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PINNACLE COLLEGE
COPPERLEAF

MATHEMATICS: PAPER 1
PRELIMINARY EXAMINATION 2024
23 November 2024

GRADE 12

Name & Surname: Memo

Class: _____

Examination Number:

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Examiner: C. Pretorius

Time: 3 hours

Moderator: S. Colley

Total: 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the session. Remember to write your name and surname in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator.
6. Ensure that your calculator is in DEGREE mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE**, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. **TWO** blank pages (page 24-25) are included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
14	11	25	21	5	5	13	12	21	13	10	150

SECTION A

QUESTION 1

(a) Solve for x:

(1) $\sqrt{x^2 - 5} = 2\sqrt{x}$

$x^2 - 5 = 4x$ ✓m

$x^2 - 4x - 5 = 0$ ✓a

$(x - 5)(x + 1) = 0$

$x = 5$ ✓ca or $x \neq -1$ ✓a
N/A

(4)

(2) $-x^2 + 4 > 0$

$-x^2 > -4$ ✓

$x^2 < 4$ ✓m

$-2 < x < 2$ ✓a



(3)

(3) $\log x^2 + \log x^3 = 10$

$\log(x^2 \times x^3) = 10$ ✓m

$\log x^5 = 10$

$x^5 = 10^{10}$ ✓m

$x = \sqrt[5]{10^{10}}$

$x = 10^2$

$x = 100$ ✓a

(3)

(b) Discuss the nature of the roots of $x(x + 4p) = p^2$.

$$x^2 + 4xp - p^2 = 0 \quad \checkmark \text{ simplified}$$

$$\Delta = b^2 - 4ac$$

$$= (4p)^2 - 4(1)(-p^2) \quad \checkmark \text{ m + sub}$$

$$= 16p^2 + 4p^2$$

$$= 20p^2 \quad \checkmark \text{ a}$$

Real roots $\checkmark \text{ ca}$

(4)

[14]

QUESTION 2

(a) Given:

$$\sum_{n=1}^k (5x - 3)^n$$

(1) If $k = \infty$, for which value(s) of x will the given sum converge?

$$-1 < r < 1$$

$$-1 < 5x - 3 < 1 \quad \begin{matrix} \checkmark m \\ \checkmark sub \end{matrix}$$

$$2 < 5x < 4$$

$$\frac{2}{5} < x < \frac{4}{5} \quad \checkmark a$$

(3)

(2) If $k = 10$ and $x = 1$, evaluate the given expression.

$$\sum_{n=1}^{10} 2^n$$

$$\begin{matrix} a = 2 \\ r = 2 \end{matrix}$$

$$S_{10} = \frac{2(2^{10} - 1)}{2 - 1} \quad \begin{matrix} \checkmark a & \checkmark r \\ \checkmark m \end{matrix}$$

$$= 2046 \quad \checkmark a$$

(4)

- (b) Due to increased internet sales globally, harbours around the world have found themselves ill-equipped to deal with the influx of imports. On average it takes 5 days to offload a container ship and a new one arrives every 2 days.

Tabulated below is the data captured from a harbour.

Day (n)	0	10	20	30	40	n
Ships unloaded	0	2	4	6	8	$T_n = \frac{1}{5}n$
Ships arrived	0	5	10	15	20	$T_n = \frac{1}{2}n$
Ships waiting	0	3	6	9	12	$T_n = \dots$

- (1) If the general rule for ships unloaded is $T_n = \frac{1}{5}n$ and the general rule for ships arrived is $T_n = \frac{1}{2}n$, find the general rule for ships waiting.

$$T_n = \frac{1}{2}n - \frac{1}{5}n \quad \checkmark a$$

$$= \frac{3}{10}n \quad \checkmark ca$$

(2)

- (2) Assuming this pattern continues, how many ships will be waiting at the end of a full year?

$$T_{365} = \frac{3}{10}(365) \quad \checkmark sub$$

$$= 109,5$$

$$\approx 109 \text{ waiting} \quad \checkmark ca$$

(2)

[11]

QUESTION 3

(a) Given $f(x) = \frac{a}{x+p} + q$ with axes of symmetry $y = x + 3$ and $y = -x + 1$.

(1) Determine the coordinates of the point of intersection for the axes of symmetry.

$$x + 3 = -x + 1 \quad \checkmark m$$

$$2x = -2$$

$$x = -1 \quad \checkmark ca$$

$$y = -1 + 3$$

$$y = 2 \quad \checkmark ca$$

$$(-1; 2)$$

(3)

(2) Hence, determine the values of p and q .

$$p = 1 \quad \checkmark a$$

$$q = 2 \quad \checkmark a$$

(2)

(3) It is further given that the y -intercept of $f(x)$ is 4, determine the value of a .

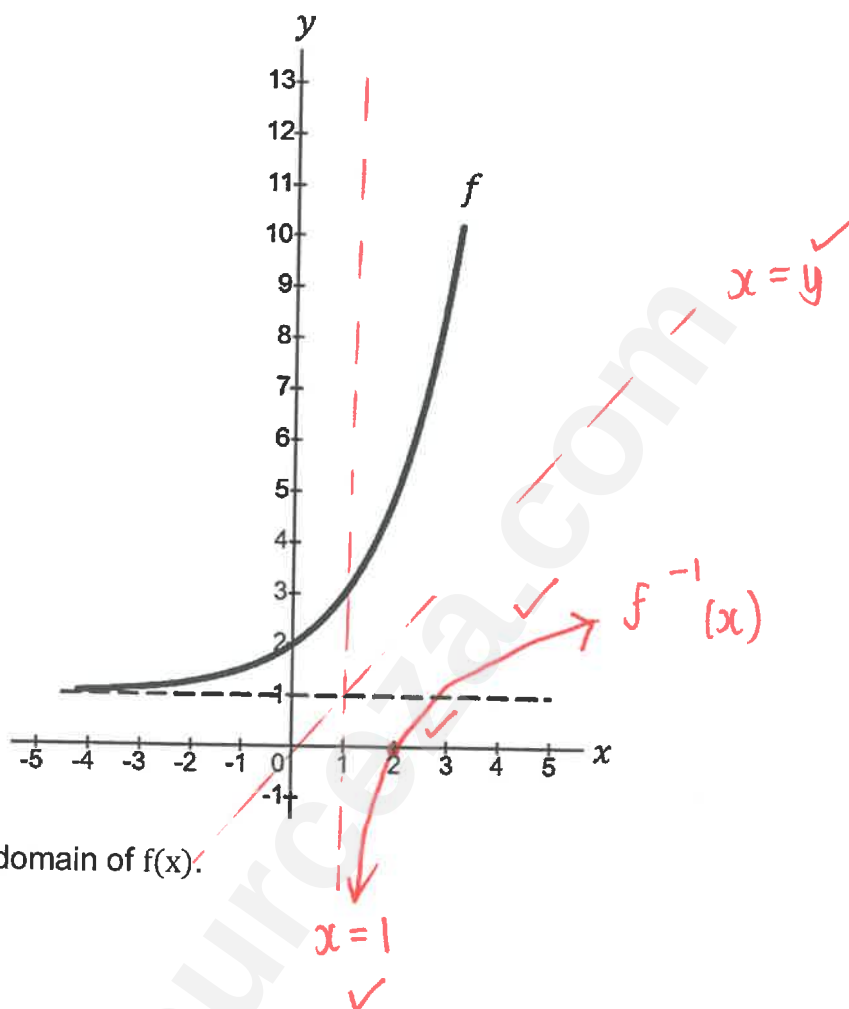
$$4 = \frac{a}{0+1} + 2 \quad \checkmark m \quad \checkmark sub$$

$$4 = \frac{a}{1} + 2$$

$$2 = a \quad \checkmark a$$

(3)

- (b) On the Cartesian Plane below the graph of $f(x) = 2^x + 1$ is drawn to scale.



- (1) Write down the domain of $f(x)$.

$$x \in \mathbb{R} \quad \checkmark$$

(1)

- (2) Write down the range of f .

$$y \in (1; \infty) \text{ or } y > 1 \quad \checkmark$$

(1)

- (3) Find the equation of $f^{-1}(x)$ in the form $y = \dots$

$$y = 2^x + 1 \quad \checkmark$$

$$x = 2^y + 1 \quad \checkmark m$$

$$x - 1 = 2^y$$

$$y = \log_2(x - 1) \quad \checkmark$$

(3)

- (4) Sketch the graph of $f^{-1}(x)$ on the same diagram. Clearly show all intercept(s) with the axis/es and/or asymptote(s), if applicable.

(4)

- (5) f^{-1} is translated to become a new function $h(x) = f^{-1}(x - p)$. Determine the value(s) of p so that h will have a y -intercept.

$$p \leftarrow -1$$

(2)

- (6) If $g(x) = 2^{-x+2} + 4$, show all calculations in solving algebraically for x where $f(x) = g(x)$.

$$2^x + 1 = 2^{-x+2} + 4 \quad \checkmark \text{sub}$$

$$2^x + 1 = \frac{1}{2^x} \cdot 2^2 + 4$$

$$2^x = \frac{4}{2^x} + 3 \quad \checkmark \text{cd}$$

$$2^{2x} = 4 + 3 \cdot 2^x \quad \checkmark \quad \text{let } 2^x = k$$

$$k^2 = 4 + 3k$$

$$k^2 - 3k - 4 = 0 \quad \checkmark$$

$$(k - 4)(k + 1) = 0$$

$$k = 4 \quad \text{or} \quad k = -1$$

$$2^x = 4 \quad 2^x \neq -1 \quad \checkmark$$

$$x = 2 \quad \checkmark \quad \text{No solution}$$

(6)

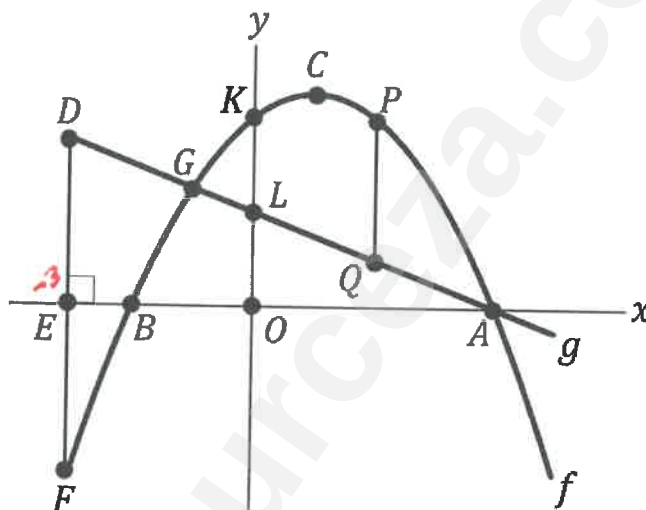
[25]

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QUESTION 4

In the diagram alongside:

- $f(x) = -x^2 + 2x + 8$ and $g(x) = -x + 4$.
- O is the origin.
- K and L are y-intercepts of f and g respectively.
- A is a shared x-intercept of f and g .
- G is a second point of intersection.
- B is the other x-intercept of f .
- C is the turning point of f .
- Line DEF is perpendicular to the x-axis.
- PQ is a vertical line that can move between the functions where $f(x) \geq g(x)$.



(a) Determine the length of KL .

$$KL = 4 \text{ units} \checkmark \checkmark$$

(2)

(b) If $OE = 3$ units, determine the length of DF .

$$\begin{aligned} f(3) &= -(-3)^2 + 2(-3) + 8 \checkmark \\ &= -7 \checkmark \end{aligned}$$

$$\begin{aligned} g(3) &= 3 + 4 \checkmark \\ &= 7 \checkmark \end{aligned}$$

$$\therefore DF = 14 \text{ units} \checkmark$$

OR

$$\begin{aligned} DF &= -x + 4 - (-x^2 + 2x + 8) \checkmark m \\ &= -x + 4 + x^2 - 2x - 8 \checkmark \\ &= x^2 - 3x - 4 \checkmark \\ &= (-3)^2 - 3(-3) - 4 \checkmark \text{sub} \\ &= 14 \checkmark \end{aligned}$$

(5)

(c) If $OB = 2$ units and $OA = 4$ units, use this information to find the coordinates of C.

$$x = \frac{-2+4}{2} = 1$$

$$y = -(1)^2 + 2(1) + 8$$

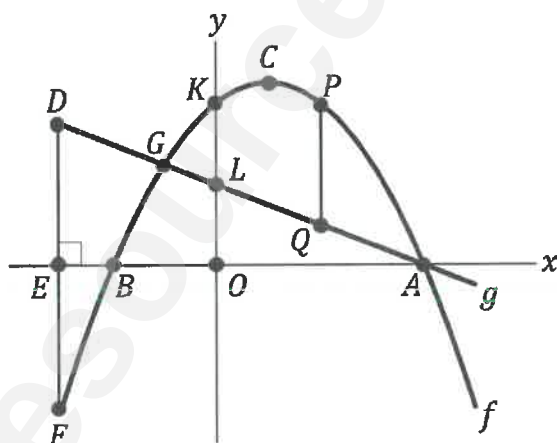
$$y = 9$$

$$C(1; 9)$$

(4)

(d) This is the same diagram as page 9: $f(x) = -x^2 + 2x + 8$ and $g(x) = -x + 4$.

- O is the origin.
- G is a second point of intersection.
- PQ is a vertical line that can move between the functions where $f(x) \geq g(x)$.



(1) For which value(s) of x will $f(x) \leq g(x)$?

$$-x^2 + 2x + 8 = -x + 4$$

$$-x^2 + 3x + 4 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$G(-1; y) \quad A(4; 0)$$

$$\therefore x \leq -1 \text{ and } x \geq 4$$

(5)

(2) Determine the maximum distance of PQ.

$$\begin{aligned} PQ &= -x^2 + 2x + 8 - (-x + 4) \quad \checkmark m \\ &= -x^2 + 2x + 8 + x - 4 \\ &= -x^2 + 3x + 4 \end{aligned}$$

$$PQ' = -2x + 3 \quad \checkmark m$$

$$0 = -2x + 3$$

$$-3 = -2x$$

$$\frac{3}{2} = x \quad \checkmark d$$

$$\begin{aligned} PQ &= -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 4 \quad \checkmark \text{sub} \\ &= 6, 75 \text{ units} \quad / \quad \frac{25}{4} \quad \checkmark \text{ca} \end{aligned}$$

(5)

[21]

QUESTION 5

If $f(x) = 2x^2 - 3x + 1$, determine $f'(x)$ using first principles.

$$2(x+h)^2 - 3(x+h) + 1$$

$$2x^2 + 4xh + h^2 - 3x - 3h + 1$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4xh + h^2 - 3h}{h}$$

$$= \lim_{h \rightarrow 0} 4x + h - 3$$

$$= 4x - 3$$

[5]

78 marks

SECTION B

QUESTION 6

Use the rules of differentiation to determine:

$$D_x \left[\sqrt[3]{x^5} - \frac{3x}{4} + \frac{2}{x^2} - 3x^{-1} \right]$$

Express your final answer with positive exponents and/or in surd form, where applicable.

$$\begin{aligned} & D_x \left[x^{\frac{5}{3}} - \frac{3}{4}x + 2x^{-2} - 3x^{-1} \right] \\ &= \frac{5}{3}x^{\frac{2}{3}} - \frac{3}{4} - 4x^{-3} + 3x^{-2} \\ &= \frac{5\sqrt[3]{x^2}}{3} - \frac{3}{4} - \frac{4}{x^3} + \frac{3}{x^2} \end{aligned}$$

[5]

QUESTION 7

- (a) A company buys a truck for R2 000 000. Due to the truck's components, it depreciates at 30% p.a. on the reducing balance method. The inflation rate on a new truck is 27% per annum. The company plans on setting up a sinking fund over 5 years (calculated at 15% p.a. compounded monthly) and will use the trade in value of the old Truck towards purchasing a new one.

- (1) Determine the trade in value of the old truck after 5 years.

$$A = 2\,000\,000 (1 - 30\%)^5 \checkmark m$$

$$= R\,336\,140 \checkmark a$$

(2)

- (2) Determine the value of a new replacement truck in 5 years' time.

$$A = 2\,000\,000 (1 + 27\%)^5 \checkmark m$$

$$= R\,6\,607\,673,88 \checkmark a$$

(2)

- (3) Determine the monthly payment into the sinking fund.

$$R\,6\,607\,673,88 - 336\,140 = R\,6\,271\,533,88 \checkmark a$$

$$6\,271\,533,88 = \frac{x \left[\left(1 + \frac{0,15}{12} \right)^{60} - 1 \right]}{\frac{0,15}{12}} \checkmark sub$$

$$\checkmark m$$

$$x = R\,70\,805,18 \checkmark ca$$

(4)

- (1) Calculate the total value of the loan.

$$= 21013821,44 \text{ Va}$$

- 2
(3)

$$OB = 1013821,44 \left(1 + \frac{0,104}{12}\right)^{205} - \frac{9500 \left[\left(1 + \frac{0,104}{12}\right)^{205} - 1\right]}{\frac{0,104}{12}}$$

$$= R613\,269,70$$

(3)

[13]

QUESTION 8

(a) Consider the word DAUGHTER.

How many ways can the letters be arranged:

(1) In any order?

$$8! = 40320 \checkmark$$

(1)

(2) Such that the first letter is a vowel?

$$3! \times 7! = 15 \times 5040 = 75600$$

$\checkmark m$ $\checkmark a$

(2)

(3) Calculate the probability that in a particular arrangement of the 8 letters, the letters T, P and R will be next to each other, in any order.

$$6! \times 3! = 720 \times 6 = 4320$$

$$\frac{4320}{40320} = \frac{3}{28} \approx 0,1$$

$$2! \times 7! = 10080$$

$$\frac{10080}{40320} = \frac{1}{4}$$

(2)

(b) Every South African banknote has a unique serial number with the following features:

- Begins with two letters (repetition allowed)
- Followed by a maximum of 8 digits (0 – 9, repetition allowed)
- Ends with either an A, B, C, D or E depending on the denomination.
(i.e. A = R10, B = R20, C = R50, D = R100 and E = R200).

(1) Assuming the SA Mint printed the same number of each banknote, how many bank notes are potentially in circulation? Show all calculations and reasons and leave your final answer in scientific notation.

$$26^2 \times 10^8 \times 5$$

$$= 3,38 \times 10^{11} \text{ notes}$$

(4)

(2) How much currency (total amount of money) is in circulation?

$$26^2 \times 10^8 \times 10$$

$$26^2 \times 10^8 \times 20$$

$$26^2 \times 10^8 \times 50$$

$$26^2 \times 10^8 \times 100$$

$$26^2 \times 10^8 \times 200 = R 2,6 \times 10^{13}$$

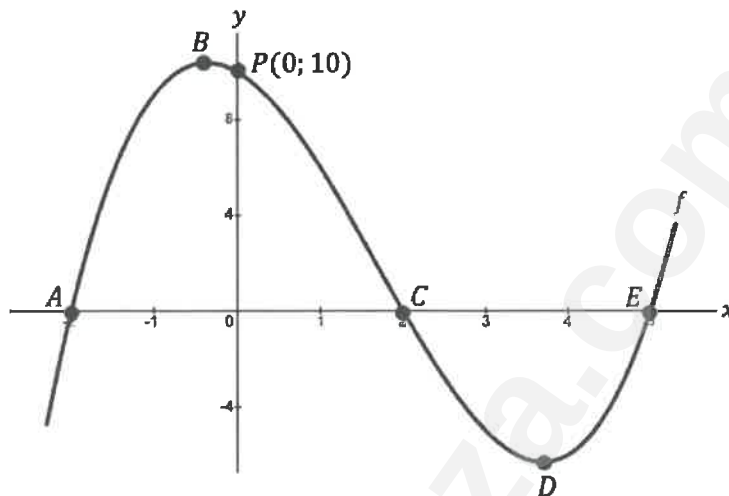
(3)

[12]

QUESTION 9

In the diagram alongside is the graph of $f(x) = ax^3 + bx^2 + cx + 10$.

- The x-intercepts are at $A(-2; 0)$, $C(2; 0)$ and $E(5; 0)$.
- B and D are the local maximum and local minimum respectively.
- Point $P(0; 10)$ the y-intercept of f .



- (a) Show that $a = \frac{1}{2}$, $b = -\frac{5}{2}$ and $c = -2$.

$$f(x) = a(x+2)(x-2)(x-5) \quad \checkmark_m$$

$$10 = a(0+2)(0-2)(0-5) \quad \checkmark_{sub}$$

$$10 = a(20)$$

$$\frac{1}{2} = a \quad \checkmark$$

$$f(x) = \frac{1}{2}(x+2)(x-2)(x-5) \quad \checkmark_{sub}$$

$$= \frac{1}{2}(x^2 - 4)(x - 5)$$

$$= \frac{1}{2}(x^3 - 5x^2 - 4x + 20) \quad \checkmark$$

$$= \frac{1}{2}x^3 - \frac{5}{2}x^2 - 2x + 10 \quad \checkmark$$

(b) Determine the coordinates of B and D.

$$f'(x) = 0 \quad \text{m}$$

$$0 = \frac{3}{2}x^2 - 5x - 2 \quad \text{f'}$$

$$0 = 3x^2 - 10x - 4$$

$$x = 3,7 \quad \text{or} \quad x = -0,4$$

$$f(3,7) = -6,3$$

$$D(3,7; -6,3)$$

$$f(-0,4) = 10,4$$

$$B(-0,4; 10,4)$$

and

(6)

(c) Determine the numerical value of $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle CDE}$. Show all calculations.

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 10,4 \times 4 = 20,8$$

$$\text{Area of } \triangle CDE = \frac{1}{2} \times 6,3 \times 3 = 9,45$$

$$\frac{20,8}{9,45} = 2,2 \text{ units}^2$$

(4)

(d) Determine the x -value of the point of inflection of f .

$$f''(x) = 0 \quad \checkmark$$

$$0 = 3x - 5 \quad \checkmark$$

$$5 = 3x$$

$$\frac{5}{3} = x \quad \checkmark$$

(3)

(e) For which values of x will $f(x) \cdot f''(x) > 0$?

$$x > 5 \quad \checkmark$$

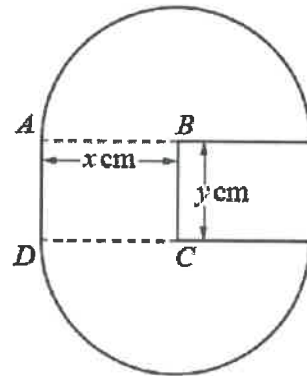
$$x < -2 \quad \checkmark$$

(2)

[21]

QUESTION 10

- (a) The diagram shows a badge made of thin sheet metal, consisting of two identical semi-circular pieces with centres B and C with radius x . The semi-circles are connected by a rectangular section, ABCD, such that $AB = x$ and $BC = y$.



- (1) Given that the area of the badge is 20 cm^2 , show that P , the perimeter of the badge, is given by:

$$P = 2x + \frac{40}{x}$$

$$\text{Area} = \text{O} + \square$$

$$20 = \pi x^2 + xy \quad \checkmark m$$

$$\frac{20 - \pi x^2}{x} = y \quad \checkmark d$$

$$p = 2\pi x + y + y + x + x \quad \checkmark m$$

$$p = 2\pi x + 2x + 2\left(\frac{20 - \pi x^2}{x}\right) \quad \checkmark \text{sub}$$

$$p = 2\pi x + 2x + \frac{40}{x} - 2\pi x$$

$$p = 2x + \frac{40}{x}$$

(4)

- (2) Hence determine the minimum value of the perimeter, leaving your answer in the simplest surd form.

$$p^2 = 2x + 40x^{-1} \quad \checkmark$$

$$0 = 2 - 40x^{-2} \quad \checkmark \checkmark$$

$$-2 = \frac{-40}{x^2}$$

$$\frac{1}{20} = \frac{1}{x^2}$$

$$\sqrt{20} = x \quad \checkmark$$

(4)

- (b) Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$.

$$f'(x) = -9x^2 + 1 \quad \checkmark m$$

$$0 = -9x^2 + 1 \quad \checkmark$$

$$9x^2 = 1$$

$$x^2 = \frac{1}{9}$$

$$x = \frac{1}{3} \quad \checkmark$$

$$y = \frac{2}{9} \quad \checkmark$$

$\therefore$ move up by $\frac{6}{9}$ units to $\frac{8}{9}$

$$\therefore q = \frac{6}{9} \quad \checkmark = \frac{2}{3}$$

(5)

[13]

QUESTION 11

Given: $f(x) = x^2 - 2x$

The minimum values of $f(x) + 2f(x) + 4f(x) + 8f(x) + \dots$ form a geometric series.

(a) Determine the first 4 terms in the series.

$$f'(x) = 2x - 2$$

$$0 = 2x - 2 \quad \checkmark m$$

$$2 = 2x$$

$$1 = x \quad \checkmark a$$

$$y = -1$$

$$\therefore -1, -2, -4, -8$$

$\checkmark a$ $\underbrace{\hspace{1cm}}_{\checkmark a}$

(4)

(b) If you were to find the sum of the minimum values to n terms, between which two consecutive terms in the series will the difference of their sums be $-536\,870\,912$?

$$S_{n+1} - S_n = -536\,870\,912 \quad \checkmark m$$

$$\frac{-1(2^{n+1} - 1)}{2 - 1} - \frac{-1(2^n - 1)}{2 - 1} = -536\,870\,912$$

$\checkmark \text{sub } a = -1 \quad \checkmark \text{sub } r = 2$

$$-(2^{n+1}) + 1 + 2^n - 1 = -536\,870\,912$$

$$-(2^n \cdot 2^1) + 1 + 2^n - 1 = -536\,870\,912 \quad \checkmark$$

$$-2 \cdot 2^n + 2^n = -536\,870\,912$$

$$2^n(-2 + 1) = -536\,870\,912$$

$$2^n = 536\,870\,912$$

$$2^n = 2^{29}$$

$$n = 29 \quad \checkmark$$

(6)

[10]

78 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED
THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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