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MATHEMATICS: PAPER 2
PRELIMINARY EXAMINATION 2024
30 August 2024

GRADE 12

Name & Surname: Memo

Class: _____

Examination Number:

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Examiner: S. Colley

Time: 3 hours

Moderator: C. Pretorius

Total: 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the session. Remember to write your name and surname in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator.
6. Ensure that your calculator is in DEGREE mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE**, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. **TWO** blank pages (page 27-28) are included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

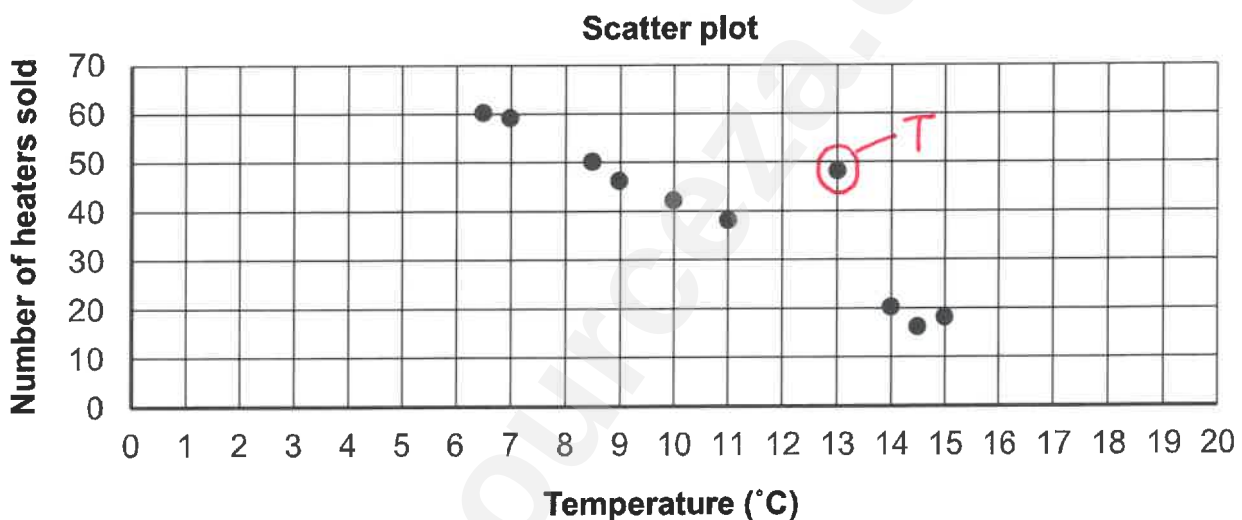
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
21	22	17	18	12	12	9	8	7	13	11	150

SECTION A

QUESTION 1

- (a) A shop in Johannesburg recorded the number of heaters sold in a given day and the temperature at noon on that day. The results are shown in the table and scatter plot below.

Temperature at noon in °C	9	8,5	10	14,5	11	15	13	14	6,5	7
Number of heaters sold	46	50	42	16	38	18	48	20	60	59



- (1) Calculate the correlation coefficient for the given set of data and use it to describe the relationship between the noon temperature and the number of heaters sold.

$$r = -0,9$$

Strong negative linear correlation

(3)

- (2) Determine the equation for the least squares regression line (line of best fit) for the given set of data. Round off your answers to **FOUR decimal places**.

$$\hat{y} = 91,3547 - 4,7608x$$

- (3) Hence, determine the number of heaters, correct to the nearest integer that should be sold if the noon temperature on a day is 12°C .

$$\begin{aligned}\hat{y} &= 91,3547 - 4,7608(12) \\ &= 34 \text{ heaters}\end{aligned}$$

- (4) The noon temperature on a certain day is 20°C . Can one use the equation above, obtained in number (2), to predict the number of heaters that will be sold? Motivate your answer.

Not reliable, extrapolation

- (5) Identify a possible outlier in the given set of data and label it on the scatter plot with the letter T. Give a reason why this outlier might have occurred.

Any valid reason

↳ clearance sale

↳ real-feel could be colder etc.

✓label ✓reason

(6) If the outlier was removed from the given set of data, how will this affect:

(i) the correlation coefficient?

Stronger, closer to -1

(2)

(ii) the gradient of the line of best fit?

gradient will become less, $\therefore$ steeper

(2)

(b) For the box-and-whisker diagram below, invent a sample of data containing nine different integer values if the mean of the data is 68. Write the nine numbers in **ascending** order.



✓ must be 52

✓ must be 72

✓ must be 77

52 — 60 — 72 — 75 — 77

remaining results must total 612 ✓

if numbers are repeated (-2 each time)

(6)
[21]

QUESTION 2

- (a) If $x = 3 \sin \theta$ and $y = 3 \cos \theta$, determine the value of $x^2 + y^2$.

$$\begin{aligned} &= 9 \sin^2 \theta + 9 \cos^2 \theta \checkmark \\ &= 9 (\sin^2 \theta + \cos^2 \theta) \\ &= 9(1) \\ &= 9 \checkmark \end{aligned}$$

- (b) Prove the identity: $\cos^2(180^\circ + x) + \tan(x - 180^\circ) \cdot \sin(720^\circ - x) \cdot \cos x = \cos 2x$ (2)

$$\begin{aligned} &= \cos^2 x + (\tan x)(-\sin x)(\cos x) \\ &= \cos^2 x + \left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x) \\ &= \cos^2 x - \sin^2 x \checkmark \\ &= \cos 2x \end{aligned}$$

(5)

(c) Simplify without the use of a calculator: $\sin(-70^\circ) \cos(-20^\circ) - \cos^2 470^\circ$

$$\begin{aligned}
 &= (-\sin 70^\circ)(\cos 20^\circ) - \cos^2(180^\circ - 70^\circ) \\
 &= (-\sin 70^\circ)(\sin 70^\circ) - \cos^2 70^\circ \\
 &= -\sin^2 70^\circ - \cos^2 70^\circ \\
 &= -(\sin^2 70^\circ + \cos^2 70^\circ) \\
 &= -1
 \end{aligned}$$

(5)

(c) Determine the general solution of: $\tan x \cdot \sin x + \cos x \cdot \tan x = 0$

$$\tan x (\sin x + \cos x) = 0$$

$$\tan x = 0 \quad \text{or} \quad \sin x + \cos x = 0$$

$$\sin x = -\cos x$$

$$x = k \cdot 180^\circ$$

$$\tan x = -1$$

$$\text{ref } L = 45^\circ$$

$$\therefore x = 180^\circ + 45^\circ + k \cdot 180^\circ$$

$$= 225^\circ + k \cdot 180^\circ$$

or

$$x = 45^\circ + k \cdot 180^\circ$$

$$315^\circ$$

(5)

(d) Show that $\tan(x - 45) = \frac{\sin x - \cos x}{\sin x + \cos x}$

RHS

$$= \frac{\sin(x - 45^\circ)}{\cos(x - 45^\circ)} \quad \checkmark$$

$$= \frac{\sin x \cdot \cos 45^\circ - \cos x \cdot \sin 45^\circ}{\cos x \cdot \cos 45^\circ + \sin x \cdot \sin 45^\circ} \quad \checkmark$$

$$= \frac{\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x}{\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x} \quad \checkmark$$

$$= \frac{\sin x - \cos x}{\sin x + \cos x}$$

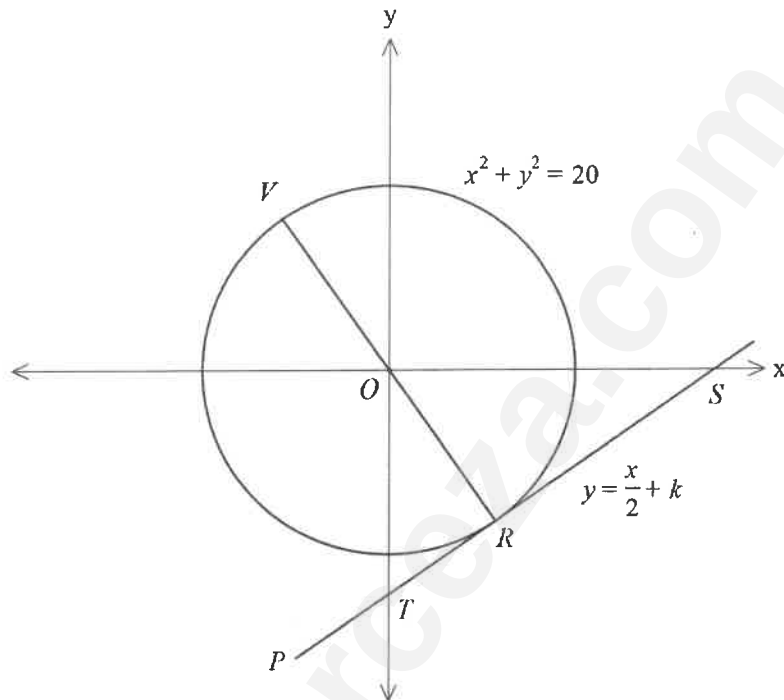
(5)
[22]

QUESTION 3

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$.

The tangent PRS to the circle R has the equation $y = \frac{x}{2} + k$.

PRS cuts the y-axis at T and the x-axis at S.



- (a) Determine, giving reasons, the equation of OR in the form $y = mx + c$.

$$m_{OR} = -2 \text{ (tan } \perp \text{ rad)} \checkmark$$

$$y = -2x + c$$

$$0 = -2(0) + c \checkmark$$

$$c = 0$$

$$y = -2x \checkmark$$

(3)

- (b) Determine the coordinates of R.

$$y^2 = 20 - x^2 \checkmark m$$

$$y = \sqrt{20 - x^2}$$

$$\sqrt{20 - x^2} = -2x \checkmark$$

$$20 - x^2 = 4x^2$$

$$20 = 5x^2$$

$$4 = x^2$$

$$x = 2$$

$$y = -2(2) \\ = -4 \checkmark$$

$$\therefore R(2; -4)$$

(4)

(c) Determine the area of $\triangle OTS$, given that $R(2; -4)$.

$$-4 = \frac{2}{2} + k \checkmark$$

$$-4 = 1 + k$$

$$-5 = k$$

$$y = \frac{x}{2} - 5 \checkmark$$

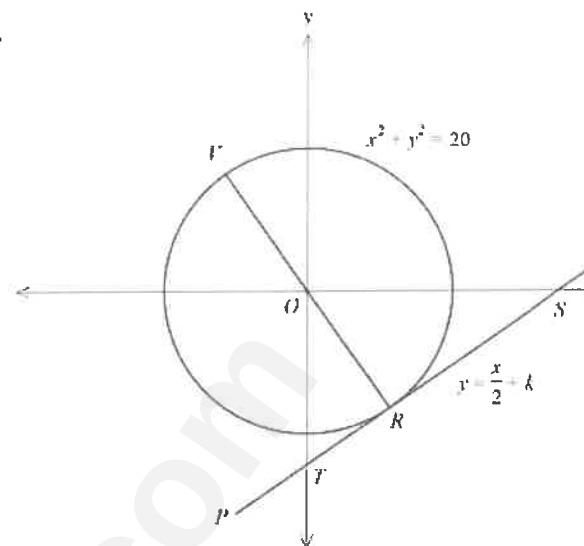
$$T(0; 5) \checkmark \text{ and } S: 5 = \frac{x}{2}$$

$$10 = x$$

$$(10; 0) \checkmark$$

$$\text{Area} = \frac{1}{2} \times 10 \times 5 \checkmark$$

$$= 25 \text{ units}^2 \checkmark$$



(6)

(d) Calculate the length of VT.

$$V(-2; 4) \checkmark$$

$$VT = \sqrt{(4 - (-5))^2 + (-2 - 0)^2} \checkmark \checkmark$$

$$= \sqrt{85}$$

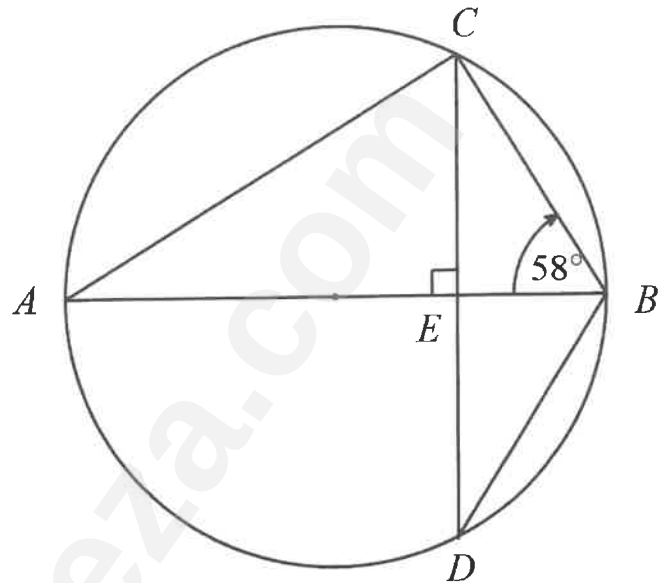
$$= 9,2 \text{ units} \checkmark$$

(4)
[17]

QUESTION 4

(a) In the diagram below,

- AB is a diameter of the circle
- Chord CD is perpendicular to AB and cuts AB at E
- AC, CB and BD are drawn
- $\angle ABC = 58^\circ$



Prove that

(1) $\angle EBD = 58^\circ$

$\angle ECB = 32^\circ$ ✓ (int L's of $\triangle BCE$)

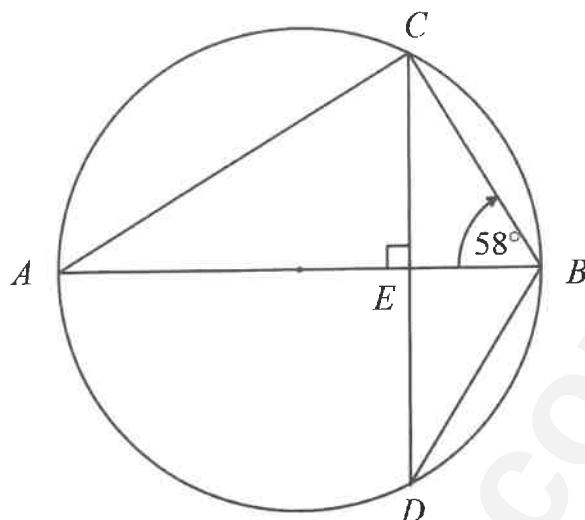
$\angle ACB = 90^\circ$ (L in semi circle) ✓

$\therefore \angle ACD = 58^\circ$ ✓

$\therefore \angle EBD = 58^\circ$ (L's in same seg) ✓

or
congruency

(4)



(2) $\triangle ABC \parallel \triangle DBE$

In $\triangle ABC$ and $\triangle DBE$

- ① $\angle CBA = \angle EBD$ (proven) ✓
 - ② $\angle A = \angle D$ (proven / L's in same seg) ✓
 - ③ $\angle CEA = \angle BED$ (remaining L's in Δ) ✓
- $\therefore \triangle ABC \parallel \triangle DBE$ (AAA)

(3)

(3) $\frac{AC}{AB} = \frac{CE}{DB}$

$CE = ED$ (line from centre $\perp$ to chord) ✓

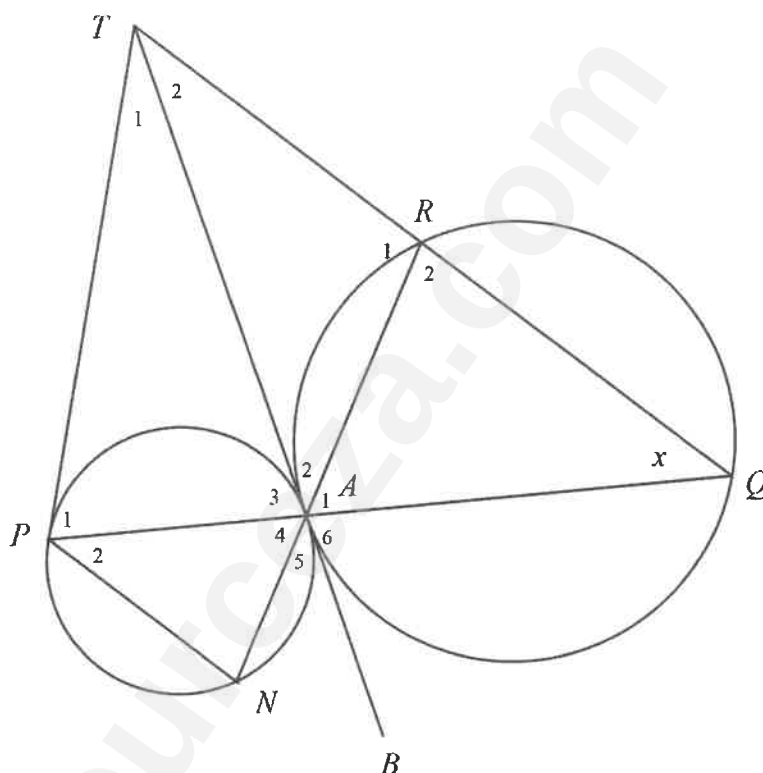
$\frac{AC}{AB} = \frac{DE}{DB}$ ✓

$\therefore \frac{AC}{AB} = \frac{CE}{DB}$ ✓

(3)

(b) In the diagram below.

- The two circles have a common tangent TAB
- PT is a tangent to the smaller circle PAN
- QRT and NAR are straight lines
- $\hat{Q} = x$.



(1) Name with reasons THREE other angles equal to x .

$$\hat{A}_2 = x \quad (\text{tan chord}) \quad \checkmark$$

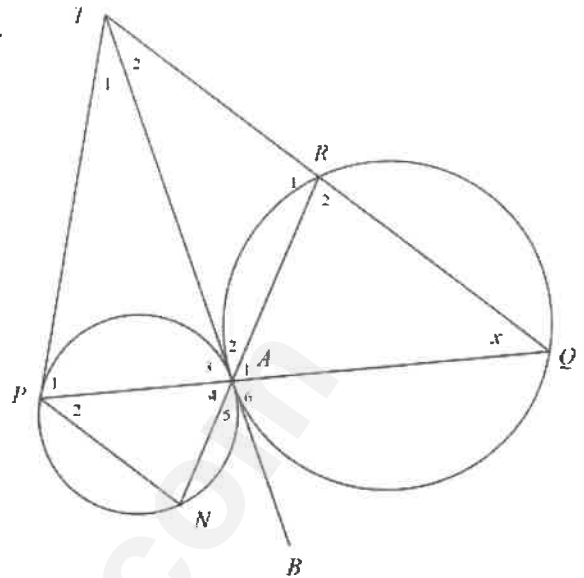
$$\hat{A}_5 = x \quad (\text{vert opp } \angle\text{'s}) \quad \checkmark$$

$$\hat{P}_2 = x \quad (\text{tan chord}) \quad \checkmark$$

(3)

(2) Prove that APTR is a cyclic quadrilateral.

$TQ \parallel PN$ (alt L's =)
 $\hat{R}_2 = \hat{N}$ (alt L's ; $TQ \parallel PN$)
 $\hat{P}_1 = \hat{N}$ (tan chord)
 $\therefore$ APTR is a cyclic quad
 (ext. L's of cyclic quad)



OR

$\hat{P}_1 = \hat{A}_3$ (tans from same pt)
 $\hat{A}_3 = \hat{A}_6$ (vert opp L's)
 $\hat{A}_6 = \hat{R}_2$ (tan chord)
 $\therefore$ APTR is a cyclic quad (ext L's of cyclic quad)

OR

$\hat{R}_2 = 180^\circ - x - \hat{A}_1$ (int L's of Δ)
 $\hat{P}_1 = \hat{A}_3$ (tans from same pt)
 $\hat{A}_3 = 180^\circ - x - \hat{A}_1$ (L's on str line)
 $\therefore \hat{R}_2 = \hat{P}_1$

$\therefore$ APTR is a cyclic quad (ext L's of cyclic quad)

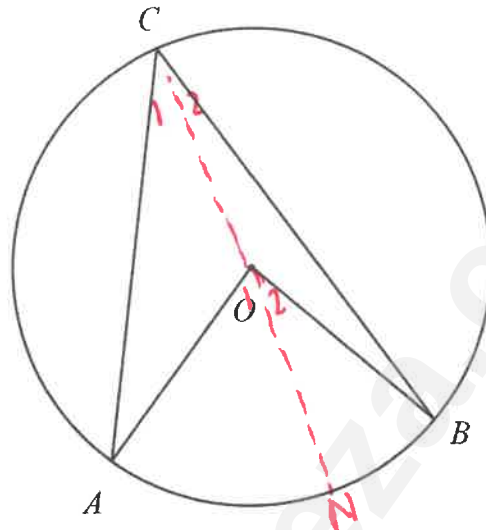
(5)
[18]

78 marks

SECTION B

QUESTION 5

- (a) Prove the theorem which states: "The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle."



Required to prove: $\angle AOB = 2\angle ACB$

Construction: Join CO and produce to N ✓

Proof:

in $\triangle ACO$

$$\angle AON = \hat{A} + \hat{ACO} \quad (\text{ext } \angle \text{ of } \triangle ACO) \quad \checkmark$$

$$\text{But } \hat{A} = \hat{ACO} \quad (AO = CO, \text{ radii}) \quad \checkmark$$

$$\therefore \angle AON = 2\hat{ACO} \quad \checkmark$$

Similarly in $\triangle BCO$

$$\angle BON = 2\hat{BCO} \quad \checkmark$$

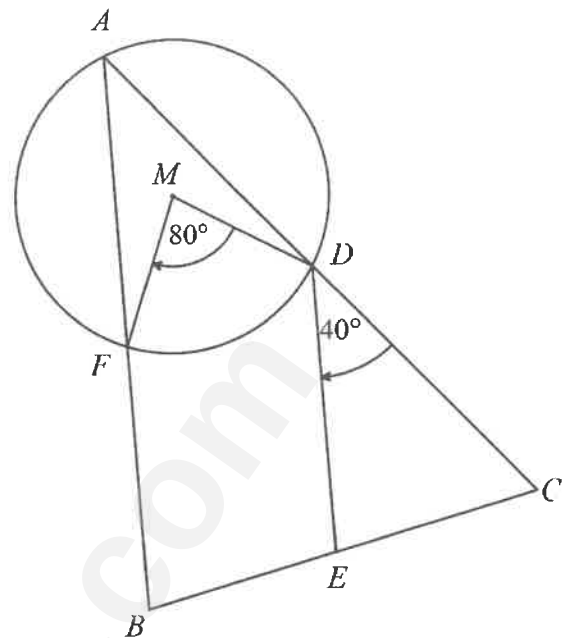
$$\therefore \angle AON + \angle BON = 2\hat{ACO} + 2\hat{BCO}$$

$$\therefore \angle AOB = \angle AON + \angle BON = 2(\hat{ACO} + \hat{BCO}) \quad \checkmark$$

$$\therefore \angle AOB = 2\angle ACB$$

(6)

- (b) In the given figure, M is the centre of circle AFD. Chords AF and AD are produced to B and C respectively and BC is joined. $\widehat{EDC} = 40^\circ$ and $\widehat{FMD} = 80^\circ$.



- (1) Prove that $AB \parallel DE$.

$$\hat{A} = 40^\circ \quad (\angle \text{at centre} = 2 \times \angle \text{at circ})$$

$$\therefore AB \parallel DE \quad (\text{corr } \angle's =)$$

(3)

- (2) If $\frac{BE}{EC} = \frac{7}{8}$ and $AC = 22,5\text{cm}$, calculate the length of AD.

$$\frac{AD}{AC} = \frac{BE}{BC} \quad (AB \parallel DE \text{ in } \triangle ABC)$$

$$\therefore \frac{AD}{22,5} = \frac{7}{15}$$

$$AD = 10,5$$

(3)
[12]

QUESTION 6

Given the equation: $\sin(x + 60^\circ) + 2 \cos x = 0$

- (a) Show that the equation can be written as $\tan x = -4 - \sqrt{3}$

$$\sin x \cos 60^\circ + \cos x \cdot \sin 60^\circ + 2 \cos x = 0 \quad \checkmark$$

$$\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x + 2 \cos x = 0 \quad \checkmark$$

$$\sin x + \sqrt{3} \cos x + 4 \cos x = 0 \quad \checkmark$$

$$\sin x + \cos x (\sqrt{3} + 4) = 0 \quad (\div \cos x)$$

$$\tan x + \sqrt{3} + 4 = 0 \quad \checkmark$$

$$\tan x = -\sqrt{3} - 4$$

(4)

- (b) Hence determine the solutions to the equation $\sin(x + 60^\circ) + 2 \cos x = 0$ in the interval $-180^\circ \leq x \leq 180^\circ$.

$$\text{ref } L = 80, 103, 90, 93, 6 \quad \checkmark$$

$$\therefore x = 180^\circ - 80, 103 \dots + k \cdot 180^\circ \quad k \in \mathbb{Z}$$

$$x = 99, 9 + k \cdot 180^\circ \quad \checkmark$$

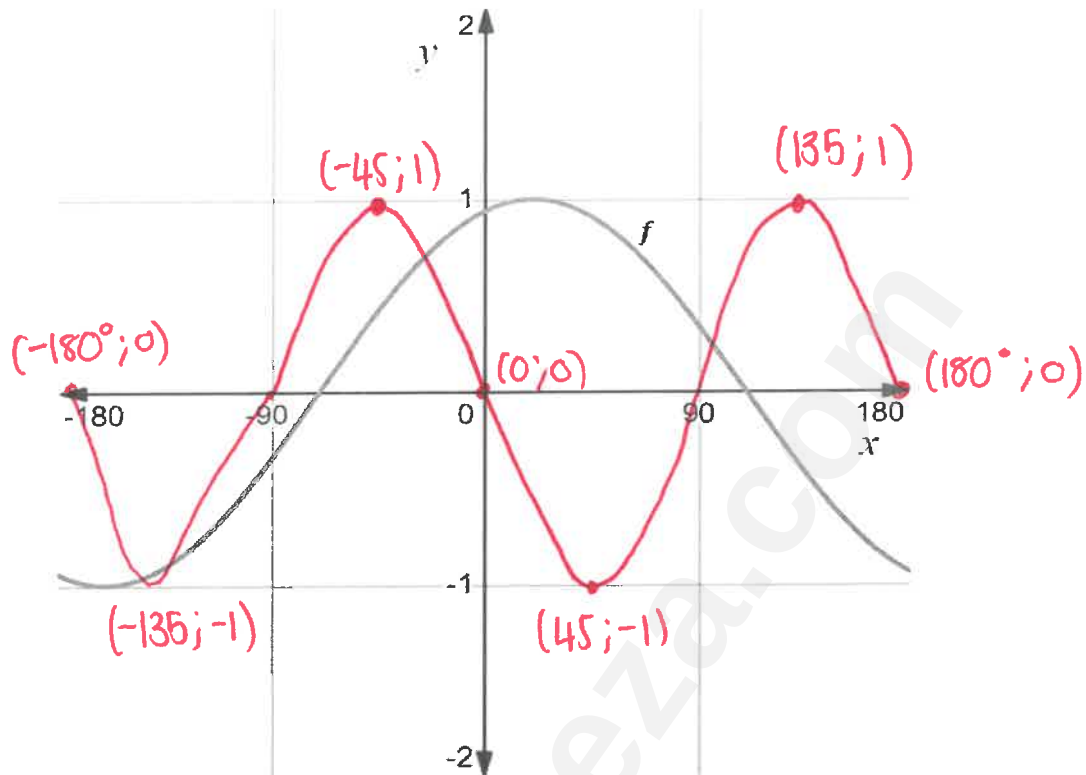
$$x = 360^\circ - 80, 103 \dots + k \cdot 180^\circ$$

$$= 279, 9 + k \cdot 180^\circ \quad \checkmark$$

$$x \in \{99, 9; 80, 1\} \quad \checkmark$$

(4)

(c) In the diagram below, the graph of $f(x) = \cos(x - 20^\circ)$ is drawn for $-180^\circ \leq x \leq 180^\circ$.



- (1) Draw the graph of $g(x) = -\sin 2x$ for $-180^\circ \leq x \leq 180^\circ$ on the same set of axes. Indicate the coordinates of all turning points, endpoints and intercepts of axes.

x -int ✓

y -int ✓

turning pt ✓

end points ✓

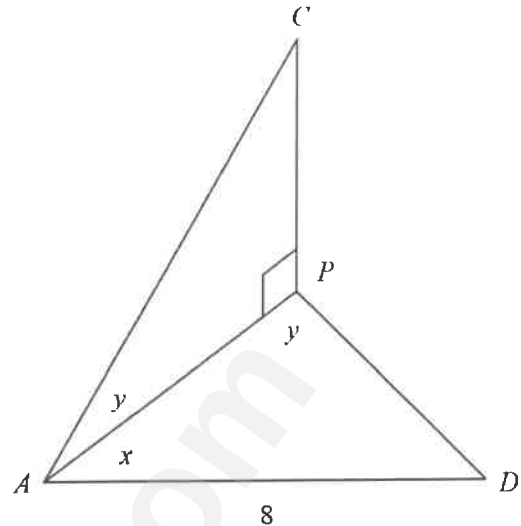
(4)
[12]

QUESTION 7

In the diagram below A, D and P are points in the same horizontal plane. CP is a vertical pole.

The angle of elevation of C from A is y .

$\widehat{PAD} = x$, $\widehat{DPA} = y$ and $AD = 8$ metres.



- (a) Show that $CP = \frac{8 \sin(x+y)}{\cos y}$

$$\widehat{APC} = 180^\circ - (x+y) \quad \checkmark$$

$$\frac{AP}{\sin(x+y)} = \frac{8}{\sin y} \quad \checkmark$$

$$\tan y = \frac{CP}{AP}$$

$$\begin{aligned} \therefore CP &= \tan y \times AP \quad \checkmark \\ &= \frac{\sin y}{\cos y} \times \frac{8 \sin(x+y)}{\sin y} \quad \checkmark \end{aligned}$$

$$= \frac{8 \sin(x+y)}{\cos y} \quad \checkmark$$

- (b) If $x = y$, what are the possible values of CP? (5)

$$CP = \frac{8 \sin 2y}{\cos y} \quad \checkmark$$

$$= \frac{8 \times 2 \sin y \cos y}{\cos y} \quad \checkmark$$

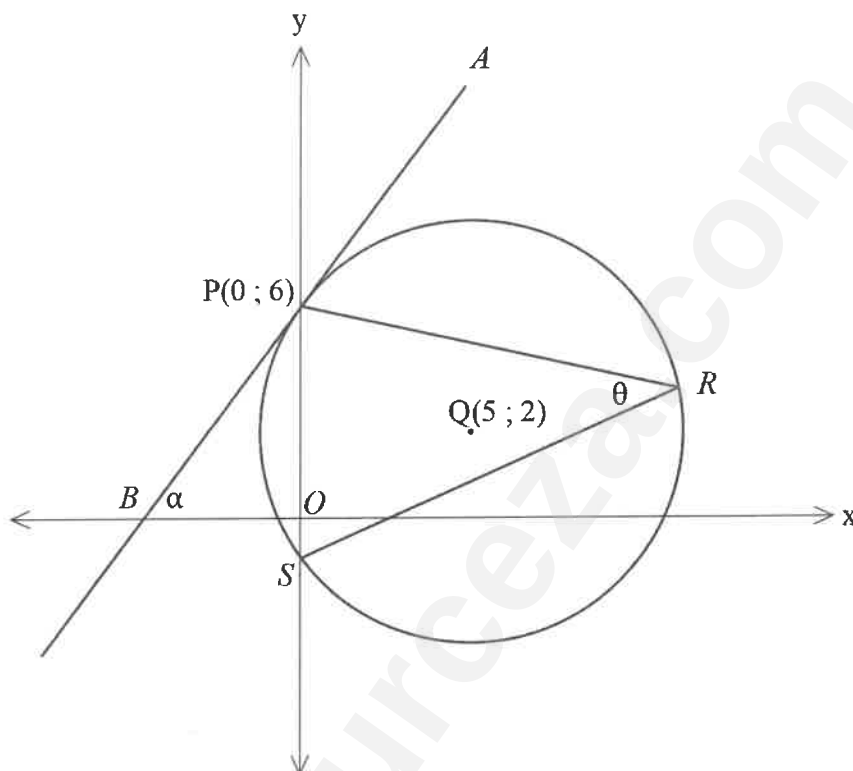
$$= 16 \sin y \quad \checkmark$$

$$0 < CP < 16 \quad \checkmark$$

(4)
[9]

QUESTION 8

In the diagram below, $Q(5; 2)$ is the center of a circle that intersects the y-axis at $P(0; 6)$ and S . The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $\widehat{PRS} = \theta$.



- (a) Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$.

$$QP = \sqrt{(2-6)^2 + (5-0)^2} \checkmark$$

$$= \sqrt{41} \checkmark$$

$$(x-5)^2 + (y-2)^2 = 41 \checkmark$$

(3)

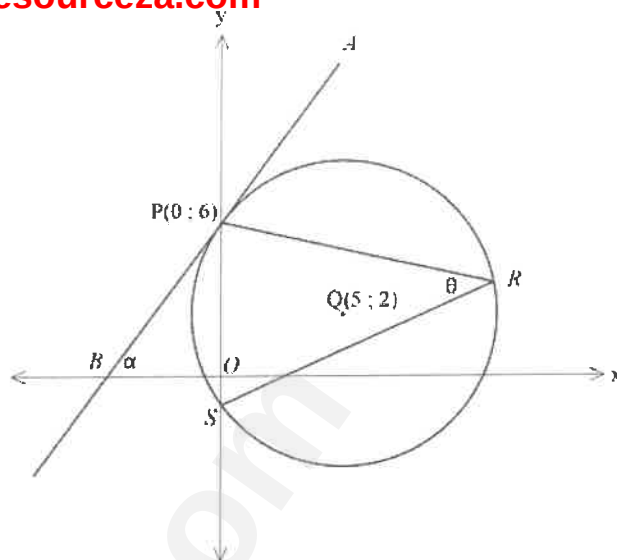
(b) Calculate the size of α .

$$m_{QP} = \frac{2-6}{5-0} \\ = -\frac{4}{5} \checkmark$$

$$m_{AB} = \frac{5}{4} \checkmark$$

$$\tan \alpha = \frac{5}{4}$$

$$\alpha = 51,3^\circ \checkmark$$



(c) Calculate, with reasons the size of θ .

$$\hat{BPQ} = 38,66^\circ$$

$$\theta = 38,7^\circ$$

(int $L's$ of Δ) $\checkmark$

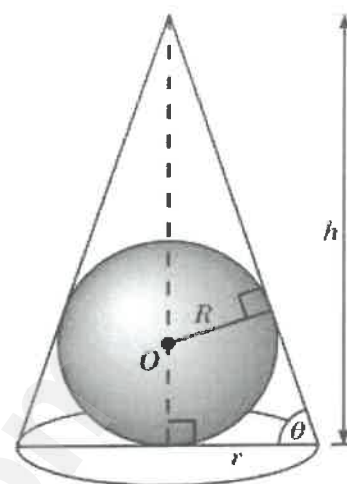
(tan chord) $\checkmark$

(3)

(2)
[8]

QUESTION 9

- (a) The diagram below shows a sphere placed inside a cone which has a radius (r) of 7 cm. The total surface area of the sphere is $100\pi \text{ cm}^2$ and the height (h) of the cone is three and a half times the length of the radius of the sphere (R).



$SA = \pi r^2 + \pi rs$	$V = \frac{1}{3} \pi r^2 h$
$SA = 4\pi R^2$	$V = \frac{4}{3} \pi R^3$

Calculate the volume of the empty space in the cone, to one decimal.

$$100\pi = 4\pi R^2 \quad \checkmark$$

$$25 = R^2$$

$$R = 5 \quad \checkmark$$

$$h = 5 \times 3,5$$

$$= 17,5 \quad \checkmark$$

$$V = \frac{1}{3} \pi (7)^2 (17,5) - \frac{4}{3} \pi (5)^3 \quad \checkmark$$

$$= 374,4 \text{ cm}^3 \quad \checkmark$$

(5)

- (b) Nandi has two spheres in her office, a small marble and a large bowling ball. If the bowling ball has a volume which is 2744 times the volume of the marble, how many times larger is the radius of the bowling ball than the radius of the marble? Show all working.

$$2744 \times \text{Vol of M} = \text{Vol of BB}$$

$$2744 \times \frac{4}{3} \pi m^3 = \frac{4}{3} \pi b^3 \quad \checkmark$$

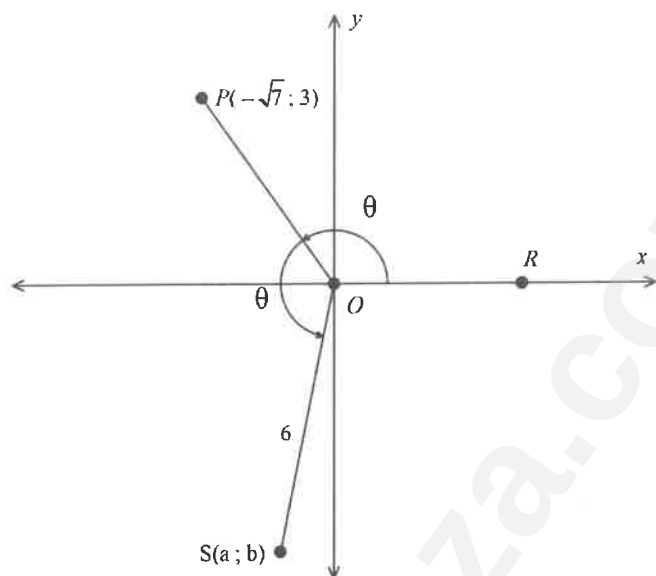
$$14m = b$$

$\therefore$ the radius must be 14 times larger $\checkmark$

(2)
[7]

QUESTION 10

- (a) $P(-\sqrt{7}; 3)$ and $S(a; b)$ are points on the Cartesian plane, as shown below
 $\widehat{POR} = \widehat{POS} = \theta$ and $OS = 6$ units.



Determine, without a calculator, the value of a .

$$\tan \theta = \frac{3}{-\sqrt{7}} \quad \checkmark$$

$$\theta = 180^\circ - 48,5 \dots$$

$$= 131,4096221 \quad \checkmark$$

$$\cos 2\theta = \frac{a}{6} \quad \checkmark$$

$$\cos 2(131,409 \dots) = \frac{a}{6} \quad \checkmark$$

$$a = -0,8 \text{ or } -\frac{3}{4} \quad \checkmark$$

$$OP^2 = (-\sqrt{7})^2 + 3^2$$

$$OP^2 = 16$$

$$OP = 4$$

$$\cos 2\theta = \frac{a}{6}$$

$$a = 6(1 - 2\sin^2 \theta) \quad | \quad a = 6(2\cos^2 \theta - 1)$$

$$a = 6 - 12\left(\frac{3}{4}\right)^2$$

$$a = -\frac{3}{4}$$

$$a = -\frac{3}{4}$$

(5)

- (b) Determine the value of $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x}$.

$$\lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{\sin x} \checkmark$$

$$\lim_{x \rightarrow 0} 2 \cos x \checkmark$$

$$= 2 \checkmark$$

(3)

- (c) For which values in the interval $0^\circ \leq x \leq 180^\circ$ will the geometric sequence $\sin x ; \sin 2x ; \dots$ converge?

$$r = 2 \cos x \checkmark$$

$$-1 < 2 \cos x < 1 \checkmark$$

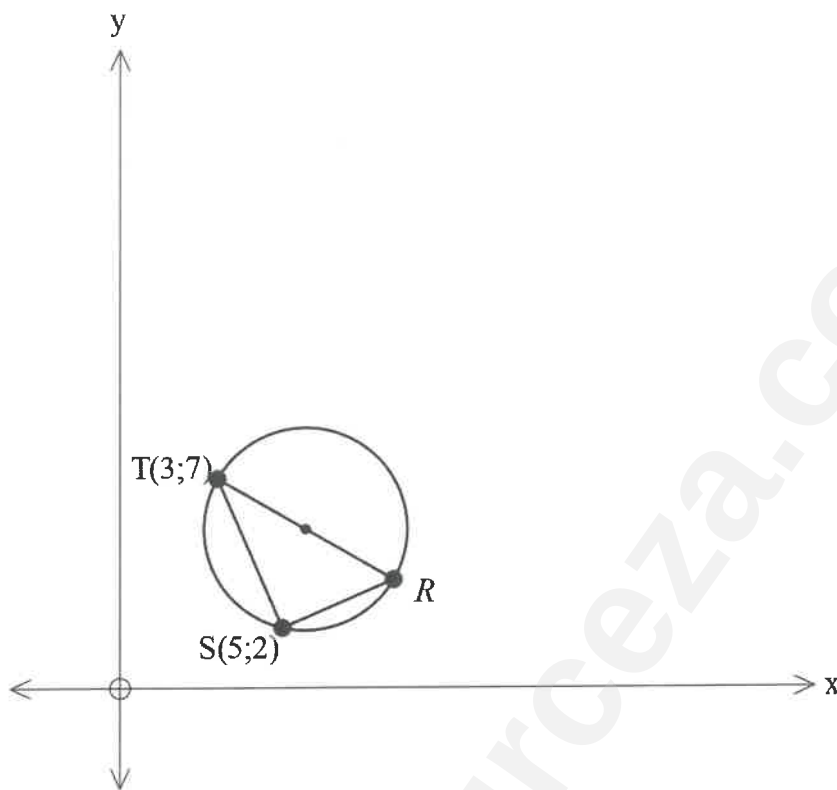
$$-\frac{1}{2} < \cos x < \frac{1}{2} \checkmark$$

$$60^\circ < x < 120^\circ \checkmark$$

(5)
[13]

QUESTION 11

In the diagram is a small circle which passes through $T(3;7)$, $S(5;2)$ and R . RT is a diameter of the circle.



- (a) Show that the equation of SR is $y = \frac{2}{5}x$.

$$m_{TS} = \frac{7-2}{3-5}$$

$$= \frac{5}{-2}$$

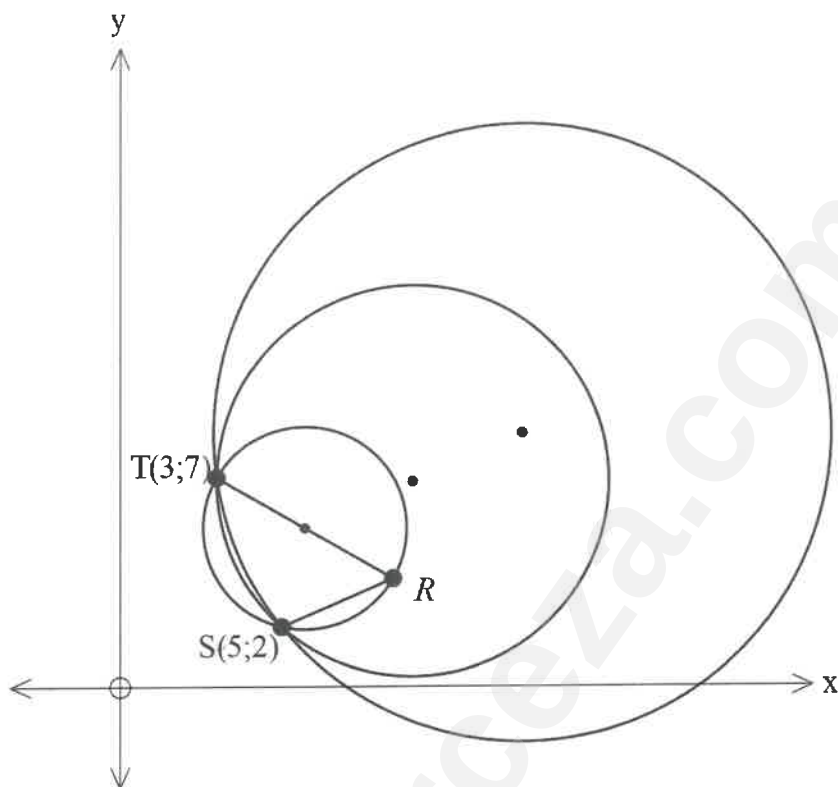
$$\therefore m_{SR} = \frac{2}{5} \quad (\angle \text{ in semi circle})$$

$$y-2 = \frac{2}{5}(x-5)$$

$$y = \frac{2}{5}x$$

(4)

- (b) This small circle now begins to grow but at the same time continues to pass through the fixed positions of $T(3;7)$ and $S(5;2)$. RT remains the diameter of the growing circle.



Determine the new coordinates of R if:

- (1) TR becomes parallel to the x -axis.

Sub $R(x; 7)$ into $y = \frac{2}{5}x$

$$7 = \frac{2}{5}x \quad \checkmark$$

$$x = \frac{35}{2} \quad \checkmark$$

$$\therefore R\left(\frac{35}{2}; 7\right) \quad \checkmark$$

(3)

- (2) the equation of the circle becomes $(x - 9)^2 + \left(y - \frac{13}{2}\right)^2 = \frac{145}{4}$.

(centre is $(9; \frac{13}{2})$) ✓

$$\frac{x+3}{2} = 9 \quad \checkmark$$

$$\therefore x+3 = 18$$

$$x = 15$$

$$\frac{y+7}{2} = \frac{13}{2} \quad \checkmark$$

$$y+7 = 13$$

$$y = 6$$

$$\therefore R(15; 6) \quad \checkmark$$

(4)
[11]

72 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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