



**[eduresourceza.com](http://eduresourceza.com)**



## 2024 PRELIMINARY EXAMINATION PAPER I

**MATHEMATICS**

## GRADE 12

Time: 3 hours

Marks: 150

Examiner: I. Marais

Moderators: M. Olayiwola

L. Manson

NAME: \_\_\_\_\_

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This paper consists of 26 pages and an Information Sheet of 2 pages. Please check that your paper is complete.
2. Read the questions carefully and answer them all.
3. Write your answers on this question paper in the spaces provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphic calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **TWO DECIMAL PLACES** unless otherwise stated.
9. You must write legibly and present your work neatly.
10. Additional Space has been provided at the end of the question paper. If you run out of space for a question, use the Additional Space. Clearly label your answer and indicate at the question that you have used the Additional Space.

| SECTION A |    |    |    |    |    | SECTION B |    |    |     |     | TOTAL |
|-----------|----|----|----|----|----|-----------|----|----|-----|-----|-------|
| Q1        | Q2 | Q3 | Q4 | Q5 | Q6 | Q7        | Q8 | Q9 | Q10 | Q11 |       |
|           |    |    |    |    |    |           |    |    |     |     |       |
| 22        | 12 | 10 | 14 | 6  | 7  | 9         | 13 | 29 | 16  | 12  | 150   |

**SECTION A****[71 marks]****QUESTION 1**

- (a) Solve for  $x$  in each of the following, clearly showing all algebraic working:

(1)  $\sqrt{x-1} = 3\sqrt{5}$

(2)

(2)  $\frac{1}{3}x^2 = -6x$

(3)

(3)  $(2^x - 1)(2^x + 4)(2x - 1) = 0$

(3)

$$(4) \quad \log x^3 - \log x = 4$$

$$(5) \quad x^2 \leq 0$$

(3)

(1)

(b) Solve simultaneously for  $x$  and  $y$  if:

$$(x - 3)(y + 1) = 18$$

$$x - 4y = 1$$

(6)

- (c) Given  $x^2 + kx = 15 - x^2$ . If 5 is a root of the equation, determine the value of  $k$ , and hence, the other root of the equation.

(4)

[22]

**QUESTION 2**

- (a) Determine the sum of the first 90 terms of the following series:

$$4 + 15 + 26 + 37 + 48 + \dots$$

(2)

- (b) Calculate, clearly showing all working:

$$\sum_{n=3}^{\infty} 24 \cdot \left(\frac{1}{6}\right)^{n-2}$$

(3)

(c) The fourth term of a geometric series is 56.

(1) If the first term is 7, determine the common ratio.

(3)

(2) If  $S_n$  is the sum of the first  $n$  terms of the series, determine the largest value of  $n$  such that  $S_n < 350\,000$ .

(4)

[12]

**QUESTION 3**

Given the function  $p(x) = -\frac{1}{2}x^2 - 2x + 6$ .

- (a) Write down the domain of  $p(x)$ .

(1)

- (b) By completing the square, show that  $p(x)$  can be written as

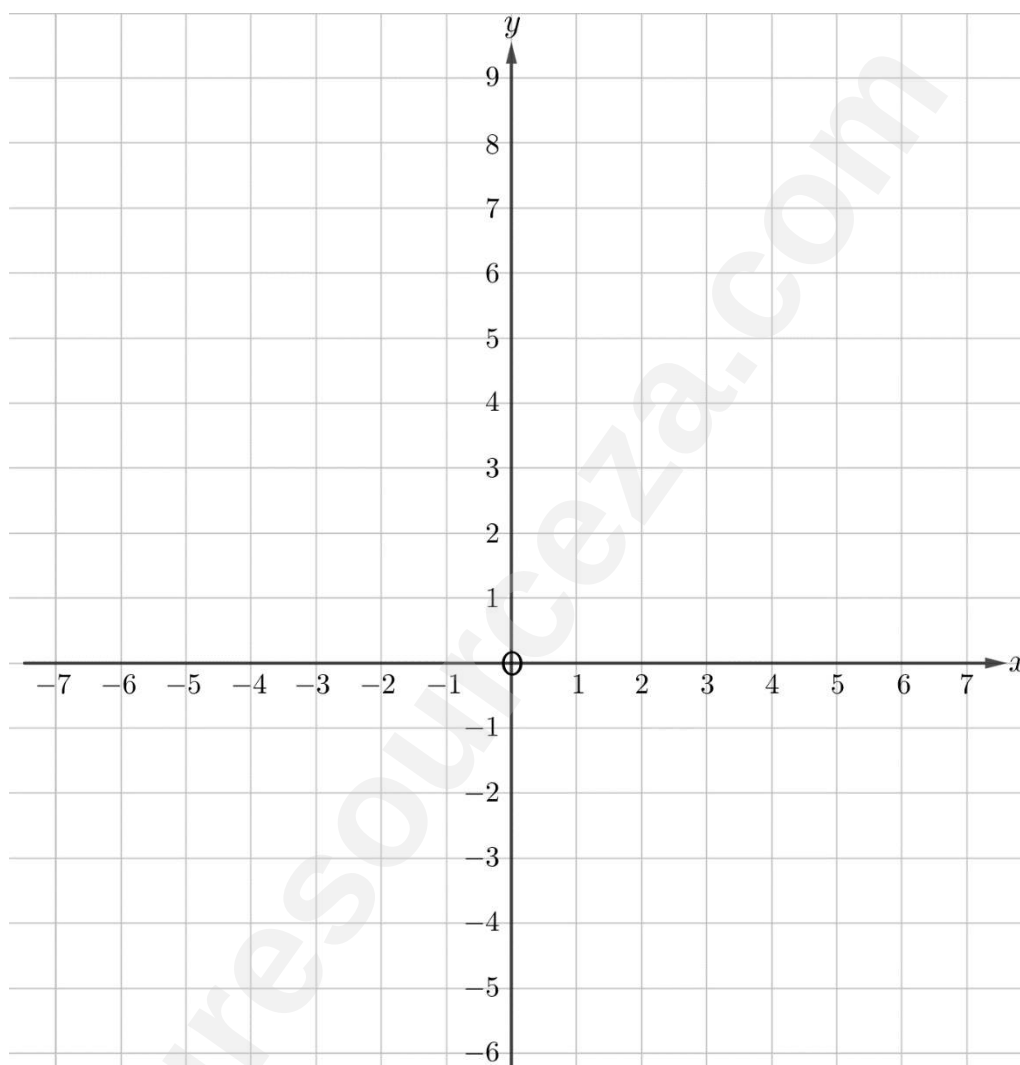
$$p(x) = -\frac{1}{2}(x + 2)^2 + 8.$$

(2)

- (c) Determine the  $x$  – intercepts of  $p(x)$ .

(4)

- (d) On the grid provided below, neatly sketch the graph of  $p(x)$ . Clearly indicate all intercepts with the axes as well as the coordinates of the turning point of  $p(x)$ .



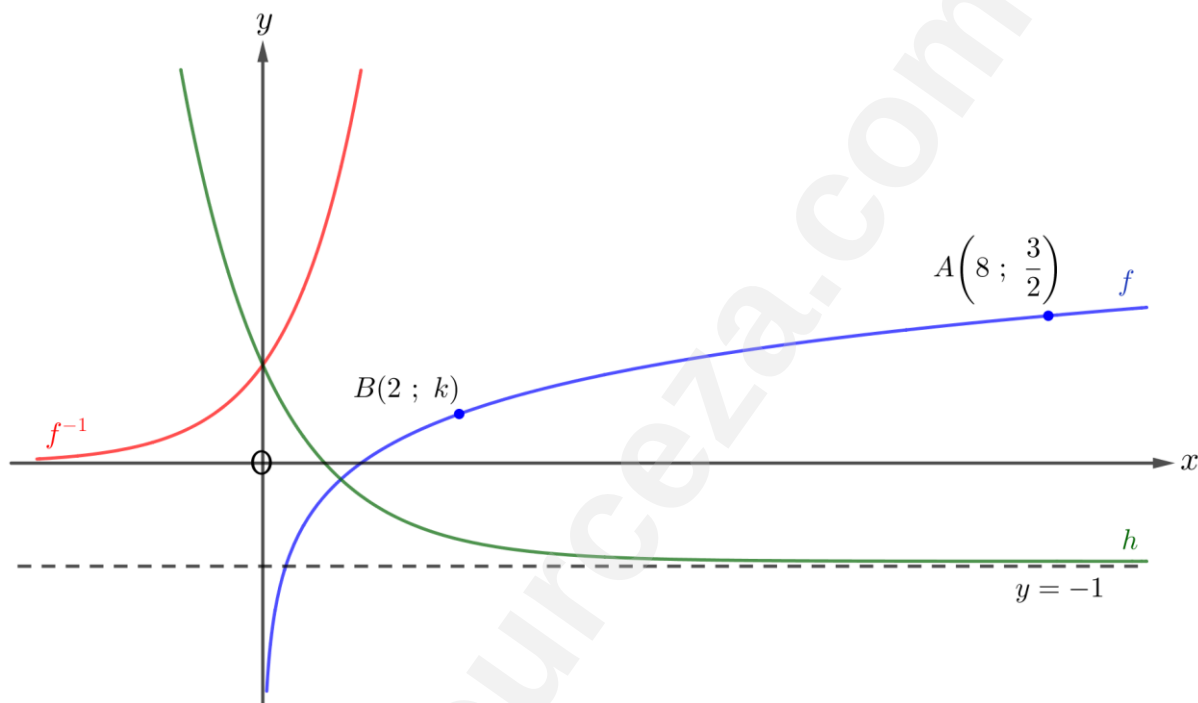
(3)

[10]

**QUESTION 4**

The figure shows the curve of  $f$  described by  $f(x) = \log_m x$  as well as its inverse  $f^{-1}(x)$ .

$A\left(8; \frac{3}{2}\right)$  is a point on the curve of  $f$ .



(a) Show that  $m = 4$ .

(2)

(b) If  $B$  is the point  $(2; k)$ , determine the numerical value of the average gradient between  $B$  and  $A$ .

(3)

(c) Write down the equation of  $f^{-1}(x)$ .

(1)

(d) Write down the equation of the line about which the graph of  $f(x)$  and the graph of  $f^{-1}(x)$  are symmetrical.

(1)

(e) Write down the equation of a new function  $g(x)$ , which has not been sketched, if  $g(x)$  is a reflection of  $f^{-1}(x)$  in the  $y$  – axis.

(1)

(f) Write down the equation of the vertical asymptote of  $f(x - 5)$ .

(1)

(g) For which value(s) of  $x$  is  $f(x) \leq 0$ ?

(2)

(h) The graphs of  $f^{-1}(x)$  and  $h(x)$  intersect on the  $y$  – axis. Furthermore,  $y = -1$  is an asymptote of  $h(x)$ . If  $h(x) = a \cdot 3^{-x} + q$ , find  $a$  and  $q$ .

(3)

[14]

**QUESTION 5**

- (a) Find the derivative from first principles of  $f(x) = 3x^2$

(4)

- (b) Determine  $g'(x)$  if  $g(x) = \frac{1}{\sqrt[5]{x^2}}$

(2)

**[6]**

### QUESTION 6

You invest in two different investment schemes for a period of 5 years.

- *Investment 1*

You place R3 500 into an investment at the end of each month.

Interest on this investment is 15% per annum compounded monthly.

- *Investment 2*

You make a deposit of R24 000 immediately and then only make a second deposit of R7 000 into the same account at the end of the third year.

Interest on this investment is 20% per annum compounded quarterly.

If you combine the value of the two investments at the end of the five-year period, then what is the value of the lump sum?

[7]

**SECTION B****[79 marks]****QUESTION 7**

You approach a bank for a home loan to the value of R850 000. This needs to be paid off monthly over a period of twenty years. The interest rate offered on the home loan is 9% per annum compounded monthly.

- (a) What are the minimum monthly payments required to pay off the loan over the 20 year-period?

(4)

- (b) If the monthly payment is changed to R9 000 per month, then what will be the outstanding balance on your loan at the end of 12 years?

*(This is directly after the payment is made at the end of the 12 years.)*

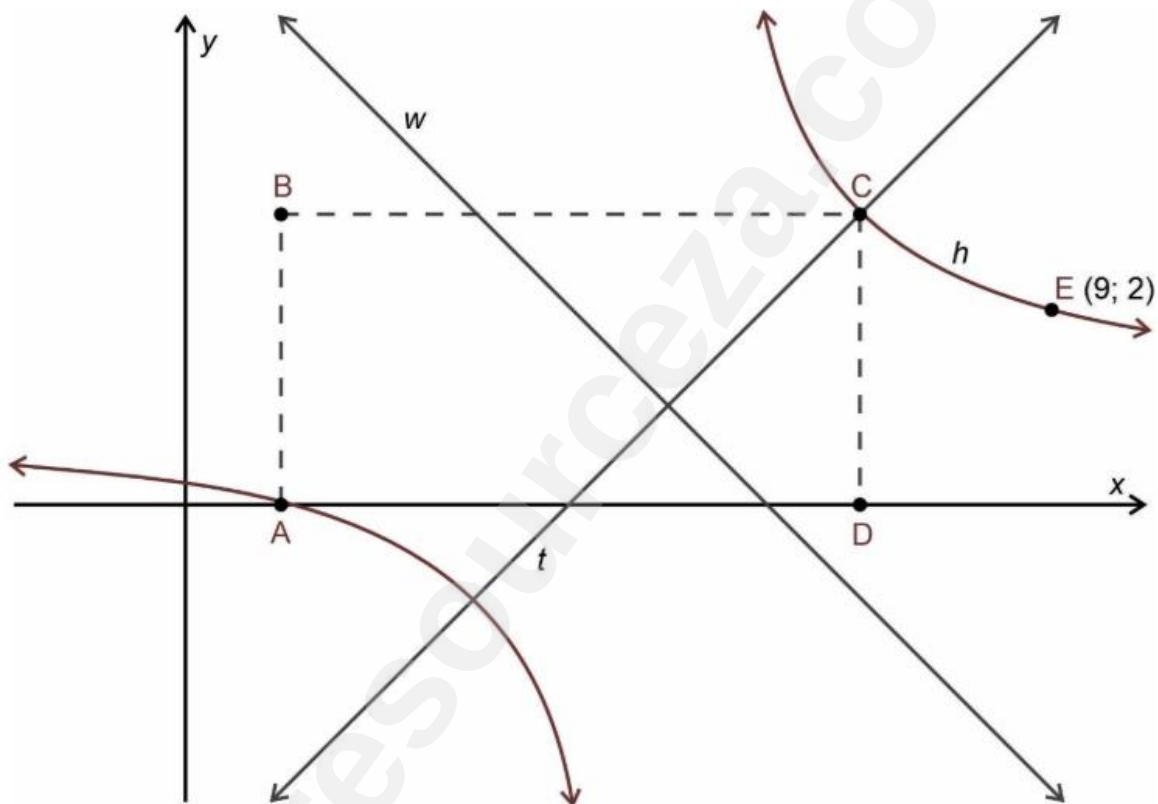
(5)

**[9]**

### QUESTION 8

In the diagram below:

- $w(x) = -x + 6$  and  $t(x) = x - 4$  are the axes of symmetry for the given graph  $h$ .
- $E(9; 2)$  lies on  $h$ .
- $ABCD$  is a rectangle with  $AD$  on the  $x$  - axis.



- (a) If  $h(x) = \frac{a}{x-p} + q$ , find the values of  $a$ ,  $p$  and  $q$ .

(6)

- (b) Determine the area of rectangle  $ABCD$ .

eduresourceza.com

(7)

[13]

### QUESTION 9

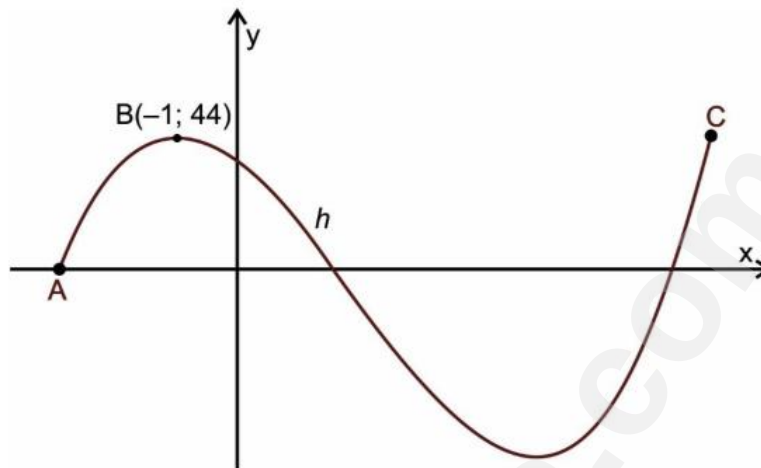
(a) You are given the following functions:

- $g(x) = 3x + c$ , and
- $f(x) = -2x^2 + 15x$ .

If  $g$  is a tangent to  $f$ , then calculate the coordinates of the point where  $g$  cuts the  $x$  – axis.

(5)

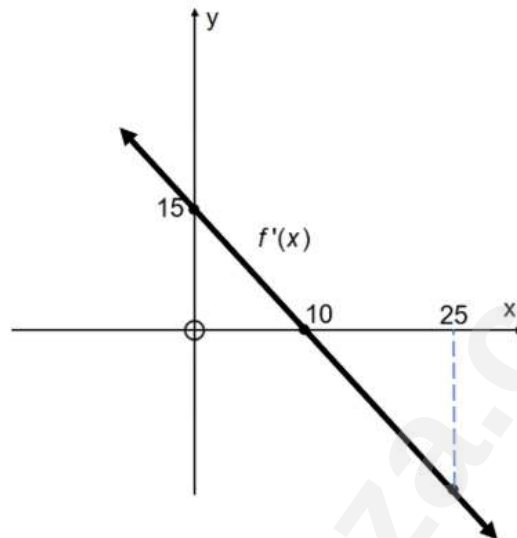
- (b) In the diagram below,  $B$  is one of the turning points of the cubic function  $h$ .



Find the values of  $a$  and  $b$  if  $h(x) = x^3 + ax^2 + bx + 36$ .

- (c) In the diagram, the graph of  $f'(x)$  is sketched for  $x \in \mathbb{R}$ .

**Note:** O represents the origin.



- (1) Write down the gradient of  $f$  at  $x = 0$ .

(1)

- (2) Write down the  $x$  — coordinate of the turning point of  $f$ .

(1)

- (3) What is the gradient of  $f$  at  $x = 1$ ?

(4)

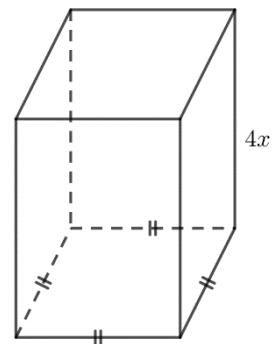
(4) Calculate the point of inflection of  $f$ .

(1)

(d) A wire, 12 metres long, is cut into two pieces. One piece is bent to form an equilateral triangle and the other piece is bent to form a square. A side of the triangle has a length of  $2x$  metres.

(1) Determine the length of a side of the square in terms of  $x$ .

(2) If the square is now used as the base of a rectangular prism with a height of  $4x$  metres (pictured alongside), determine the maximum volume of the rectangular prism.



(2)

(7)

[29]

**QUESTION 10**

- (a) The probability that a soccer player scores a goal during a match is  $2x$ . The probability that the same soccer player does not score a goal during a match is  $13x$ . Predict the number of goals the soccer player can expect to score in the next 60 matches.

(4)

- (b) Three items from each of four different departments of a major grocery store shall be featured in a one-page newspaper advertisement. The page layout for the advertisement is shown in the diagram below. One item is placed in each block  $A - L$ .

|       | Column 1 | Column 2 | Column 3 |
|-------|----------|----------|----------|
| Row 1 | $A$      | $B$      | $C$      |
| Row 2 | $D$      | $E$      | $F$      |
| Row 3 | $G$      | $H$      | $I$      |
| Row 4 | $J$      | $K$      | $L$      |

- (1) In how many different ways may the twelve items be arranged in the advertisement?

(2)

- (2) A specific item from the twelve MUST be placed in block  $A$ , another MUST be placed in block  $F$  and a third MUST be placed in block  $K$ . In how many different ways may the items now be arranged.

(2)

- (3) In how many different ways may the items be arranged in the advertisement if the items from the same department are grouped together in the same row? Note: the items may be arranged in any order in their specific row.

(3)

- (b) In the diagram below, a square dartboard is shown. The dartboard is divided into 25 identical smaller squares, some of which are shaded and some of which are numbered.

|  |   |   |   |  |
|--|---|---|---|--|
|  |   |   |   |  |
|  | 4 | 9 | 2 |  |
|  | 3 | 5 | 7 |  |
|  | 8 | 1 | 6 |  |
|  |   |   |   |  |

The number shown in each square represents the score obtained by a dart landing in that square.

A dart landing in the shaded region scores 0.

If two darts are thrown and hit the dartboard, calculate the probability that the sum of the scores is exactly 9 points.

(5)

[16]

**QUESTION 11**

- (a) Three consecutive terms of an arithmetic sequence are given as follows:

$$\frac{1}{b+c}; \frac{1}{c+a}; \frac{1}{a+b}$$

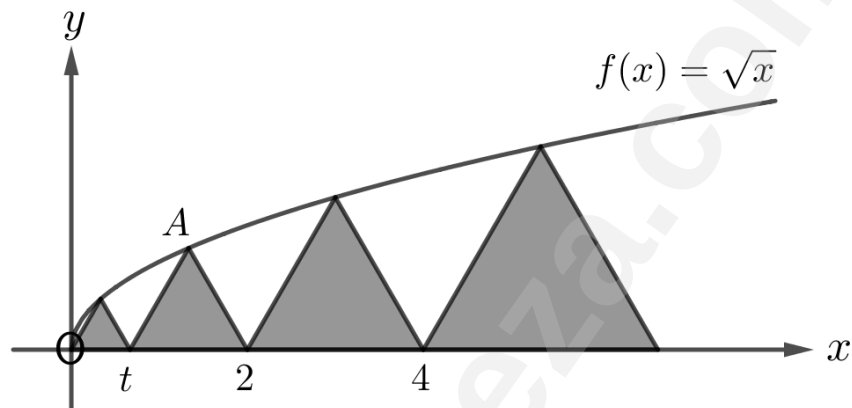
Show that  $a^2$ ,  $b^2$  and  $c^2$  are also three consecutive terms of an arithmetic sequence.

(5)

- (b) The graph of  $f(x) = \sqrt{x}$  has been sketched in the diagram below and a sequence of points on the  $x$  – axis are  $\{0 ; t ; 2 ; 4\}$ .

The points are drawn such that a sequence of equilateral triangles result.

The points on the  $x$  – axis form a quadratic sequence.



- (1) Determine the value of  $t$ .

(4)

- (2) Hence, or otherwise, find the coordinates of the point  $A$ .

(3)

[12]

**ADDITIONAL SPACE (ALL QUESTIONS)**

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE  
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

eduresourceza.com