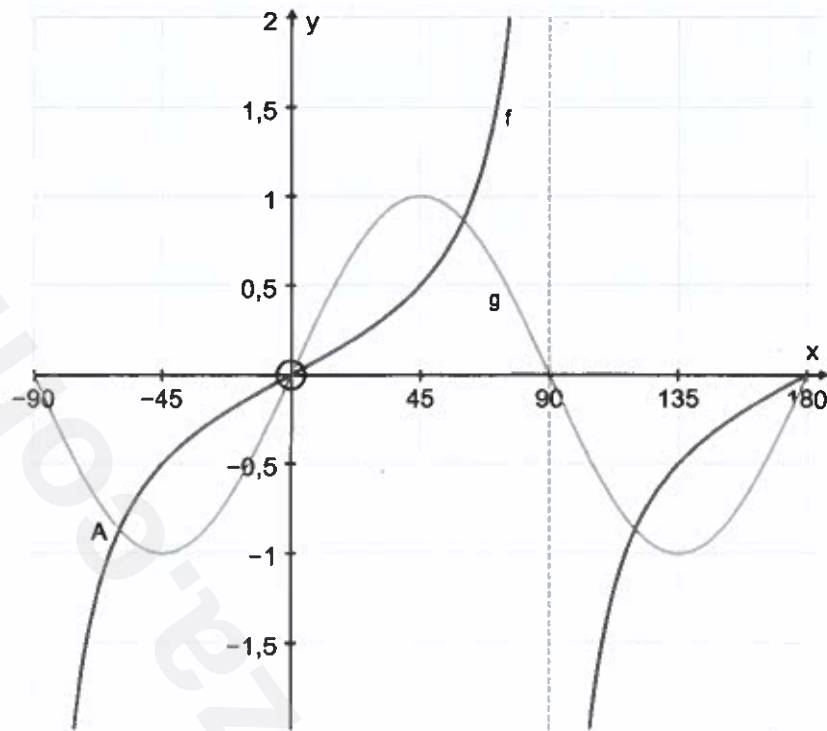
$$\textcircled{3} \quad CA = 120^\circ$$

$$\text{OR} \quad x = 120^\circ + k360^\circ$$

$$x = -120^\circ + k360^\circ$$

$$k \in \mathbb{Z}$$

- b) Sketched below are the graphs of $f(x) = \frac{1}{2} \tan x$ and $g(x) = \sin 2x$, where the x -coordinate at point A is -60°



Use your solutions obtained in (a) as well as the graphs to determine the value(s) of $x \in [-90^\circ; 180^\circ]$, for which:

1) $f(x) \geq g(x)$

$$x \in [-60^\circ; 0^\circ] \cup [60^\circ; 90^\circ) \cup [120^\circ; 180^\circ]$$

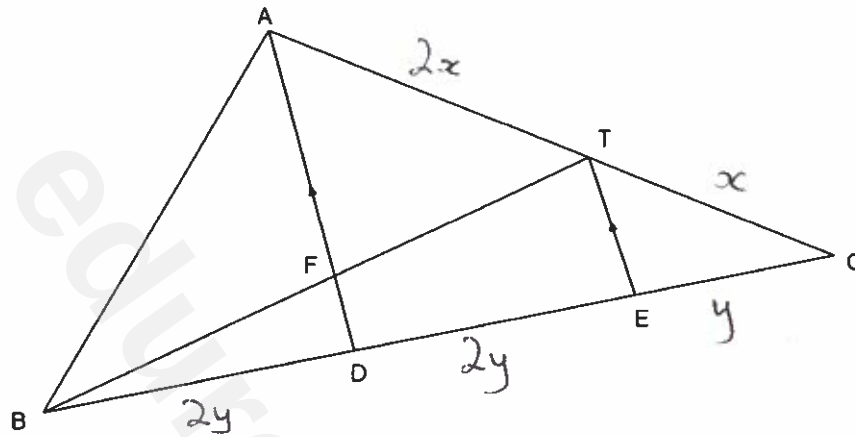
2) $f'(x) > 0$ and $g'(x) > 0$

$$x \in (-45^\circ; 45^\circ) \cup (135^\circ; 180^\circ)$$

QUESTION 11

8 MARKS

In the figure, $\triangle ABC$ has D and E on BC such that $BD = 6\text{cm}$ and $DC = 9\text{cm}$. $AT:TC = 2:1$ and $AD \parallel TE$.



c) Write down the numerical value of $\frac{CE}{ED}$.

$$\frac{CE}{ED} = \frac{CT}{TA} = \frac{1}{2} \quad \text{Prop theorem}$$

Show that D is the midpoint of BE .

$$DE:CD = 2y:3y = DE:9\text{cm}$$

$$\therefore DE = 6\text{cm}$$

$$DE = 6\text{cm} = BD$$

$\therefore D$ is the midpoint.

If $FD = 2\text{cm}$, calculate the length of TE .

$$TE = 4\text{cm} \quad \text{converse Midpoint theorem.}$$

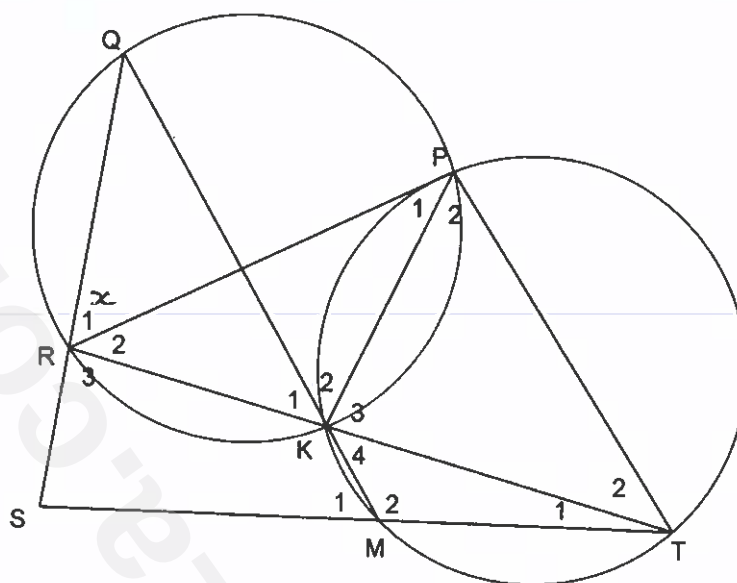
d) Calculate the numerical value of:

$$\begin{aligned} \frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD} &= \frac{\frac{1}{2} h(DC)}{\frac{1}{2} h(BD)} \\ &= \frac{9\text{cm}}{6\text{cm}} = \frac{3}{2} \end{aligned}$$

QUESTION 12

7 MARKS

In the diagram, ^{equal} circles $QRKP$ and $PKMT$ intersect at P and K . RKT , QKM , QRS and SMT are straight lines. $RK = PK$. Let $\hat{R}_1 = x$.



- a) Show that $\hat{MTP} = x$.

$\hat{K}_2 = x$ ✓ $\angle$ is in same segment.

$\hat{MTP} = x$ ✓ Ext $\angle$ of cyclic quad. ✓

- b) Hence, or otherwise, prove that $PRST$ is a cyclic quadrilateral.

$PRST$ is cyclic, Converse of Ext $\angle$ of cyclic quad) ✓

- c) Prove that RP is a tangent to circle $PKMT$ at P .

$$\hat{P}_1 = \hat{R}_2$$

$$\hat{R}_2 = \hat{T}_2$$

$$\therefore \hat{P}_1 = \hat{T}_2$$

✓ Isos Δ , $PK = RK$

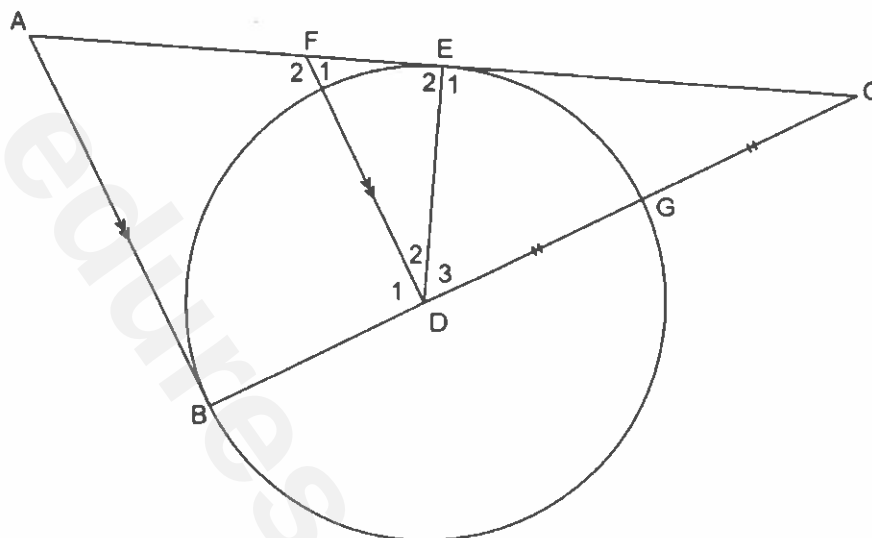
✓ Equal circles, equal chords

$\therefore PR$ is a tangent (converse tan chord) ✓

QUESTION 13

13 MARKS

In the diagram, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C . $DG = CG$. F is a point on AC such that $DF \parallel AB$.



Find, with reasons, the ratio of $\frac{AC}{FC}$.

$$\frac{AC}{FC} = \frac{BC}{CD} = \frac{3r}{2r} = \frac{3}{2}$$

line $\parallel$ to side of $\triangle$

b) Prove, giving reasons, that $\triangle ABC \parallel \triangle DEC$.

in $\triangle ABC$ and $\triangle DEC$

$$\hat{C} = \hat{C} \quad \text{common}$$

$$\hat{E}_1 = \hat{B} = 90^\circ \quad \text{tan } \perp \text{ rad}$$

$$\hat{A} = \hat{D}_3 \quad \text{L's in a } \Delta$$

$$\therefore \triangle ABC \parallel \triangle DEC$$

c) Prove that $DE^2 = \frac{AE \cdot EC}{3}$.

(5)

$$\therefore \frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC} \quad \checkmark \quad \triangle ABC \parallel \triangle DEC$$

$$\frac{AB}{DE} = \frac{BC}{EC} \quad \checkmark$$

$$AB = AE \quad \checkmark \quad \text{tangents from the same point}$$

$$BC = 3ED \quad \checkmark \quad \text{radii}$$

$$\therefore \frac{AE}{DE} = \frac{3ED}{EC} \quad \checkmark$$

$$\therefore \frac{AE \cdot EC}{3} = ED^2$$

EXTRA WORKING:

eduresourceza.com