



eduresourceza.com



Crawford
International

**CRAWFORD INTERNATIONAL PRELIMINARY EXAMINATION
SEPTEMBER 2024
MATHEMATICS: PAPER II**

Time: 3 hours

150 marks

MARKING GUIDELINES

MARKER TO ENTER MARKS:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
8	14	15	6	13	10	10	27	16	12	19	150

QUESTION 1

- (a) Determine the equation of the least squares regression line. (3)
 $y = 161,26 + 26,88x$ 1A for a
1A for b
1 for equation
- (b) Calculate the correlation coefficient for this data set. (1)
 $r = 0,98$ 1A
- (c) Estimate on which day in January one can expect that 618 patients will be treated. (2)
Show your calculation to justify your conclusion. 1M
 $618 = 161,26 + 26,88(x)$ 1CA
 $x = 17$
or $\hat{x} = 17$
- (d) The medical staff predict that 995 patients will be treated on 31 January. (2)
Discuss the validity of this prediction.
Extrapolation . Any valid reason. 1A
extrapolation
1M valid
reason
- [8]

QUESTION 2

The time taken, in minutes, by a group of students to complete their mathematics homework is summarised in the frequency table below.

Time, t (minutes)	Frequency
$5 \leq t < 15$	3
$15 \leq t < 25$	f
$25 \leq t < 35$	7
$35 \leq t < 45$	6
$45 \leq t < 55$	4

- (a) The estimated mean of the grouped data is 30 minutes. (3)
Calculate the value of the missing frequency, f .

$$\frac{10(3) + 20f + 30(7) + 40(6) + 50(4)}{20 + f} = 30 \quad 2A$$

$$f = 8 \quad 1CA$$

- (b) Calculate the estimated standard deviation of the given data. (2)

$$sd = 12,2 \quad 2A$$

- (c) Comment on the meaning of a standard deviation and explain how it relates to the data presented above. (2)

On average, how does each data entry differ from the mean. 1M

It describes the spread the data values from the mean.

The time taken by the students to complete homework varies significantly. 1M

- (d) Determine the estimated number of values that lie above one standard deviation of the mean. Show your working. (3)

$$\begin{aligned} 30 + 12.2 & \\ = 42,2 & \quad 1M \\ \pm 6 \text{ students} & \quad 1CA \end{aligned}$$

(e)

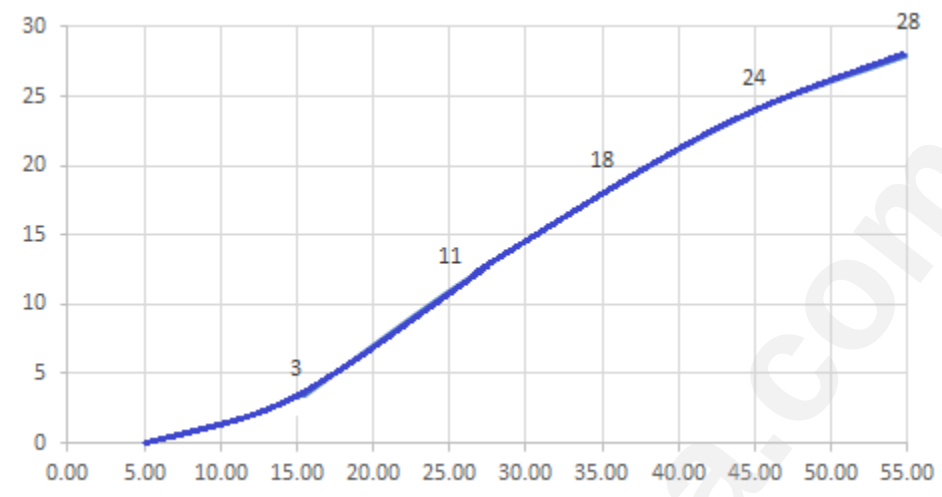
On the grid provided below, draw an ogive which represents the data above.

(4)

2A correct points

1A (5; 0)

1M correct shape



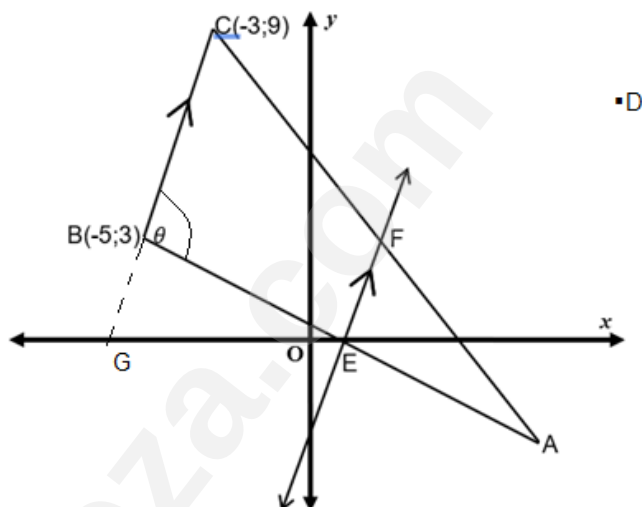
[14]

QUESTION 3

Refer to the diagram below:

- $EF \parallel BC$.
- the vertices of $\triangle ABC$ are A, B $(-5; 3)$ and C $(-3; 9)$.
- The x-intercept of line AB at point E is $(1; 0)$.
- $\hat{ABC} = \theta$

This diagram is NOT drawn to scale.



- (a) Show that the equation of EF is $y = 3x - 3$ (3)
- $$m_{EF} = m_{BC} = \frac{9-3}{-3+5}$$
- $$= 3$$
- $$y - 0 = 3(x - 1)$$
- $$\therefore y = 3x - 3$$
- 1A for gradient
2CA sub into correct formula
- (b) Calculate the value of θ (5)
- $$\tan \hat{BGO} = 3$$
- $$\hat{BGO} = 71,6^\circ$$
- $$\tan \hat{BEX} = -\frac{1}{2}$$
- $$\hat{BEX} = 153,4^\circ$$
- $$\hat{GBE} = 153,4^\circ - 71,6^\circ$$
- $$= 81,8^\circ$$
- $$\theta = 98,2^\circ$$
- 1M
1A
1A
1CA
1CA
- (c) If $AE:BE$ is $1:1$, determine the coordinates of A. (2)
- A $(7; -3)$
2A any method

- (d) Calculate the length of FE. (3)
 $BQ = \sqrt{(-3+5)^2 + (9-3)^2}$ 1M
 $= 2\sqrt{10}$ 1A
 $FE = \sqrt{10}$ midpoint theorem 1CA
 3/3 if reason is omitted
 (2)
- (e) Determine the coordinates of D if ABCD is a parallelogram.
 $D(9;3)$ 2A any method
[15]

QUESTION 4

Simplify the following trigonometric expression as far as possible:

$$\frac{\sin(2\theta)}{\tan(\theta)} - 2\cos(\theta + 90^\circ) \cdot \sin(360^\circ + \theta)$$

(6)

$$\begin{aligned} & \frac{2\sin\theta \cdot \cos\theta}{\frac{\sin\theta}{\cos\theta}} + 2\sin\theta \cdot \sin\theta \\ &= 2\cos^2\theta + 2\sin^2\theta \\ &= 2 \end{aligned}$$

4A

1CA

1A

[6]**QUESTION 5**

- (a) Showing all calculations, determine the general solution for the equation $\sin 2x = -2\cos x$.

(6)

$$\begin{aligned} 2\sin x \cdot \cos x + 3\cos x &= 0 \\ \cos x(2\sin x + 3) &= 0 \\ \cos x &= 0 \text{ only} \\ x &= 90^\circ + k(180^\circ) \end{aligned}$$

1A ; 1M

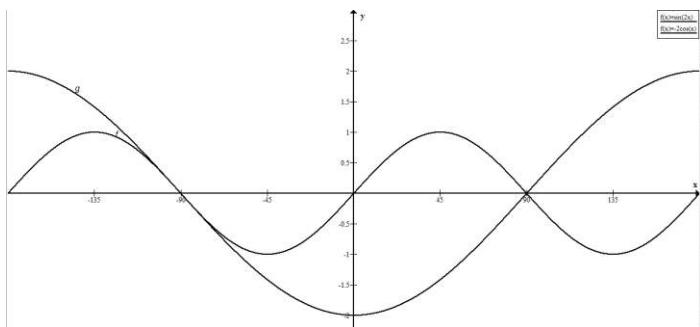
1CA

factorising

2CA

1CA

- (b) (1) On the same axes provided below, sketch the graphs of $f(x) = \sin 2x$ and $g(x) = -2\cos(x)$ for $-180^\circ \leq x \leq 180^\circ$. (4)



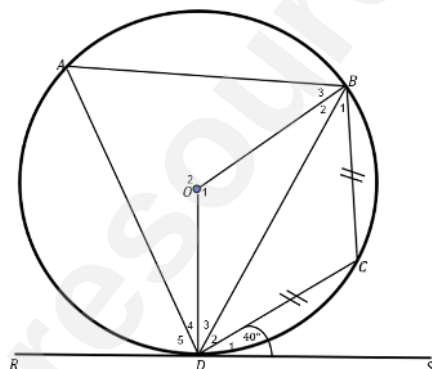
4A

- (2) Use your graphs of f and g to determine the value(s) of x for which $\sin 2x + 2\cos x > 0$ for $-180^\circ \leq x \leq 180^\circ$ (3)
 $\sin 2x > -\cos 2x$
 $x \in (-90^\circ; 90^\circ)$

2CA for intervals
1CA for brackets
[13]

QUESTION 6

In the figure below, RDS is a tangent to the circle centre O at D .
 $BC = DC$ and $\widehat{CDS} = 40^\circ$.



- (a) Determine the size of each of the following angles. State reasons in each case. (2)

$$\widehat{B}_1 = 40^\circ \quad \text{tan chord theorem}$$

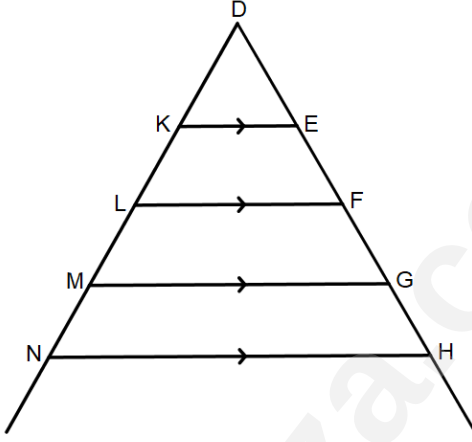
$$\therefore \widehat{D}_1 = 40^\circ \quad \angle s \text{ opp equal sides}$$

1A $\widehat{B}_1 = 40^\circ$
 1CA $\widehat{D}_1 = 40^\circ$
 statement and reason 1 mark each

- (b) $\hat{C} = 100^\circ$ int $\angle$ s of Δ
 $\hat{A} = 80^\circ$ opp $\angle$ s of cyclic quad
- 1A $\hat{C} = 100^\circ$
 1CA $\hat{A} = 80^\circ$
 1A reasons (3)
- (c) $\hat{O}_1 = 160^\circ$ $\angle$ at centre = $2 \times \angle$ at circumference
- 1A $\hat{O}_1 = 160^\circ$
 1A reason (2)
- (d) $OB = OD$ radii
 $\therefore \hat{D}_3 = \hat{B}_2$ $\angle$ s opp equal sides
 $\hat{D}_3 = \frac{180^\circ - 160^\circ}{2} = 10^\circ$ int $\angle$ s of Δ
- 1A
 1A
 1A (3)
[10]

QUESTION 7

In the diagram below, $KE \parallel LF \parallel MG \parallel NH$ and so on.
 It is also given that $DK:KL = 2:1$, $DL:LM = 2:1$, $DM:MN = 2:1$ and so on.
 $KE = 2$ units



- (a) Calculate the lengths of LF, MG and NH. Show your calculations. (6)
 You do NOT have to provide reasons

In $\triangle DLF$:

$$\frac{DL}{DK} = \frac{LF}{KE} = \frac{3}{2}$$

1A

$$\therefore \frac{LF}{2} = \frac{3}{2}$$

1M

$$\therefore LF = 3$$

1A

In $\triangle DMG$:

$$\frac{DM}{DL} = \frac{MG}{LF} = \frac{3}{2}$$

$$\therefore \frac{MG}{3} = \frac{3}{2}$$

1CA

$$\therefore MG = 4,5$$

1CA

In $\triangle DNH$:

$$\frac{DM}{MN} = \frac{NH}{MG} = \frac{3}{2}$$

$$\therefore \frac{NH}{4,5} = \frac{3}{2}$$

1CA

$$\therefore NH = 6,75$$

1CA

- (b) If this pattern continues, calculate the lengths of the tenth parallel line.

Sequence: 2; 3; 4,5; 6,25;

$$T_{10} = ar^9$$

$$T_{10} = 2\left(\frac{3}{2}\right)^9$$

$$T_{10} = 76,9$$

(4)

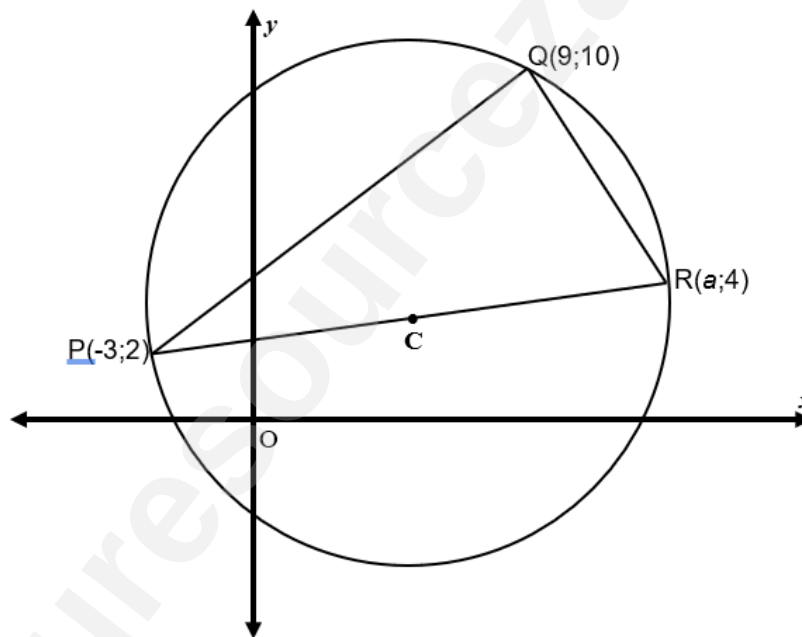
1A for a
1 M for
formula
1CA for r
1CA

[10]

QUESTION 8

The points P(-3; 2), Q(9; 10) and R(a; 4) lie on the circumference of the circle, as shown in the figure below.

- PR is a diameter of the circle with centre C.
- M is a point on PQ so that PM : PQ is 3 : 4.



- (a) Show all calculations to prove that $a = 13$

(4)

$$m_{PQ} = \frac{2}{3}$$

1A

$$\hat{PQR} = 90^\circ \quad \text{angle in semi circle}$$

1M

$$\therefore m_{QR} = -\frac{3}{2}$$

1CA

$$\frac{10-4}{9-a} = -\frac{3}{2}$$

1CA

$$a = 13$$

- Downloaded from eduresourceza.com
- (b) Determine the equation of the circle. (6)
 $C\left(\frac{13-3}{2}; \frac{4+2}{2}\right)$ 1M; 1A
 $\therefore C(5; 3)$ 1A
 $\therefore (x-5)^2 + (y-3) = r^2$ 1CA
 Sub $Q(9; 10): (9-5)^2 + (10-3)^2 = r^2$ 1CA
 $r^2 = 65$ 1CA
 $\therefore (x-5)^2 + (y-3) = 65$
- (c) Calculate the coordinates on M. (4)
 Midpoint PQ: (3;6) 1M 1A
 Midpoint top half: (6;8) 2CA
- (d) Calculate the size of $\hat{PKO}$ (3)
 $m_{pq} = \frac{2}{3}$ 1A
 $\text{Incl}(PQ) = 33,7^\circ$ 1CA
 $\hat{PKO} = 90^\circ - 33,7^\circ$
 $= 56,3^\circ$ 1CA
- (d) Consider a second circle with equation $x^2 - 28x + y^2 - 14y + 241 = 0$. (5)
 Determine the coordinates of the centre and the radius.
 $x^2 - 28x + 196 + y^2 - 14y + 49 = -241 + 196 + 49$ 2A
 $(x-14)^2 + (y-7)^2 = 4$ 1CA
 Centre: (14; 7) ; $r = 2$ 2CA
- (e) Prove that the two circles intersect in two places.. (5)
 Distance between centres: 1M
 $\sqrt{(14-5)^2 + (7-3)^2}$ 1CA
 $= 9,8$ 1CA
 $r + R = \sqrt{65} + 2$ 1CA
 $= 10,1$
 $r + R > \text{distance between centres}$ 1M
 $\therefore \text{circles intersect.}$

[27]

QUESTION 9

- (a) Determine the values of a and b ($a, b > 0$) if $a \sin x + b$ has a maximum value of 5 and a minimum value of -1 (4)

$$a(1) + b = 5 \quad \dots\dots(A) \quad 1A$$

$$a(-1) + b = -1 \quad \dots\dots(B) \quad 1A$$

$$(A) + (B) \quad 2b = 4$$

$$b = 2 \quad 1CA$$

$$a = 3 \quad 1CA$$

Answer only: 4/4

- (b) A and B are acute angles with $\cos(A+B) = 0,8$ $\cos(A-B) = -0,2$. Determine the value of $\tan A \tan B$, without the use of a calculator (5)

$$\cos A \cos B - \sin A \sin B = 0,8 \quad (A) \quad 1A$$

$$\cos A \cos B + \sin A \sin B = -0,2 \quad (B) \quad 1A$$

$$(A) + (B) \quad 2 \cos A \cos B = 0,6$$

$$\cos A \cos B = 0,3 \quad 1CA$$

$$(A) - (B) \quad -2 \sin A \sin B = 1$$

$$\sin A \sin B = -0,5 \quad 1CA$$

$$\tan A \tan B = -\frac{0,5}{0,3} = -\frac{5}{3} \quad 1CA$$

- (c) (1) Prove that $\frac{\sin 3A}{\sin A} = 4 \cos^2 A - 1$

$$\text{LHS} \quad \frac{\sin(A+2A)}{\sin A} \quad 1M$$

$$= \frac{\sin A \cdot \cos 2A + \cos A \cdot \sin 2A}{\sin A} \quad 1A \text{ compound}$$

$$= \frac{\sin A \cdot \cos 2A + 2 \sin A \cdot \cos^2 A}{\sin A} \quad 1CA \text{ double}$$

$$= \cos 2A + \cos^2 A \quad 1CA \text{ simplify}$$

$$= 2 \cos^2 A - 1 + 2 \cos^2 A \quad 1CA \text{ double}$$

$$= 4 \cos^2 A - 1$$

$$= \text{LHS} \quad (5)$$

- (2) Hence, or otherwise, determine the maximum and minimum value of $\frac{\sin 3A}{\sin A}$

$$\text{Max value: } 3$$

$$\text{Min value: } -1$$

$$2A$$

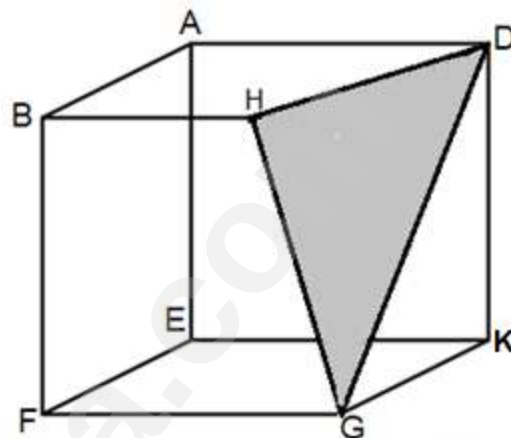
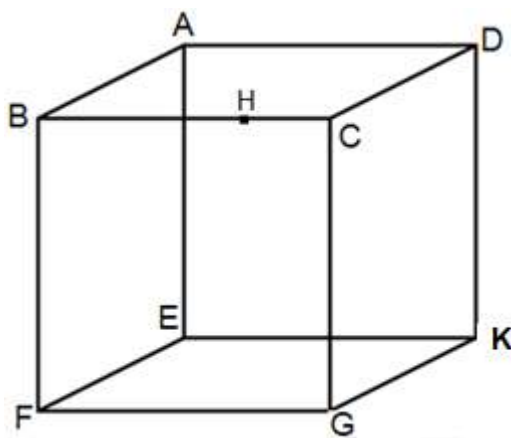
[16]

QUESTION 10

The diagram below represents a cube with vertices A, B, C, D, E, F, G and K. H divides the side BC in the ratio 3 : 1.

A corner section with base HDG is cut out of the cube.

Let $HC = x$.



Refer to the diagram and answer the following questions. Show all calculations:

- (a) Determine the lengths of HG, HD and DG in terms of x and hence prove that

$$\hat{D}HG = 86,63^\circ.$$

(8)

$$HG^2 = x^2 + (4x)^2$$

1A

$$HG = \sqrt{17}x$$

1CA

$$HD = \sqrt{17}x$$

1CA

$$DG^2 = \sqrt{(4x)^2 + (4x)^2}$$

1A

$$DG^2 = 4\sqrt{2}x$$

1CA

$$(4\sqrt{2}x)^2 = (\sqrt{17}x)^2 + (\sqrt{17}x)^2 - 2\sqrt{17}x(\sqrt{17}x)\cos\hat{D}HG$$

2CA

$$\cos\hat{D}HG = \frac{1}{17}$$

1A

- (b) If the area of $\triangle HDG = 76,34 \text{ units}^2$, calculate the side lengths of cube ABCDEFG. Round your answer off to the nearest unit.

(4)

$$\frac{1}{2}\sqrt{17}x(\sqrt{17}x)\sin 86,63^\circ = 76,34$$

1 M

1CA

$$x = \sqrt{\frac{2(76,34)}{17\sin 86,63}}$$

$$x = 3$$

1CA

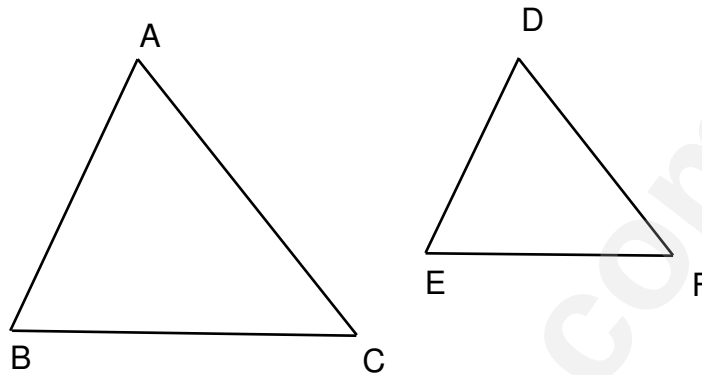
Side length: $s = 12 \text{ units}$

1CA

[12]

QUESTION 11

- (a) Using the given diagrams below, prove the theorem which states if two triangles are equiangular, then the corresponding sides are in proportion. (6)



Construction: On AB mark off AG = DE.
On AC mark off AH = DF.
Join GH

1M constr

Proof:

$\triangle AGH$ and $\triangle DEF$

(1) $AG = DE$ construction

1A

(2) $\hat{A} = \hat{D}$ given

(3) $AH = DF$ construction

$\therefore \triangle AGH \equiv \triangle DEF$ SAS

1A

$\therefore \hat{G}_1 = \hat{E}$

but $\hat{B} = \hat{E}$ given

$\therefore \hat{G}_1 = \hat{B}$

1A

$\therefore GH \parallel BC$ corresponding angles equal

$\therefore \frac{AB}{AG} = \frac{AC}{AH}$

1CA

$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ ($AG = DE, AH = DF$)

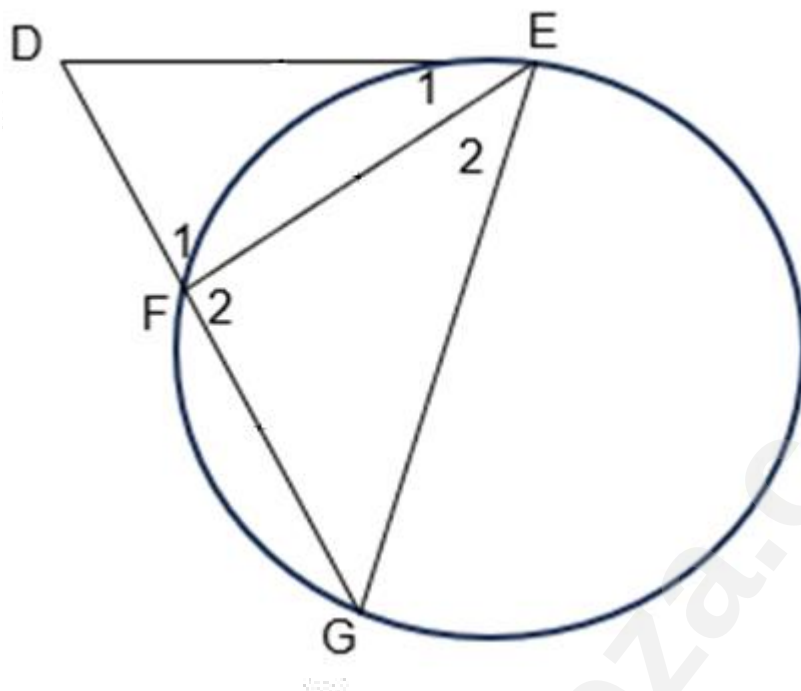
1A

Similarly it can be proven that $\frac{AB}{DE} = \frac{BC}{EF}$

$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

(b)

In the figure DE is a tangent and DFG is a secant.



(1) Prove that $DE^2 = DF \cdot DG$ (4)

In $\triangle DEF$ and $\triangle DGE$

$\hat{E}_1 = \hat{G}$ tan chord thm

$\hat{D}$ is common

$\hat{E}_1 + \hat{E}_2 = \hat{F}_1$ sum of angles of $\triangle$

$\therefore \triangle DEF \sim \triangle DGE$ AAA

$\frac{DE}{DG} = \frac{EF}{GE} = \frac{DF}{DE}$ equiangular $\triangle$ s

2A

1A with
reason
1A ratios

- (2) If it is further given that $DE = EF = FG$ and if $\frac{DF}{DE} = x$, prove that (5)

$$1 = x^2 + x$$

$$\frac{DE}{DG} = \frac{EF}{GE} = \frac{DF}{DE} = x$$

1M

$$\frac{DE}{DF + FG} = x$$

1A

$$\frac{DF + FG}{DE} = \frac{1}{x}$$

1A

$$\frac{DF}{DE} + \frac{FG}{DE} = \frac{1}{x}$$

1A

$$x + 1 = \frac{1}{x}$$

1A

$$x^2 + x = 1$$

- (3) Prove that $\cos D = \frac{1}{2}x$ (4)

$$FE^2 = DE^2 + DF^2 - 2DE \cdot DF \cos D$$

1M; 1A

$$FE^2 = FE^2 + DF^2 - 2FE \cdot DF \cos D$$

$$\cos D = \frac{DF^2}{2DF \cdot DE}$$

1CA

$$= \frac{1}{2}x$$

1CA

[19]