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Crawford International

CRAWFORD INTERNATIONAL PRELIMINARY EXAMINATION
AUGUST 2024
MATHEMATICS: PAPER I

Time: 3 hours **150 marks**

NAME OF CANDIDATE:	
NAME OF EDUCATOR:	

MARKER TO ENTER MARKS:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
16	16	7	12	17	9	7	6	13	9	13	8	7	10	150

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INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages. Please check that your question paper is complete.

2. Read all the questions carefully. **Answer ALL the questions on the question paper and hand it in at the end of the session.**

3. Five blank pages (pages 24 to 28) are included at the end of the paper. If you run out of space for a question, use these pages. **Clearly indicate the question number of your answer should you use this extra space.**

4. Diagrams are not necessarily drawn to scale.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**

7. It is in your own interest to write legibly and present your work neatly.

8. Round off to **one decimal place** unless otherwise stated.

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SECTION A (77 MARKS)

QUESTION 1

(a) Solve for x:

(1) $(x^2 - 4)(2x - 1) = 0$

(3)

(2) $4x^2 + 1 \geq 5x$

(4)

(3) $\log_2(x - 3) + \log_2(x - 2) = 1$

(4)

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(b) Given: $2^x + 2^{x+2} = -5y + 60$

(1) Express 2^x in terms of y.

(2)

(2) How many solutions for x will the equation have if $y = -4$?

(3)

[16]

QUESTION 2

(a) Given: $f(x) = x - 3x^2$. Determine $f'(x)$ from first principles.

(5)

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(b) Determine $\frac{d}{dx}\left[4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3\right]$.

(4)

(c) The equation of a tangent to the curve of $f(x) = ax^3 + bx$ is $y - x - 4 = 0$.
If the point of contact is $(-1; 3)$, calculate the values of a and b.

(7)

[16]

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QUESTION 3

A stone is thrown vertically upwards and its height h (in metres) above the ground at time t (in seconds) is given by

$$h(t) = 35 - 5t^2 + 30t.$$

(a) Find the stone's initial height above the ground.

(1)

(b) Find the initial speed with which the stone was thrown.

(2)

(c) How fast was the stone travelling when it reached a height of 60m above the ground on the way down?

(4)

[7]

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QUESTION 4

(a) Given: 0; −1; 1; 6; 14; …

(1) Show that the above sequence has a constant second difference.

(2)

(2) Determine a simplified expression for the nth term of the sequence.

(4)

(b) The eighth term of a geometric sequence is 640. The third term is 20. Determine the sum of the first seven terms.

(6)
[12]

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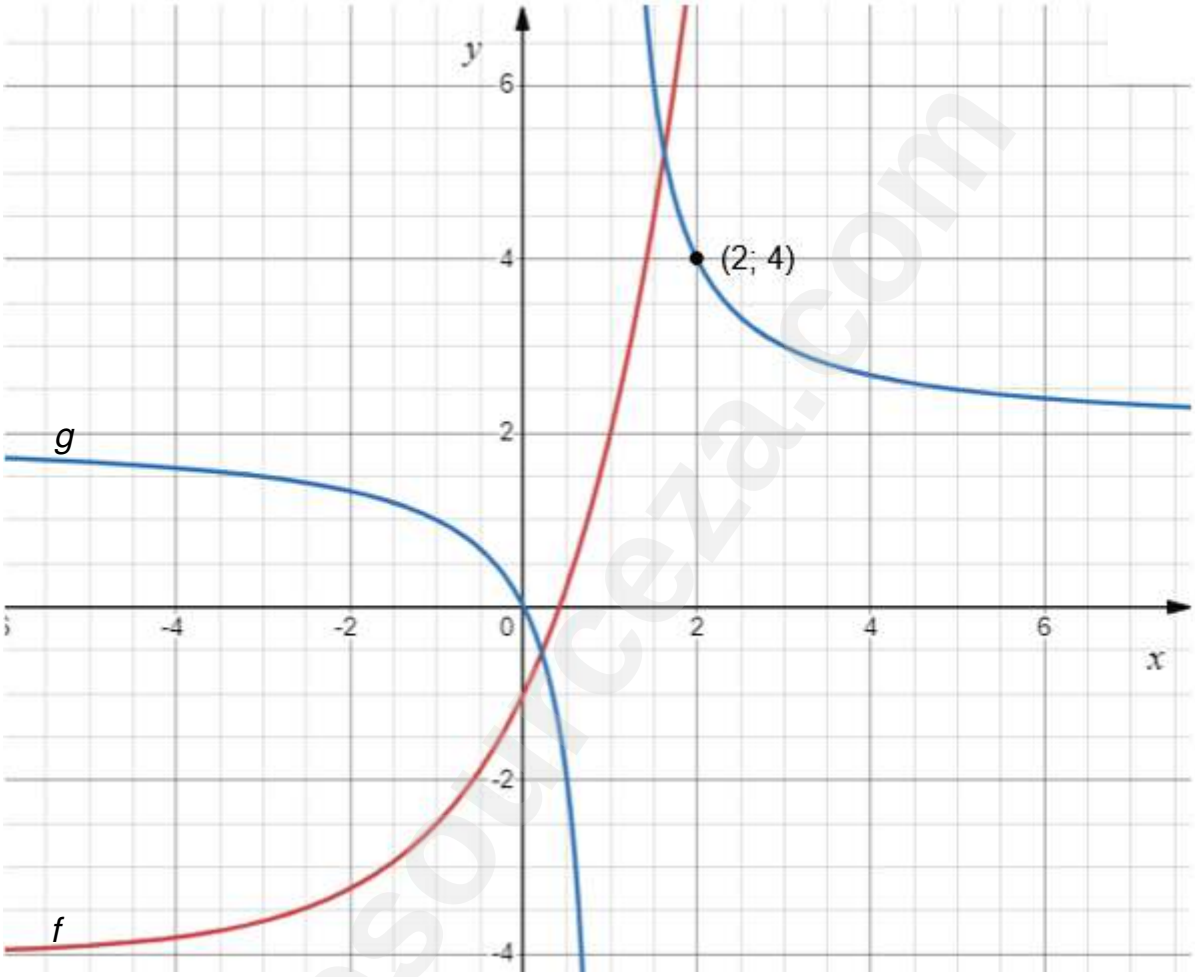
QUESTION 5

(a) On the diagram below, the graphs of $f(x) = 3.2^x - 4$ and $g(x) = \frac{2}{x-t} + 2$ are shown. The point (2; 4) lies on the graph of g.



(1) Determine the equation of $f^{-1}(x)$, the inverse of $f(x)$.

(3)

(2) Write down the domain and range of $f(x+4) - 2$.

(2)

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(3) Show that $t = 1$.

(2)

(4) Write down the coordinates of the point of intersection of the axes of symmetry of g.

(1)

(5) Given a straight line $y = mx + c$ such that

- it is parallel to the axis of symmetry of g that has a negative gradient
- it passes through the x-intercept of f.

Determine the equation of the straight line.

(4)

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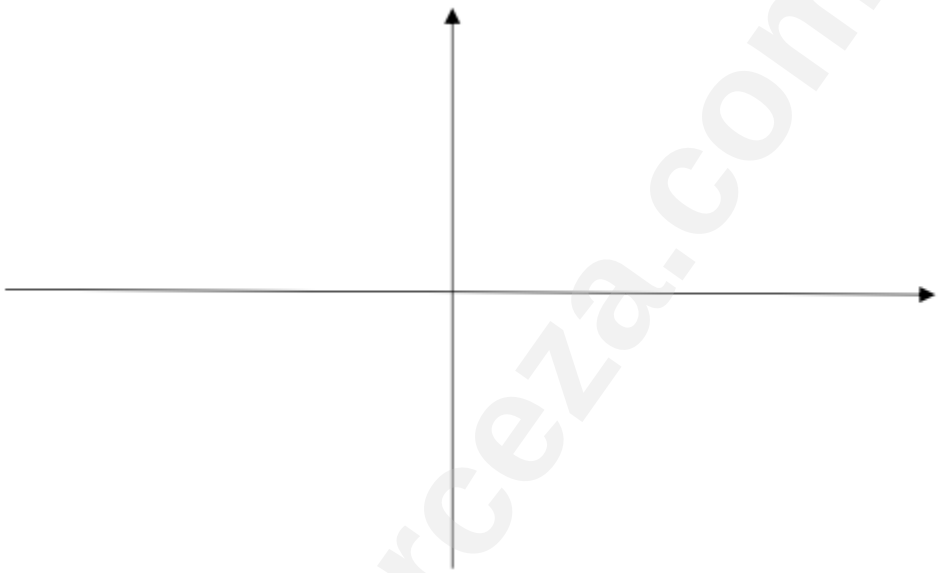
(b) The function $f(x) = ax^2 + bx + c$ has the following properties:

- The range of f is $(-\infty ;4]$
- $b = 2a$
- The distance between the x-intercepts is 2 units.

Draw a neat sketch of $f(x)$ on the set of axes below, showing ALL relevant information.



(5)
[17]

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QUESTION 6

(a) Tebogo invests R10 000 into a savings account. Three and a half years later, she invests a further R5 000, and four years after her R5 000 deposit, she withdraws R3 000.

If the interest rate for the first five years is 8,2% per annum compounded quarterly and then changes to 7,8% per annum compounded monthly, calculate the amount in the savings account 10 years after her first investment of R10 000.

(6)

(b) Ms Venter takes out a loan of R480 000 and starts paying off the loan two months later. Interest is charged at 14,25% per annum compounded monthly.

Calculate the amount she needs to pay monthly to pay off the loan in 8 years.

(3)
[9]

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SECTION B (73 MARKS)

QUESTION 7

Mr Gupta started a small business on 1 February 2023. On 1 February 2024 he invested R 150 000 into a savings account called SUPASAVA that earns him a fixed interest rate of 7,35% per annum compounded monthly.

The initial cost of machinery to run his business was R 2 380 000 and it depreciates at a rate of 18,5% per annum on a reducing balance basis. The cost of the machinery is expected to increase at an average rate of 9,7% per annum.

Mr Gupta also established a sinking fund to allow him to replace the machinery on 1 February 2030. He made the first payment into the sinking fund on 1 August 2023 and made/will continue to make monthly payments into the fund at the beginning of each month. Interest on the sinking fund is 10,37% per annum compounded monthly. His intention is that the value of the sinking fund immediately after his final payment into the fund, together with the money he will get by selling his old equipment at the depreciated value on 1 February 2030 and the money in his SUPASAVA account on 1 February 2030, will cover the cost of the new equipment fully.

Calculate Mr Gupta's required monthly contribution to the sinking fund.

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[7]

QUESTION 8

Given: $g(x) = ax^3 + bx - 2$.

$g(x)$ is exactly divisible by $(2x - 1)$, and has a remainder of 50 when it is divided by $(x + 2)$.

Determine the values of a and b.

[6]

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QUESTION 9

(a) Given: $\sum_{n=1}^{\infty}\left(k-\frac{3}{2}\right)^{n}$.

(1) For which value(s) of k does the series converge?

(3)

(2) If the series does converge, prove that

$$\sum_{n=1}^{\infty}\left(k-\frac{3}{2}\right)^{n}=\frac{2k-3}{5-2k}$$

(4)

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(b) Consider the following sum of a series:

$S_n = 1 + 13 + 106 + 1\,009 + 10\,012 + 100\,015 + \ldots$ to n terms.

Prove that $S_n = \dfrac{2.10^n + 27n^2 - 27n - 2}{18}$.

(6)

[13]

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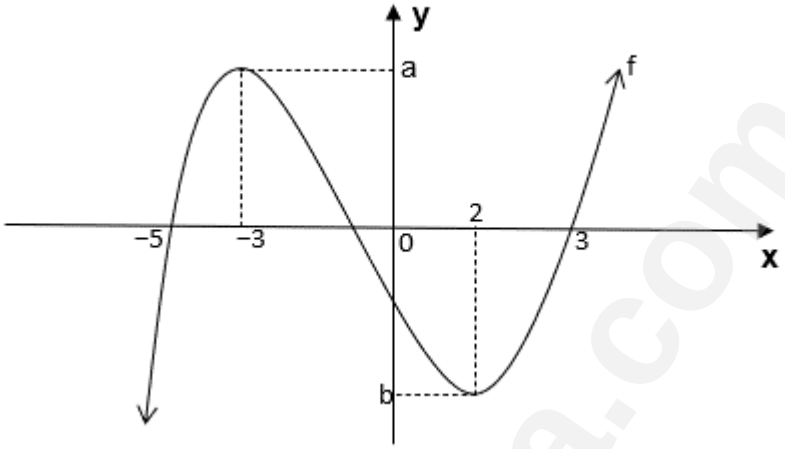
QUESTION 10

Part of the graph of the cubic polynomial f is shown below.



(a) For which values of x is $f'(x) = 0$?

(2)

(b) For which values of x is $f'(x) > 0$?

(2)

(c) **On the diagram above**, draw a sketch graph of the derivative $f'(x)$ indicating only the x-intercepts of $f'(x)$ on the graph.

(3)

(d) The equation $f(x) - k = 0$ has only one real root. What are the possible values of k?

(2)

[9]

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QUESTION 11

(a) Bongani goes to the fun fair and wins a game of darts. He goes to the booth to claim his prize from a range of sweets. There are twelve available: four that are caramel, four that are orange, and four that are toffee. He randomly chose two sweets. Determine the probability that the two sweets he chose were different flavours.

(5)

(b) How many numbers greater than 500 can be formed from the four digits 2, 3, 4 and 5 if repetition is **not** allowed.

(4)

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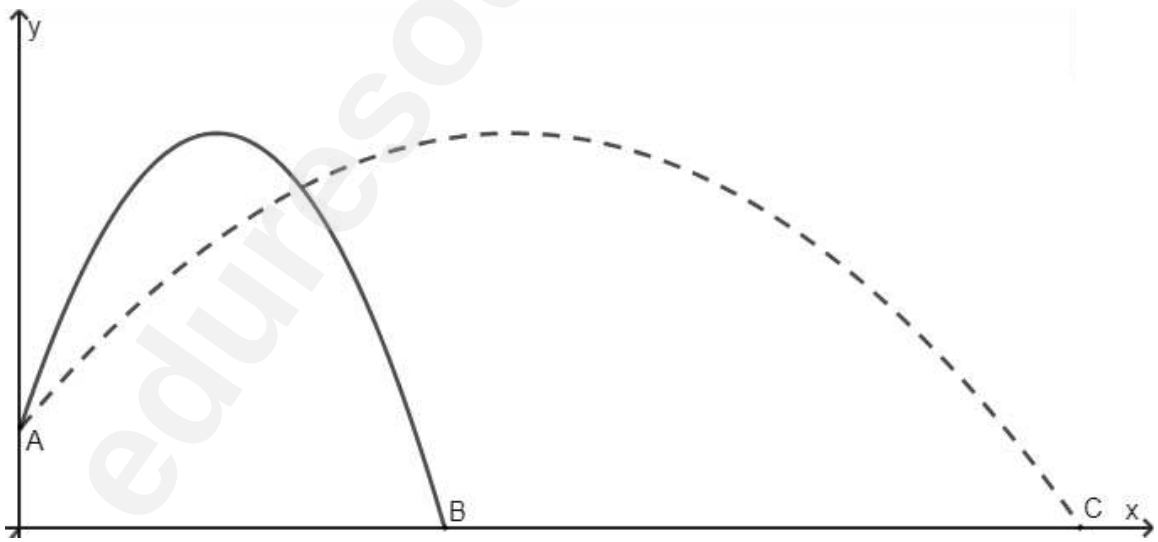
(c) Elizabeth randomly chooses three different numbers from 3 to 9. She asks her friend Ava to guess what numbers she has chosen. What is the probability that Ava guesses the correct numbers on her first try. The order of numbers is not important.

(4)
[13]

QUESTION 12

Sumaya and Diana launch two drones from a balcony of a building (A in the diagram below shows the balcony). The drones travel in the same direction, and both drones follow a parabolic path. Sumaya's drone lands at B and Diana's drone lands at C on a horizontal surface. A sketch (not drawn to scale) is given below to illustrate the situation.



Sumaya's drone reaches a maximum height of 120 m above the ground at a horizontal distance of 60 metres from the balcony.

The path of Diana's drone is modelled by $y = -\frac{1}{250}x^2 + \frac{6}{5}x + 30$.

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Calculate how far Sumaya's and Diana's drones are apart when they land on the ground. Give your answer correct to two decimal places.

[8]

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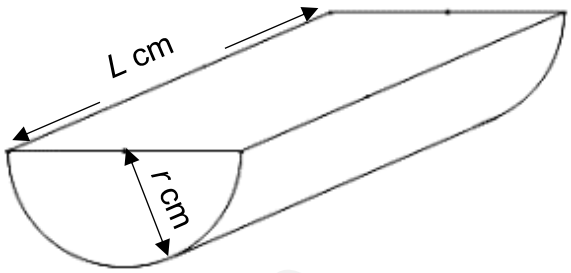
QUESTION 13

The figure alongside shows the design of an animal drinking trough in the shape of a hollow, open-topped half cylinder made of thin sheet metal. The radius of the semicircular sides is r cm and the length of the trough is L cm.



The metal used in the construction of the drinking trough is 600π cm^2.

Determine the radius of the drinking trough so that it has maximum volume.

[7]

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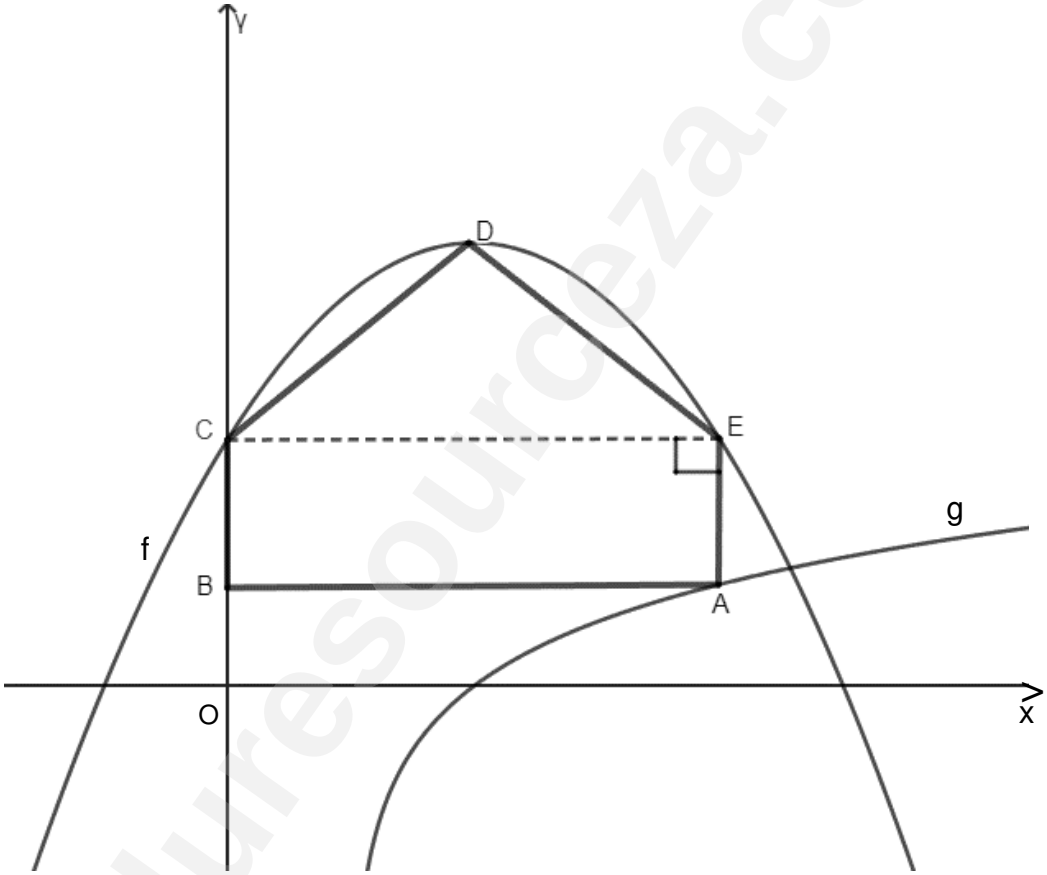
QUESTION 14

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and $g(x) = \log_{1,7}(x - 1)$ are shown.

- D is the turning point of f
- C and E lie on the graph of f, and C is the point at which f cuts the y-axis
- A lies on the graph of g
- $x_B = 0$ and BA and CE are parallel to the x-axis
- $C\hat{E}A = 90°$
- O is the origin



(a) Determine whether the y-intercept of $g^{-1}(x)$ lies on the line segment OB, the line segment BC, or on the y-axis above point C. You must show appropriate calculations and reasoning to support your answer.

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(7)

(b) Calculate the area of pentagon ABCDE.

(3)
[10]

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ADDITIONAL SPACE

REMEMBER TO INDICATE CLEARLY AT THE QUESTION THAT YOU HAVE USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

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