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$$\begin{aligned} (x-2) &= 3\sqrt{x+2} \\ (x-2)^2 &= 9(x+2) \\ x^2 - 4x + 4 &= 9x + 18 \\ x^2 - 13x - 14 &= 0 \\ (x-14)(x+1) &= 0 \\ x &= 14 \text{ or } x \neq -1 \end{aligned}$$

- b) Solve for x if:
1.  $x - 3\sqrt{x+2} = 2$ .
- (6)

$$\begin{aligned} (3x+4)(2x-1) &= 0 \\ 6x^2 + 5x - 4 &= 0 \\ \therefore p &= 5 \end{aligned}$$

- a) The quadratic equation  $6x^2 + px - 4 = 0$  has roots  $-\frac{3}{4}$  and  $\frac{1}{2}$ . Determine the value of p.
- (3)

Question 1 [27]

- Write your name, surname and class on this cover page.
- This question paper consists of 11 questions and 17 pages.
- Answer all the questions in the spaces provided on the question paper.
- You may use an approved, non-programmable and non-graphical calculator, unless otherwise stated. Round off to 2 decimal places unless the context requires rounding up or down.
- All the necessary working details must be clearly shown.
- Write legibly and present your work neatly.
- The information sheet is on page 17, and you're allowed to detach it.

INSTRUCTIONS:

NAME & CLASS:

MEMO

|                          |                 |          |            |
|--------------------------|-----------------|----------|------------|
| Examiner                 | L Mudzamiri     | Marks    | 150        |
| Moderator                | C van de Venter | Time     | 3 Hours    |
| Date handed out          | 23/08/2024      | Due date | 23/08/2024 |
| PRELIMINARY EXAM PAPER 1 |                 |          |            |
| MATHEMATICS              |                 |          |            |

HERITAGE HOUSE

$$\begin{aligned}
 y &= 3x - 4 \dots (A) \\
 \text{Subst: (A) in (B)} \\
 x^2 + 2x(3x-4) - (3x-4)^2 &= -2 \\
 x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) &= -2 \\
 x^2 + 6x^2 - 8x - 9x^2 + 24x - 16 + 2 &= 0 \\
 -2x^2 + 16x - 14 &= 0 \\
 x^2 - 8x + 7 &= 0 \\
 (x-7)(x-1) &= 0 \\
 x=7 \text{ or } x=1
 \end{aligned}$$

$y = 3(7) - 4$  or  $y = 3(1) - 4$   
 $y = 17$  or  $y = -1$   
 $(7, 17)$  or  $(1, -1)$

(d) Solve for  $x$  and  $y$  simultaneously in the following set of equations:  
 $3x - y = 4$  and  $x^2 + 2xy - y^2 = -2 \dots (B)$

(7)

$$\begin{aligned}
 \Delta &= b^2 - 4ac \\
 &= (-k)^2 - 4(k+1)(-1) \\
 &= k^2 + 4(k+1) \\
 &= k^2 + 4k + 4 \\
 &= (k+2)^2
 \end{aligned}$$

$\therefore$  roots are rational.  
 perfect square.

(c) Given that:  $(k+1)x^2 - kx - 1 = 0$  where  $k \neq -1$ , show that the roots are rational.

(4)

$$\begin{aligned}
 3. \quad x^{\frac{2}{3}} &= 36 \\
 \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} &= (36)^{\frac{3}{2}} \\
 x &= (2^2 \cdot 3^2)^{\frac{3}{2}} \\
 x &= 2^3 \cdot 3^3 \\
 x &= 216
 \end{aligned}$$

might do this in 1 step

(3)

$$\begin{aligned}
 \log_3 \left( \frac{x}{x+10} \right) &= \log_3 9 \\
 \frac{x}{x+10} &= 9 \\
 x+10 &= \frac{x}{9} \\
 x &= \frac{x}{9}
 \end{aligned}$$

(4)

Mr Mabaso starts a business and buys a delivery truck for R550 000.

- a) If the inflation rate is 6% per annum, calculate the cost of a new delivery truck after a period of five years.

$$A = P(1+i)^n$$

$$A = R550\,000(1+6\%)^5$$

$$A = R736\,024,07$$

- b) If depreciation is calculated at 20% per annum using the reducing-balance method, what will the value of Mr Mabaso's truck in five years' time?

$$A = P(1-i)^n$$

$$A = R550\,000(1-20\%)^5$$

$$A = R180\,224$$

- c) Mr Mabaso sets up a sinking fund starting *immediately* and for the next 5 years so that he can trade in his delivery truck and buy a new one by financing the difference with the sinking fund. The bank offers an interest rate of 9% per annum compounded monthly. How much should Mr Mabaso put away at the end of each month so that he has enough money to finance the difference between the new vehicle and trade-in value of his truck at the end of 5 years? The last payment into the sinking fund is made at the end of the 5 years.

$$\text{Sinking Fund} = R555\,800,07$$

$$n = 12 \times 5 + 1 = 61$$

$$F = x \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$x = \frac{R555\,800,07}{\frac{(1+\frac{9\%}{12})^{61} - 1}{\frac{9\%}{12}}}$$

$$x = R7\,219,14$$

a) If  $f(x) = 2x^2 - 3$

1) Find  $f'(x)$  from first principles.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = 2(x+h)^2 - 3$$

$$= 2x^2 + 4xh + 2h^2 - 3$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3 - (2x^2 - 3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h}$$

$$= 4x + 0 = 4x$$

(5)

2) Find the equation of a tangent to  $f(x)$  at  $x = -2$ .

$$f'(x) = m = 4x$$

$$f'(-2) = m = 4(-2) = -8$$

$$y = -8x + c$$

$$y = 2(-2)^2 - 3 = 5$$

$$\text{subst. } (-2, 5) \text{ in } y = -8x + c$$

$$5 = -8(-2) + c$$

$$5 = 16 + c$$

$$c = -11$$

$$y = -8x - 11$$

(5)

b) Determine  $g'(x)$  if  $g(x) = \frac{5}{4x} - x + \sqrt[3]{x^2}$ 

$$g(x) = \frac{5}{4}x^{-1} - x + x^{\frac{2}{3}}$$

$$g'(x) = -\frac{5}{4}x^{-2} - 1 + \frac{2}{3}x^{-\frac{1}{3}}$$

$$= -\frac{5}{4x^2} - 1 + \frac{2}{3\sqrt[3]{x}}$$

(5)

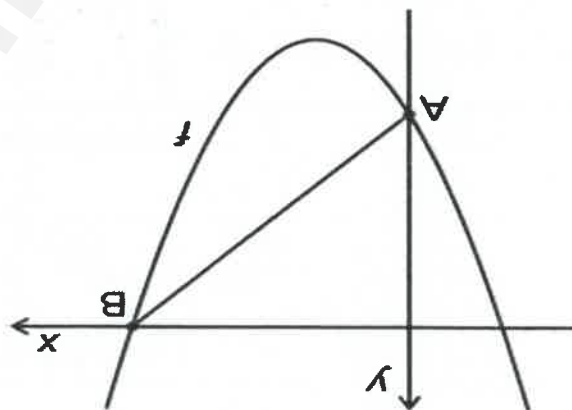
$$\begin{aligned}
 A(0, -21) \\
 B(7, 0) \\
 m_{AB} &= \frac{0 - (-21)}{7 - 0} = 3 \\
 f'(x) &= 2x - 4 \\
 2x - 4 &= 3 \\
 2x &= 7 \\
 x &= \frac{7}{2} \\
 y &= \left(\frac{7}{2}\right)^2 - 4\left(\frac{7}{2}\right) - 21 = -\frac{91}{4}
 \end{aligned}$$

$\rightarrow \left(\frac{7}{2}, -\frac{91}{4}\right)$

- (5) line AB.
- 2) Determine the coordinates of the point on  $f$  where the tangent to  $f$  is parallel to the

$$\begin{aligned}
 A(0, -21) \\
 B(x, 0) \\
 x^2 - 4x - 21 &= 0 \\
 (x - 7)(x + 3) &= 0 \\
 x &= 7 \text{ or } x = -3 \\
 B(7, 0)
 \end{aligned}$$

- (4) 1) Calculate the coordinates of A and B.



- $f(x) = x^2 - 4x - 21$
- A is the y-intercept and B is an x-intercept of  $f$ .

c) In the diagram below:

a) An arithmetic series has a first term of 3, a last term of 47 and the sum of all the terms is 300.

1) Determine the number of terms in the series.

(3)

$$S_n = \frac{n}{2}(a + l)$$

$$\frac{n}{2}(3 + 47) = 300$$

$$25n = 300$$

$$n = 12$$

2) Determine the common difference.

(3)

$$T_n = a + (n-1)d$$

$$47 = 3 + (12-1)d$$

$$44 = 11d$$

$$d = 4$$

b) Calculate:

$$\sum_{n=2}^{\infty} 4 \cdot \left(\frac{1}{2}\right)^{n-2}$$

\*

$$a = 4 \cdot \left(\frac{1}{2}\right)^{6-2}$$

$$a = \frac{1}{4}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{2}}$$

$$= \frac{1}{2}$$

c) The third term of a geometric series is 7 and the sixth term is  $-2401$ . Determine the first term

and the common ratio.

(5)

$$T_3 = ar^2 = 7$$

$$T_6 = ar^5 = -2401$$

$$\frac{T_6}{T_3} = \frac{ar^5}{ar^2} = \frac{-2401}{7} = -343$$

$$r^3 = -343$$

$$r = -7$$

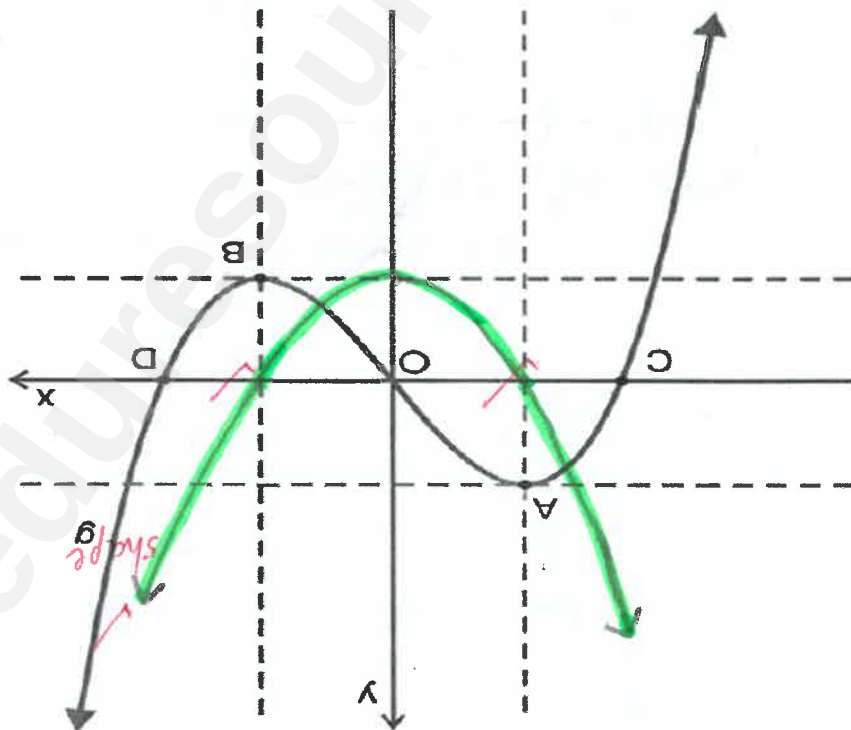
$$a(-7)^2 = 7$$

$$49a = 7$$

$$a = \frac{1}{7}$$

In the diagram below:

- $g(x) = 3x^3 - 9x$
- A and B are the turning points of  $g$ .
- C, O and D are the x-intercepts of  $g$ .



a) Determine the coordinates of A and B.

(5)

$$\begin{aligned}
 g'(x) &= 9x^2 - 9 \\
 \text{f.p.: } g'(x) &= 0 \\
 9x^2 - 9 &= 0 \\
 x^2 &= 1 \\
 x &= \pm 1 \\
 x &= +1 \\
 y &= 3(1)^3 - 9(1) = -6 \\
 \text{or } y &= 3(-1)^3 - 9(-1) \\
 y &= 6
 \end{aligned}$$

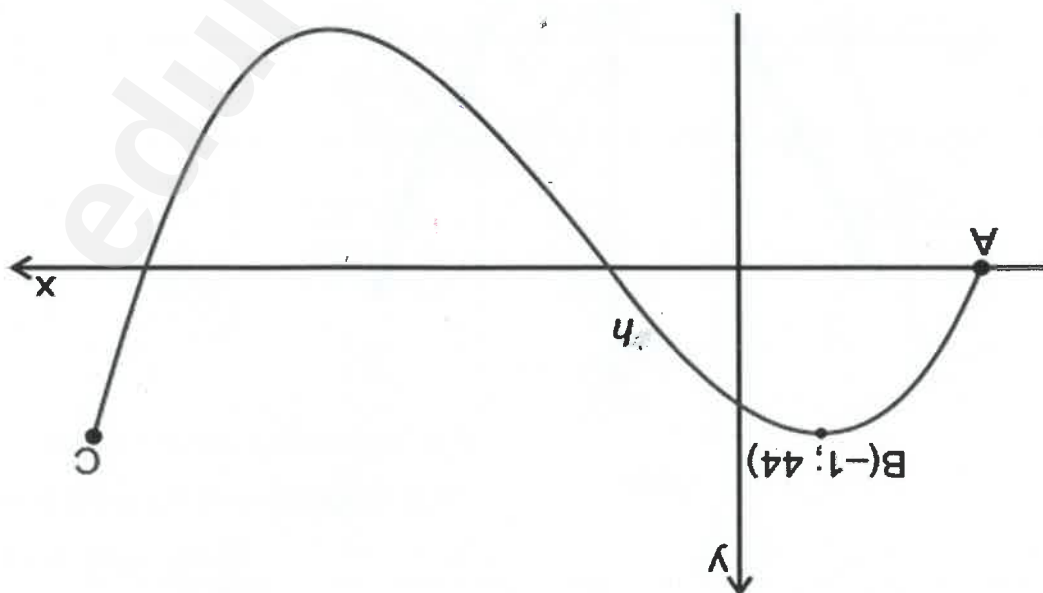
$A(-1; 6)$  ;  $B(1; -6)$

b) On the cartesian plane above, sketch  $g'(x)$ .

(3)

$$\begin{aligned}
 44 &= (-1)^3 + a(-1)^2 + b(-1) + 36 \\
 44 &= -1 + a - b + 36 \\
 9 &= a - b \\
 9 + b &= a \quad \text{--- (A)} \\
 h'(x) &= 3x^2 + 2ax + b \\
 \text{TP: } h'(x) &= 0 \Rightarrow \text{when } x = -1 \\
 3(-1)^2 + 2a(-1) + b &= 0 \\
 3 - 2a + b &= 0 \quad \text{--- (B)} \\
 \text{subst. (A) in (B)} \\
 3 - 2(9 + b) + b &= 0 \\
 3 - 18 - 2b + b &= 0 \\
 -15 - b &= 0 \\
 b &= -15 \\
 a &= 9 + (-15) \\
 a &= -6
 \end{aligned}$$

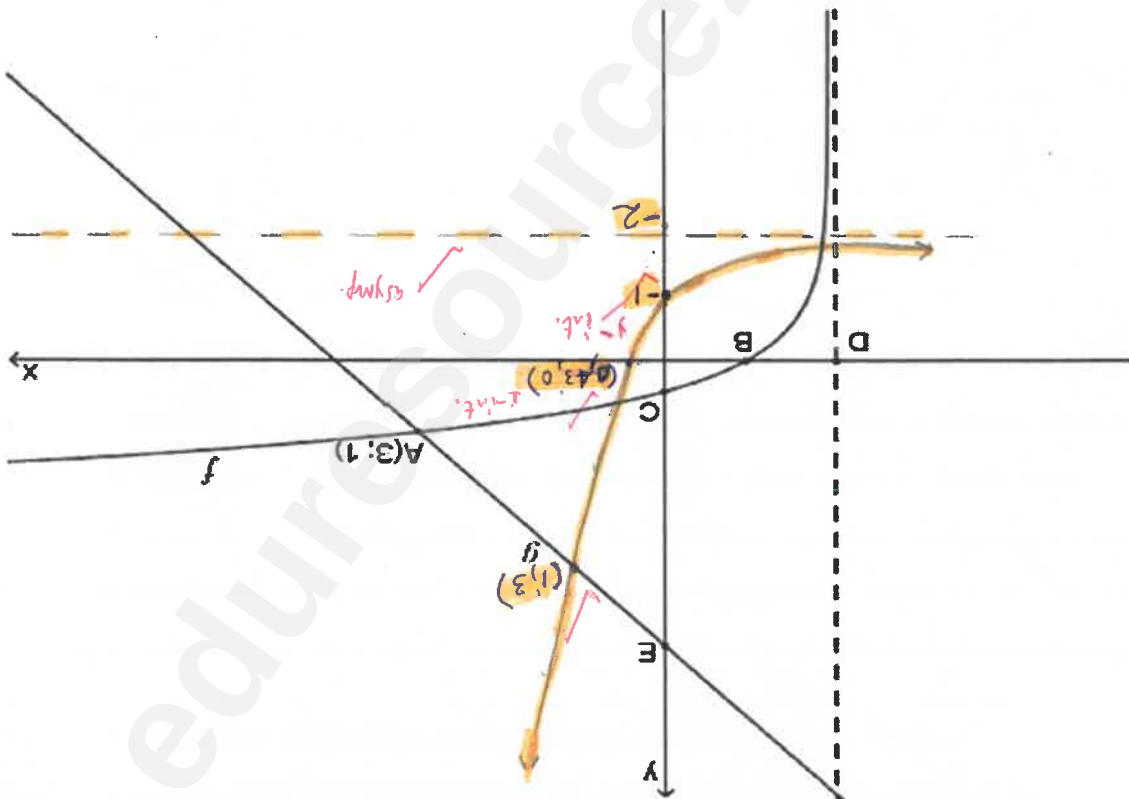
(8) Calculate the values of  $a$  and  $b$  if  $h(x) = x^3 + ax^2 + bx + 36$ .



c) In the diagram below: The turning points of the curve are  $A$  and  $B$ . The curve passes through the point  $C$ .

The graphs of  $f$  and  $g$  have been sketched on the set of axes below.

- $f(x) = \log_m(x+2)$
- $g(x) = -x + 4$
- B and C are the x-intercept and y-intercept of the graph of  $f$  respectively.
- A (3; 1) lies on both  $f$  and  $g$ .
- The dotted line through D with equation  $x = -2$  is the asymptote for  $f$ .



a)  $f(x) = \log_m(x+2)$ ; determine the value of  $m$ .

(2)

$$1 = \log_m(3+2) \quad 1 = \log_m 5 \quad m^1 = 5$$

b) Determine the coordinates of B and C if  $m = 5$ .

(4)

$$f(x) = \log_5(x+2) \quad f(0,0) = \log_5(0+2) \quad 0 = \log_5(2) \quad 5^0 = 2 \quad 1 = x+2 \quad x = -1 \quad B(-1,0)$$

$$c(0,y) \quad y = \log_5(0+2) \quad y = \log_5 2 = 0,43 \quad C(0,0,43)$$

|  |
|--|
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|  |
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|  |

- d) On the set of axes on page 9, sketch the graph of  $f^{-1}(x)$ , the inverse of  $f$ . Label the intercepts, asymptote and the point where  $f^{-1}(x) = g(x)$ . (4)

|  |
|--|
|  |
|  |
|  |
|  |
|  |

- c) For what value(s) of  $x$  is  $f(x) = g(x)$ ? (2)

$$-2 < x < 3$$

Asliya takes out a home loan of R975 000 from the bank. The fixed interest rate given by the bank is 10% per annum compounded monthly.

a) If Asliya is required to pay off the loan over a period of 20 years and her first payment is made at the end of the first month, calculate the minimum monthly payments required to pay off the

loan.

(4)

$$P = x \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

$$x = \frac{R975\,000}{\frac{1 - (1 + \frac{10\%}{12})^{-240}}{\frac{10\%}{12}}}$$

$$x = R9408,96$$

b) Calculate Asliya's balance outstanding at the end of the 14<sup>th</sup> year.

(4)

$$B_{14 \times 12} = B_{168} = R9408,96 \left[ \frac{1 - (1 + \frac{10\%}{12})^{-(240-168)}}{\frac{10\%}{12}} \right]$$

$$B_{168} = R507\,831,29$$

c) If, instead of the minimum payment, Asliya pays R20 000 per month, calculate how long she

would take for her to pay off the loan. (Round up to the nearest month)

(5)

$$R975\,000 = R20\,000 \left[ \frac{1 - (1 + \frac{10\%}{12})^{-n}}{\frac{10\%}{12}} \right]$$

$$0,40625 = 1 - (1 + \frac{10\%}{12})^{-n}$$

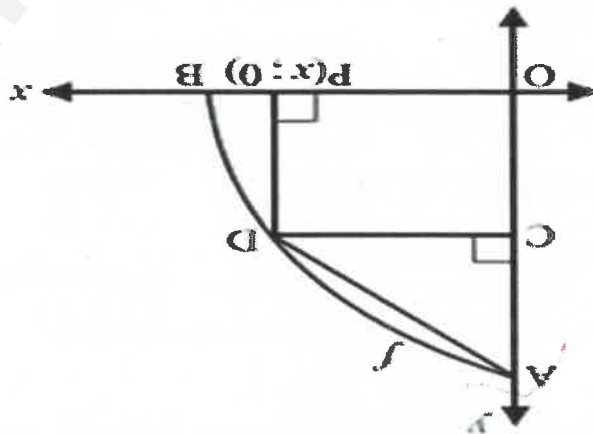
$$\left(1 + \frac{10\%}{12}\right)^{-n} = 0,59375$$

$$-n = \log_{\left(1 + \frac{10\%}{12}\right)} 0,59375$$

$$-n = -62,82$$

$$n \approx 63 \text{ months}$$

In the diagram below, the graph of the function  $f(x) = 8 - x^3$  for  $x \geq 0$  and  $y \geq 0$  is represented. The graph of  $f$  intersects the  $x$ -axis at  $B$  and the  $y$ -axis at  $A$ .  $P(x; 0)$  is any point on the interval  $OB$ .  $D$  lies on  $f$  such that  $CD \perp AO$  and  $DP \perp OB$ .



a) Determine the length of  $OA$  and  $OB$ .

$$OA = 8 \text{ units.}$$

$$B(x, 0)$$

$$0 = 8 - x^3$$

$$x^3 = 8$$

$$x = \sqrt[3]{8} = 2$$

$$OB = 2 \text{ units.}$$

b) Determine the length of  $OC$  in terms of  $x$ .

$$OC = 8 - x^3$$

c) Prove that the area of Quadrilateral  $OBDA$  is equal to:

$$1) A(x) = -x^3 + 4x + 8$$

$$A(x) = \frac{1}{2}(OB+CD)(OC) + \frac{1}{2}(CD)(CA)$$

$$= \frac{1}{2}(2+x)(8-x^3) + \frac{1}{2}x(x^3)$$

$$= \frac{1}{2}(16 - 2x^3 + 8x - x^4) + \frac{1}{2}x^4$$

$$= 8 - x^3 + 4x - \frac{x^4}{2} + \frac{x^4}{2}$$

$$= -x^3 + 4x + 8$$

(4)

(1)

(3)

2) Hence, determine the profit function  $P(x)$  if the marginal OBDA is

a maximum.

(3)

$$\begin{aligned}
 A(x) &= -x^3 + 4x^2 + 8x \\
 A'(x) &= -3x^2 + 4 \\
 A'(x) &= 0 \quad (\text{max}) \\
 -3x^2 + 4 &= 0 \\
 x^2 &= \frac{4}{3} \\
 x &= \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}
 \end{aligned}$$

### Question 9

[9]

A factory sells t-shirts in packs of 5 for R200 per pack. If the number of t-shirts purchased by a certain sporting outlet is  $(100 + 5n)$ , the price per t-shirt would be reduced to  $R(40 - n)$ , where  $n \in \mathbb{N}$ .

a) Show that the pattern that emerges for the total cost ( $T_n$ ) for the number of t-shirts purchased has a constant second difference of  $-10$ .

(3)

| $n$   | $100 + 5(1)$    | $100 + 5(2)$    | $100 + 5(3)$    | $100 + 5(4)$    |
|-------|-----------------|-----------------|-----------------|-----------------|
| Cost  | $105 \times 39$ | $110 \times 38$ | $115 \times 37$ | $120 \times 36$ |
| $d_1$ | $4095$          | $4180$          | $4255$          | $4320$          |
| $d_2$ |                 | $85$            | $75$            | $65$            |
|       |                 |                 | $-10$           |                 |

(4)

b) Determine the  $n$ th term in the form  $T_n = an^2 + bn + c$ .

$$\begin{aligned}
 2a &= -10 \\
 a &= -5 \\
 a + b + c &= 4095 \\
 -5 + 100 + c &= 4095 \\
 95 + c &= 4095 \\
 c &= 4000 \\
 T_n &= -5n^2 + 100n + 4000
 \end{aligned}$$

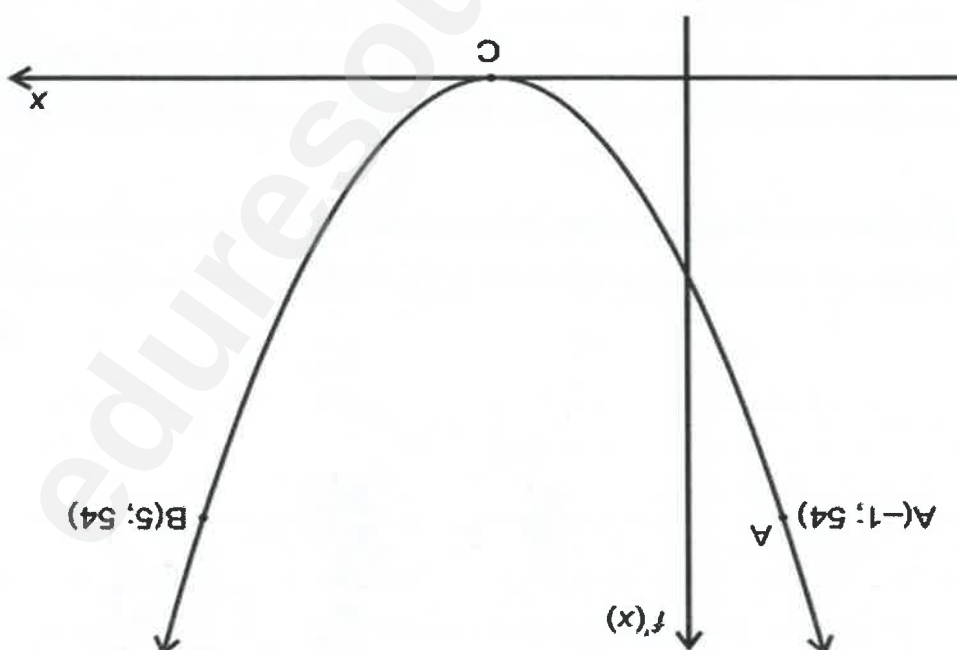
c) Explain why this business model in question 9 cannot be sustained indefinitely.

(2)

The price for a t-shirt reduces to a negative number. The factory will run at a huge loss using this model. To allow any reasonable answers.

Below is the sketch of  $f'(x)$ :

- A and B lie on  $f'(x)$  with C being the turning point.
- C lies on the x-axis.

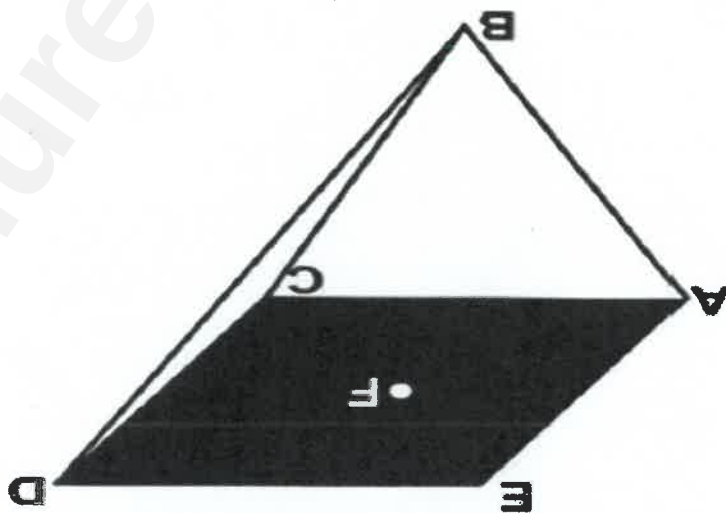


(6) If  $f(0) = -14$  and  $f(2) = 2$ , calculate the x-intercept of  $f(x)$

$$\begin{aligned}
 f'(x) &= a(x-2)^2 \\
 54 &= a(5-2)^2 \\
 54 &= 9a \\
 a &= 6 \\
 f'(x) &= 6(x-2)^2 \\
 &= 6(x^2 - 4x + 4) \\
 &= 6x^2 - 24x + 24 \\
 f(x) &= ax^3 + bx^2 + cx + d \\
 f'(x) &= 3ax^2 + 2bx + c \\
 &= 3a = 6 \quad 2b = -24 \quad c = 24 \\
 a &= 2 \quad b = -12 \\
 f(x) &= 2x^3 - 12x^2 + 24x - 14 \\
 \text{Let } y &= 0 \\
 2x^3 - 12x^2 + 24x - 14 &= 0 \\
 x^3 - 6x^2 + 12x - 7 &= 0 \\
 x &= 1
 \end{aligned}$$

The diagram below shows an inverted right square pyramid.

The length of each side of the square is 3 cm and the perpendicular height of the pyramid is 9 cm. A new pyramid is to be constructed so that the volume of the pyramid is maximised.



The new pyramid is constructed according to the following rules:

- The pyramid must remain a right square pyramid.
- The perpendicular height of the pyramid must be decreased by the same amount that the length of each side of the square is increased.

Find the ratio of the length of the side of the square to the perpendicular height when the volume is maximised.

Useful formula: Volume of Pyramid =  $\frac{1}{3} A \times H$

(8)

$$\begin{aligned}
 V &= \frac{1}{3} (3+x)(3+x)(9-x) \\
 V &= \frac{1}{3} (9-x)(9+6x+x^2) \\
 V &= \frac{1}{3} (81+54x+9x^2-9x-6x^2-x^3) \\
 V &= \frac{1}{3} (81+45x+3x^2-x^3) \\
 V &= 27+15x+x^2-\frac{1}{3}x^3 \\
 \frac{dV}{dx} &= 15+2x-x^2 \\
 \frac{dV}{dx} &= 0 \\
 15+2x-x^2 &= 0 \\
 x^2-2x-15 &= 0 \\
 (x-5)(x+3) &= 0 \\
 x &= 5 \quad x \neq -3
 \end{aligned}$$

New length =  $3+5=8$   
 New Height =  $9-5=4$   
 New length : New Height =  $8 : 4 = 2 : 1$

TOTAL = 150

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{i=n} 1 = n$$

$$T_n = a + (n-1)d$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} : r \neq 1$$

$$S_n = \frac{a}{1-r} : -1 < r < 1$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\sum_{i=1}^{i=n} i = \frac{n(n+1)}{2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1+ni)$$

$$A = P(1+i)^n$$

$$F = x \left[ \frac{i}{(1+i)^n - 1} \right]$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$P = x \left[ \frac{i}{1 - (1+i)^{-n}} \right]$$

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