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Sacred Heart College

MARIST OBSERVATORY

EDUCATION WITH HEART

PRELIMINARY EXAMINATION

MATHEMATICS PAPER 2

GRADE 12

SEPTEMBER 2024

Time: 3 hours

Marks: 150

Examiner: MF Olayiwola

Moderator: I. Marais

NAME: _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 32 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions on the question paper and hand this in at the end of the examination. Remember to write your name and surname in the space provided.
4. Write your answers in the spaces provided.
5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
6. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers. Answers only will NOT necessarily be awarded full marks.
7. Diagrams are not necessarily drawn to scale.
8. If necessary, round off answers to ONE decimal place, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. TWO blank pages (pages 31-32) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

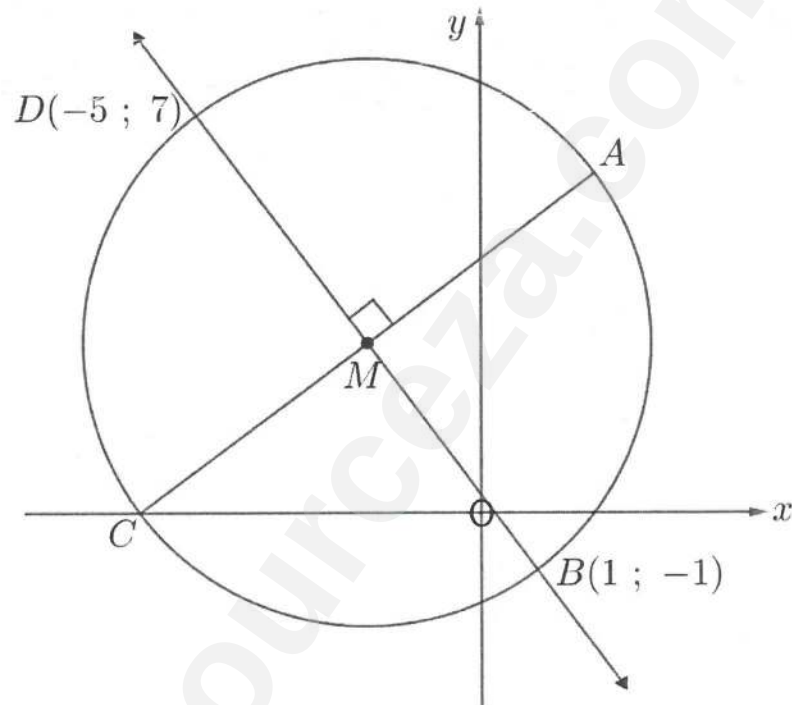
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
13	10	5	15	23	10	8	10	22	20	7	7	150

SECTION A**QUESTION 1**

In the diagram below, M is the centre of the circle $ABCD$.

Secant line BMD intersects the circle at $B(1; -1)$ and $D(-5; 7)$.

Diameter AC is drawn such that $AC \perp BD$.



- (a) Determine the coordinates of M , the centre of the circle.

(2)

- (b) Determine the length of the radius of the circle.

(2)

- (c) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

- (d) Determine the gradient of BMD .

(2)

- (e) Determine the equation of the diameter AC in the form $ax + by = c$, where a, b and c are integers.

(3)

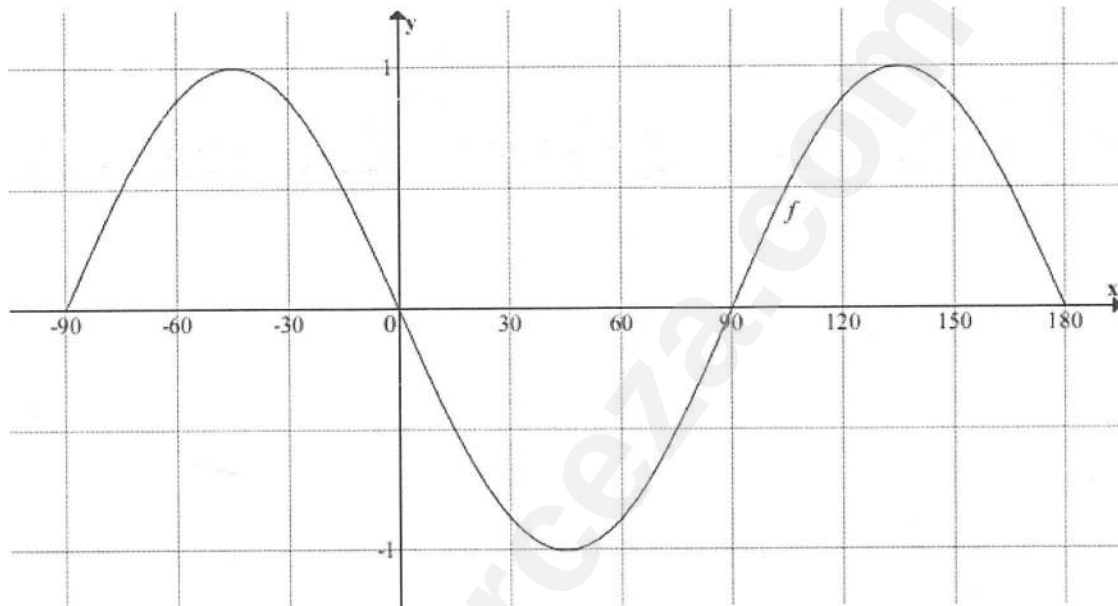
- (f) If C is an x - intercept of the circle, determine the coordinates of C .

(2)

[13]

QUESTION 2

In the diagram below, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \in [-90^\circ; 180^\circ]$.



- (a) Write down the period of $f(x)$.

(1)

- (b) On the same set of axes, draw the graph of $g(x) = \cos(x + 30^\circ)$ for the interval $x \in [-90^\circ; 180^\circ]$. Clearly show all intercepts with the axes as well as the turning and end points.

(3)

- (c) Determine the general solution of $f(x) = g(x)$.

(5)

- (d) If k is a positive real number, then for what value(s) of k will $f(x) = k$ have no real solutions?

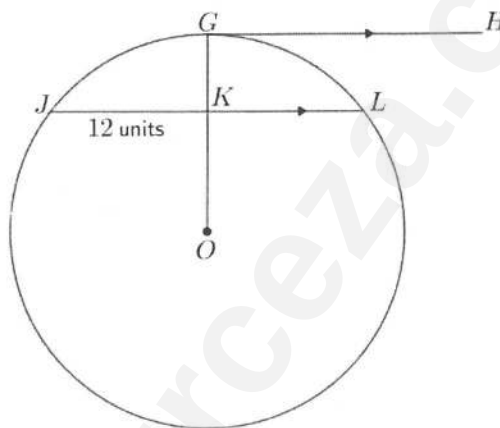
(1)

[10]

QUESTION 3

In the diagram below:

- O is the centre of circle GLJ .
- GH is a tangent to the circle at point G .
- OG intersects chord JL at K .
- $GH \parallel JL$ and $JK = 12$ units.



Determine, giving reasons, the

- (a) size of $\angle OGH$.

(2)

- (b) length of JL .

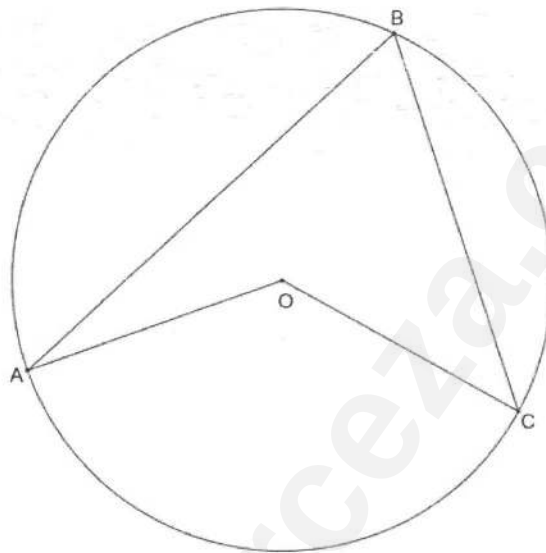
(3)

[5]

QUESTION 4

(a) Use the diagram below to prove the theorem that states:

“The angle subtended by a chord at the centre of a circle is twice the size of the angle that it subtends at the circle.”



Required to prove: $\angle AOC = 2 \times \angle ABC$.

Construction:

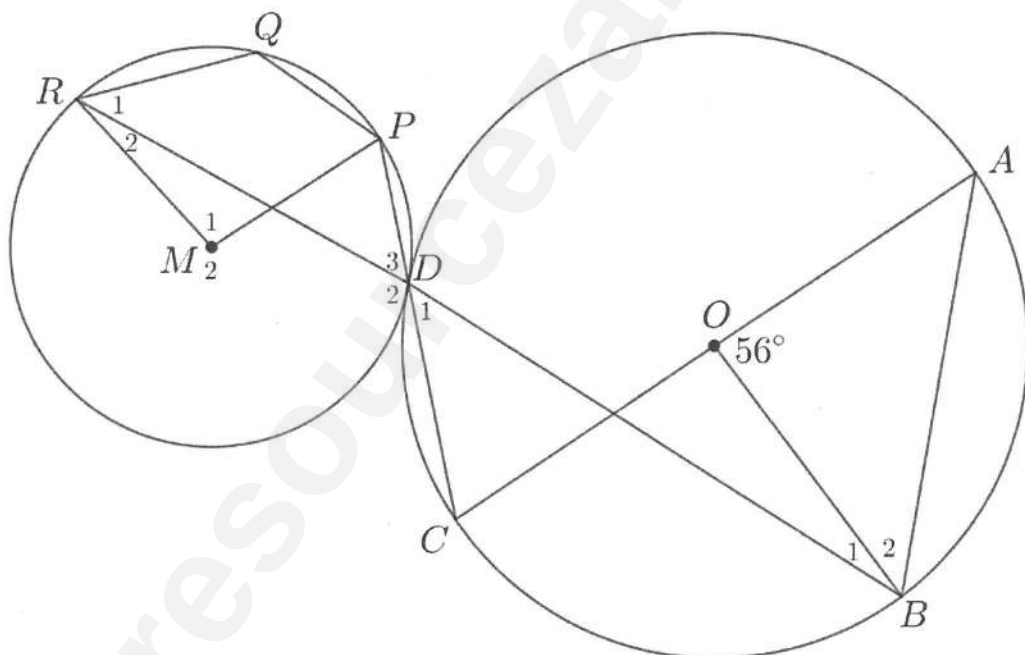
(1)

Proof:

(5)

(b) In the diagram below:

- O is the centre of the larger circle which passes through the points A, B, C and D .
- M is the centre of the smaller circle which passes through the points P, Q and R and intersects the larger circle at D .
- AC is a diameter.
- RDB and PDC are straight lines.
- $\angle AOB = 56^\circ$.



Determine, giving reasons, the size of each of the following angles:

(1) $\hat{A}$.

(3)

(2) $\hat{D}_1$.

(3) $\hat{Q}$.

(4) $\hat{M}_1$.

(2)

(2).

(2)

[15]

QUESTION 5

- (a) If $\sin 21^\circ = p$, determine, without using a calculator, the value of the following in terms of p :

(1) $\cos 21^\circ$

(2)

(2) $\sin 9^\circ$

(3)

(3) $\sin 132^\circ$

(3)

(b) Prove the identity:

$$\frac{\sin \beta}{1 + \sin(90^\circ + \beta)} - \frac{\sin(-\beta)}{1 - \cos(-\beta)} = \frac{2}{\sin \beta}$$

(5)

(c) Solve for a and b :

$$\cos(a + b) = -\frac{\sqrt{2}}{2} \text{ for } (a + b) \in [0^\circ; 180^\circ]$$

$$\cos(a - 2b) = \frac{1}{2} \text{ for } (a - 2b) \in [0^\circ; 180^\circ]$$

(4)

- (d) It is given that $\triangle XYZ$ is equilateral with an area of 20,09 units².
Determine the length of a side of $\triangle XYZ$.

(3)

- (e) Prove that

$$\sqrt{-\cos^2(90^\circ - M) - \cos M \cdot \cos(360^\circ - M)}$$

is non-real for all real angles M .

(3)

[23]

QUESTION 6

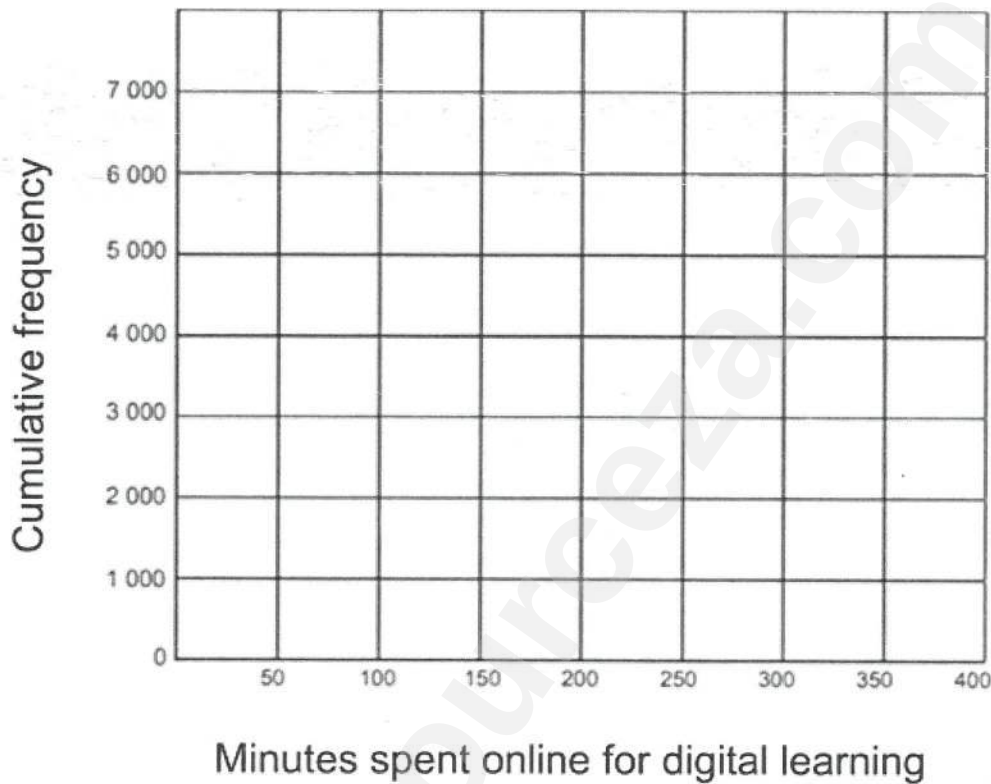
A group of 7 000 grade 12 learners were asked how many minutes they spend online for the purposes of digital learning. The results are summarised in the table below. Use this table to answer the questions that follow:

Minutes spent online for digital learning	Frequency	Midpoint
$50 \leq x < 100$	500	75
$100 \leq x < 150$	900	125
$150 \leq x < 200$	1 500	175
$200 \leq x < 250$	1 600	225
$250 \leq x < 300$	1 400	275
$300 \leq x < 350$	700	325
$350 \leq x < 400$	400	375

- (a) Calculate an estimate for the mean of minutes spent online for digital learning.

(2)

- (b) On the grid provided below, sketch a cumulative frequency curve to represent the above data:



- (c) Describe the skewness of the data. (3)

(1)

- (d) Upon further investigation, it is discovered that those learners who spend less than 150 minutes online for digital learning have limited wifi access. The Department of Education undertakes to equip these learners with wifi which will allow them to each spend 50 more minutes online for digital learning.

Explain, giving reasons, how this would effect:

- (1) the median.

(2)

- (2) the standard deviation.

(2)

[10]

76 marks

SECTION B**QUESTION 7**

Information about advertising via pamphlets and television separately is given in the table below.

- The correlation between the expenditure (in rand) on advertising via **pamphlets** (x) and sales (y) is recorded in the first row.
- The correlation between the expenditure (in rand) on **television** advertising (x) and sales (y) is recorded in the second row of the table.

	Line of best fit (including outliers)	Correlation Coefficient	The number of outliers with high expenditure and LOW sales	The number of outliers with low expenditure and HIGH sales
Pamphlets	$y = 11\,000 + 2x$	0,95	1	0
Television	$y = 8\,000 + 4x$	0,88	0	3

- (a) Which line of best fit would be the most accurate? Give a reason for your answer.

(2)

- (b) If the outliers were removed, how would this affect the:

- (1) correlation coefficient of television advertising? Give a reason for your answer.

(2)

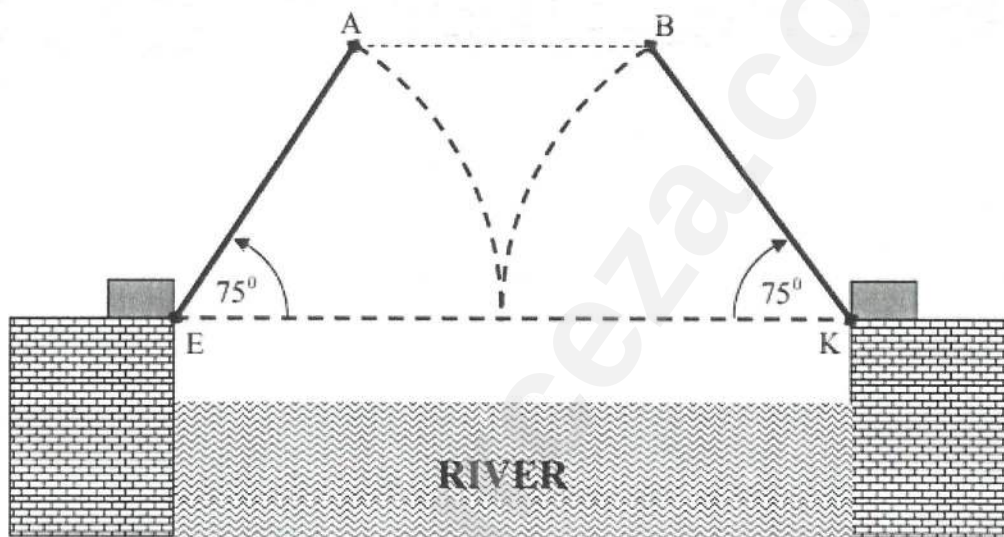
(2) gradient for each line of best fit?

(c) Using the information in the table above, explain which method of advertising you would use and why. (2)

(2)
[8]

QUESTION 8

- (a) The diagram below is an illustration of a vertical bridge positioned over a river. The bridge is made up of two arms each 25 metres in length i.e. $AE = BK = 25$ metres. To allow ships to pass through, the arms AE and BK rotate upwards about E and K respectively, until they make an angle of 75° with the horizontal.

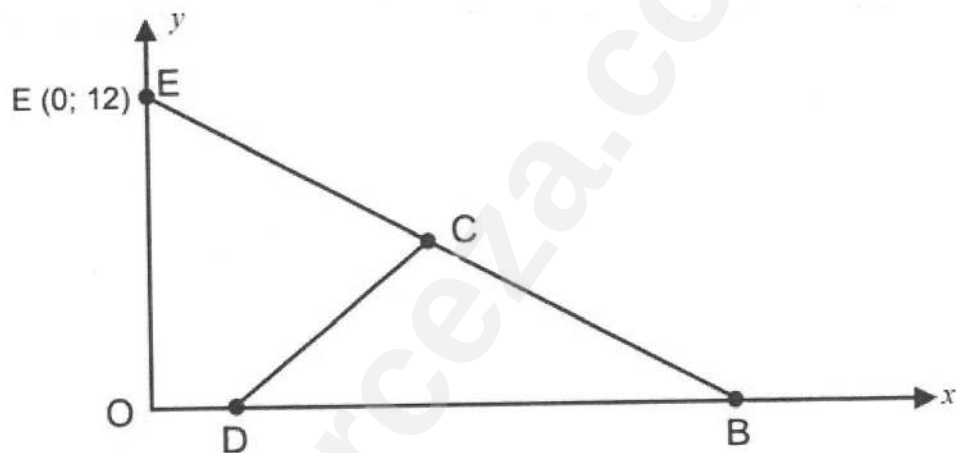


Determine the length of AB , the straight line distance between the tips of the bridge arms AE and BK .

(4)

(b) In the diagram below:

- $E(0; 12)$ is a point on the y – axis and D is a point on the x – axis.
- B lies on the x – axis and C is a point on line EB .
- $\angle ECD = 79,29^\circ$.
- The gradient of DC is $0,35$.



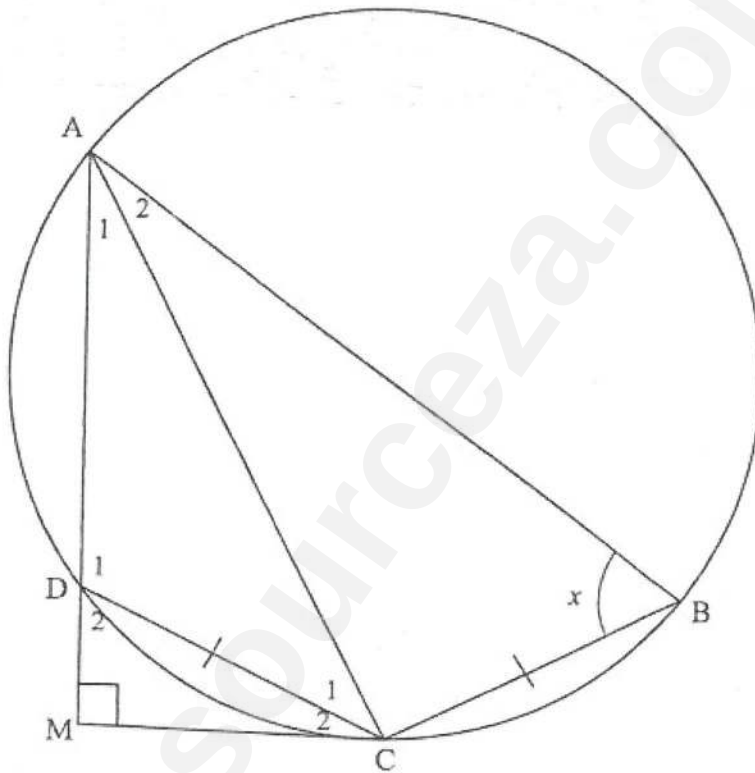
Calculate, to one decimal place, the x – coordinate of point B .

(6)
[10]

QUESTION 9

(a) In the diagram,

- $ABCD$ is a circle with AB a diameter of the circle and $BC = DC$.
- AD is produced to M such that $AM \perp CM$.
- Let $\hat{B} = x$.



- (1) Determine the size of $\hat{A}_1$ in terms of x .

(4)

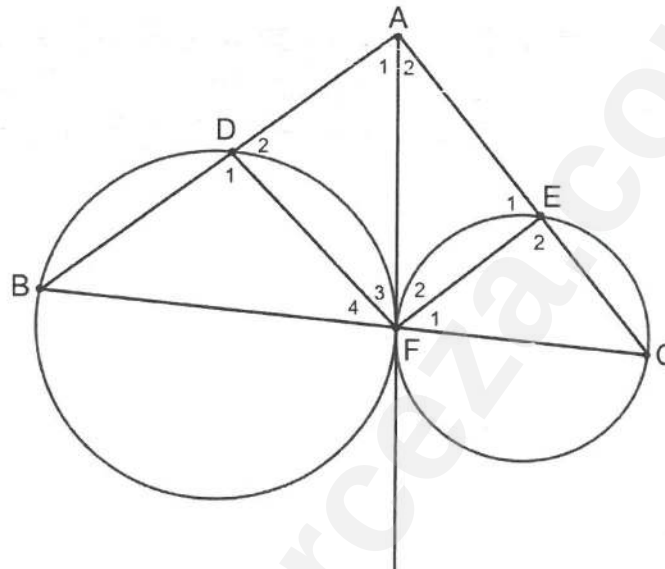
- (2) Prove that MC is a tangent to the circle at C .

- (3) Prove that $AB^2 - AC^2 = AB \cdot MD$.

(4)

(5)

- (b) In the diagram below, AF is a common tangent to the two given circles at F .
- B and D are points on the larger circle and E and C are points on the smaller circle.
 - ADB , AEC and BFC are straight lines.

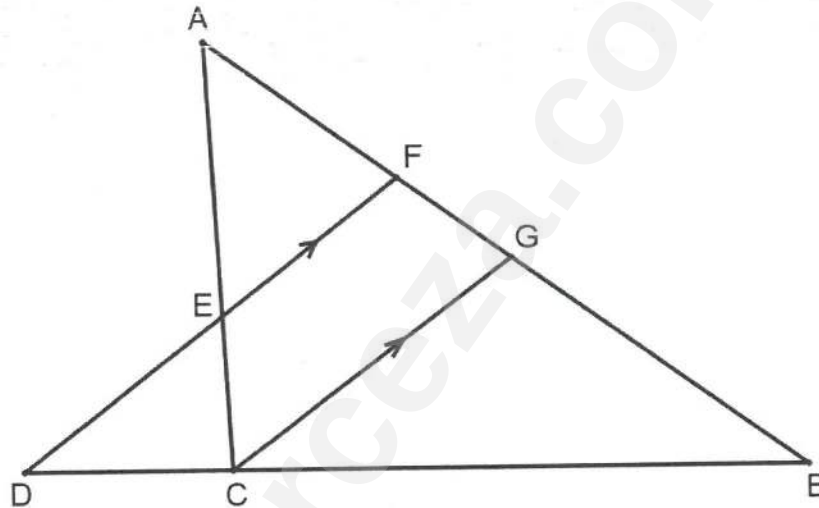


Prove that $AEFD$ is a cyclic quadrilateral.

(4)

(c) In the diagram $\triangle ABC$ is drawn.

- E and F are points on AC and AB respectively such that $\frac{AE}{EC} = \frac{3}{2}$ and $\frac{AF}{FB} = \frac{2}{5}$
- BC produced meets FE produced in D .
- G is a point on FB so that $FD \parallel GC$.



Calculate, giving reasons, $\frac{BC}{CD}$

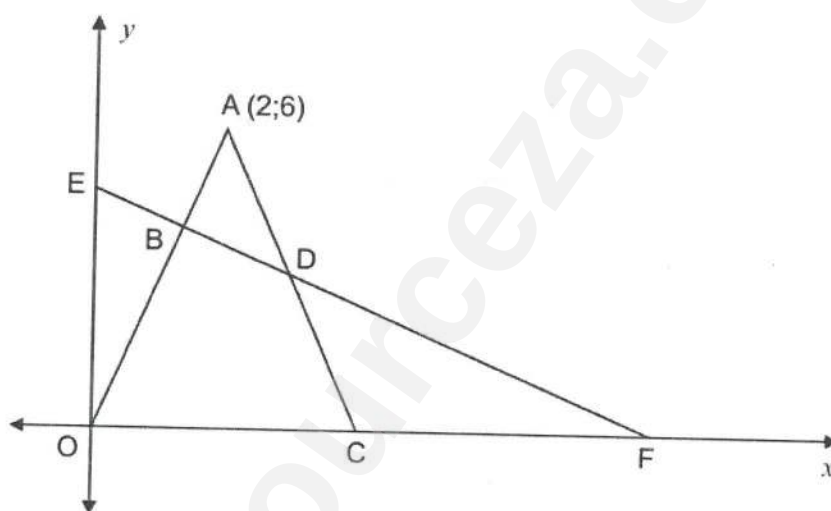
(5)

[22]

QUESTION 10

(a) In the diagram below, $\triangle AOC$ with $A(2; 6)$ and $O(0; 0)$ is drawn.

- C is a point on the x – axis.
- $AO = AC$.
- E lies on the y – axis and F lies on the x – axis.
- Line EF goes through points B and D which lie on OA and CA respectively.
- The equation of EF is given by $y = -\frac{x}{2} + 5$.



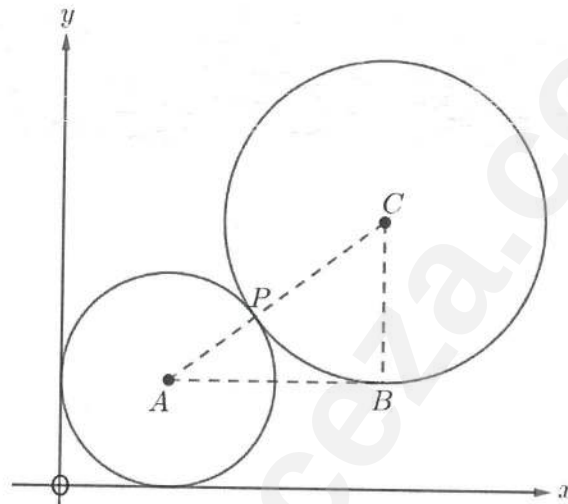
- (1) Determine the coordinates of B and hence the area of $\triangle EBO$.

(8)

- (2) If it is now given that $D\left(x; \frac{18}{5}\right)$, determine the area of $\triangle DCF$.

(4)

- (b) In the diagram below, two circles with respective centres A and C are drawn such that they are tangential at P . B is a point on the larger circle and AB is a tangent to the larger circle at B . It is given that the equation of the smaller circle is $x^2 + y^2 - 4x - 4y + 4 = 0$.



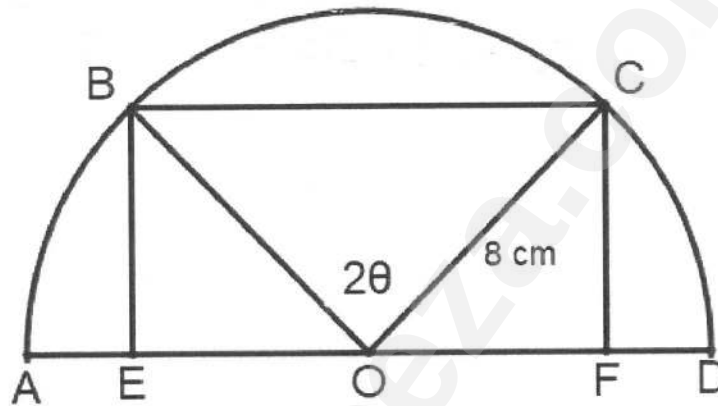
If $AB = 4$ units, determine the coordinates of point C , the centre of the larger circle.

(8)
[20]

QUESTION 11

In the diagram,

- Rectangle $EBCF$ is inscribed in a semicircle with centre O .
- The radius of the semicircle is 8 cm.
- $\widehat{BOC} = 2\theta$



- (a) If the perimeter of the rectangle $EBCF$ is represented by P , show that $P = 16\cos\theta + 32\sin\theta$.

(4)

- (b) If it is further given that $P = 16\sqrt{5}\sin(\theta + \alpha)$ and α is an acute angle, determine the value of α .

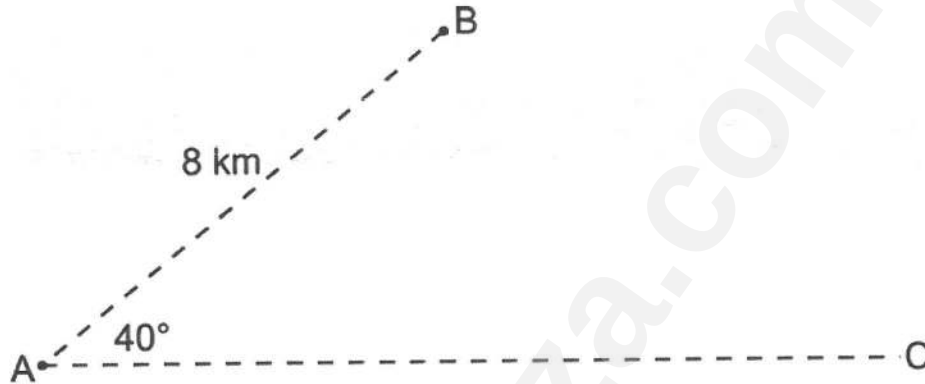
(3)

[7]

QUESTION 12

Points A and B are 8 km apart and on the same horizontal plane as point C .

The base, D , of a vertical building lies somewhere on AC . The vertical building is 7 km from point B and $\hat{BAC} = 40^\circ$.



If the vertical building is 160 metres in height, calculate the greatest possible angle of elevation from point A to the top of the building.

[7]

74 marks

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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