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CAMBRIDGE

ACADEMY

INTERNAL EXAMINATIONS

INDEPENDENT EXAMINATIONS BOARD

Name: _____

Examiner: Pretoria Cluster

Moderator: Mr. S. Nyabadza

2 September 2024

Time: 3 Hours

Total: 150 Marks

MATHEMATICS GRADE 12

Prelim Examination

Paper Two

Please read these instructions carefully:

1. This paper consists of 28 pages, 12 questions and an information sheet of 2 pages. Check that your paper is complete.
2. Answer ALL the questions on the question paper and hand it in at the end of the examination session.
3. Clearly show ALL calculations, diagrams, graphs, et cetera you have used in determining the answers.
4. Two blank pages (25-26) have been included at the end of the exam paper. If you run out of space for a question, use these pages.
5. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise. Ensure that your calculator is in **DEGREE** mode.
6. Round off to **ONE** decimal digits, unless otherwise stated.
7. Note that diagrams and graphs are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present your work neatly

Mark:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12		TOTAL
14	14	17	16	10	12	20	13	5	10	12	7		150

SECTION A**QUESTION 1****[14]**

Street vendors in the centre of Johannesburg sell cups of hot chocolate and packets of cooked chips during lunch hour every day. They kept records of the number of cups of hot chocolate and the number of packets of chips sold and the temperature at lunchtime on eight days.

Temperature ($^{\circ}\text{C}$)	12	13	14	16	18	22	24	26
Cups of Hot chocolate y_1	300	280	250	210	190	170	150	130
Packets of chips y_2	280	160	250	120	294	206	238	240

- a) The correlation coefficient of temperature and cups of hot chocolate sold is $-0,966$. Interpret this correlation coefficient in realistic terms. (2)
- b) The correlation coefficient of temperature and packets of chips sold is $0,104$. Interpret this correlation coefficient in realistic terms. (2)
- c) Determine the least squares regression lines for
- i) cups of hot chocolate and temperature (2)

ii) packets of chips and temperature. (2)

d) Hence, estimate the following:

i) the number of cups of hot chocolate sold when the temperature is 9°C . (1)

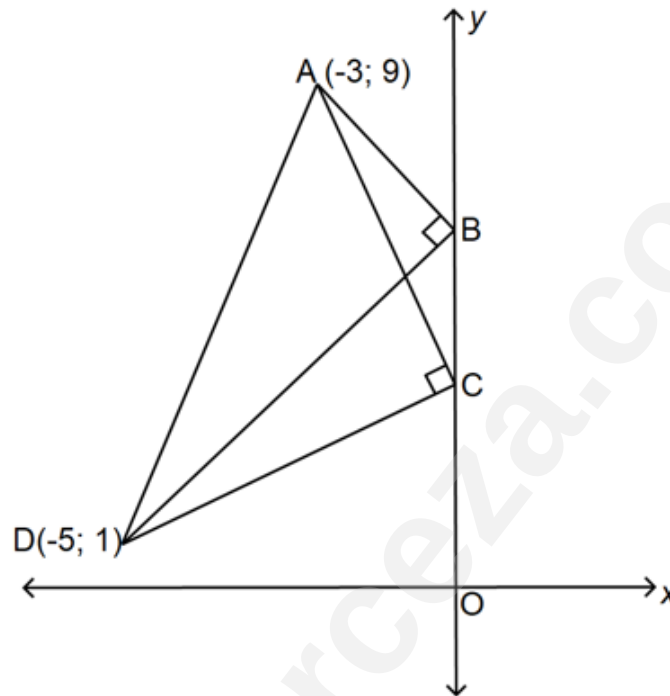
ii) the number of packets of chips sold when the temperature is 21°C . (1)

e) Comment on the reliability of the above predictions. (4)

QUESTION 2

[14]

In the diagram below, A (-3; 9) and D (-5; 1) are points on $\triangle ABD$ and $\triangle ACD$.
 B and C are points on the y-axis such that $\angle ABD = \angle ACD = 90^\circ$.



- a) Calculate the coordinates of M, the midpoint of AD. (2)

- b) Explain why AD would be the diameter of a circle passing through A, B and D. (1)

- c) Calculate the length of the radius of the circle passing through A, B and D. (2)

- d) Will point C lie on circle ABD? Give a reason for your answer. (1)
- e) Show that the coordinates of B are (0; 6). (4)
- f) Determine the equation of the straight-line BD. (2)
- g) Calculate the size of $\hat{OBD}$, correct to the nearest degree. (2)

QUESTION 3

[17]

a) Prove the identity: $\cos^2(180^\circ + x) + \tan(x - 180^\circ) \cdot \sin(720^\circ - x) \cdot \cos x = \cos 2x$ (5)

b) If $x = 3 \sin \theta$ and $y = 3 \cos \theta$, determine the value of $x^2 + y^2$. (2)

c) Without the use of a calculator determine:

i) $2 \sin 15^\circ \sin 75^\circ$ (3)

ii) $\cos 2A$ if $\cos A = -\frac{12}{13}$ and $A \in [0^\circ; 180^\circ]$ (3)

d) If $\sin 76^\circ = a$, and $\cos 76^\circ = b$, show that: $a^2 - b^2 = \sin 62^\circ$ (4)

QUESTION 4

[16]

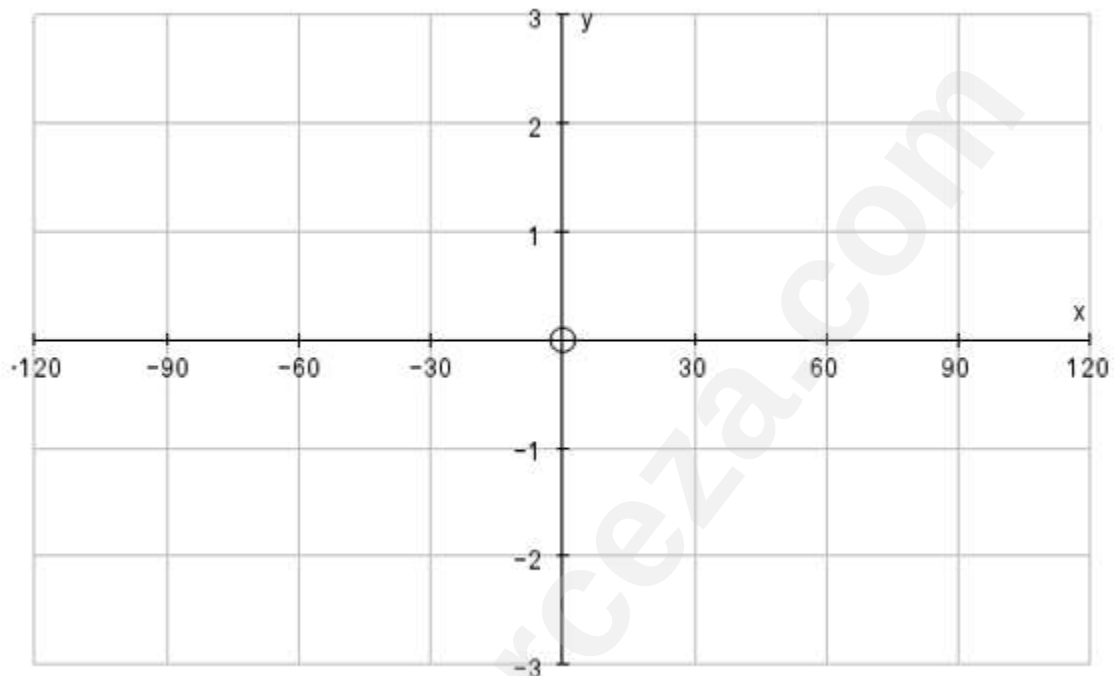
- * Circle A has the centre (2; 5).
 - * Circle B is given as: $x^2 + y^2 + 8x - 2y = -1$
 - * Circle A and Circle B have the same length radius.
 - * Tangent TP intersects **Circle B** where Circle B touches the positive y – axis, at point T.
- a) Determine the equation of Circle B in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)
- b) Write down the equation of Circle A. (2)
- c) Draw a rough sketch of the circles given. (2)

- d) i) Determine the coordinates of T. (2)
- ii) Hence, determine the equation of tangent TP. (2)
- e) Show by calculation, whether Circle A and Circle B intersect each other? (5)

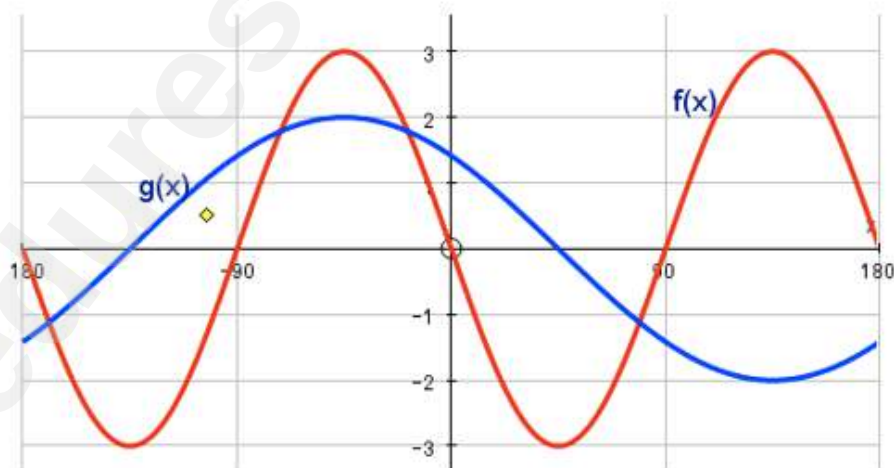
QUESTION 5

[10]

- a) On the axes below, sketch the graph of $f(x) = \sin(3x - 60^\circ)$ if $x \in [-120^\circ; 120^\circ]$.
Label all intercepts with the axes and the co-ordinates of the turning points. (4)



- b) Sketched below are the graphs $f(x) = a \sin bx$ and $g(x) = 2 \cos(x + c)$



- i) Find the values of a , b and c . (3)

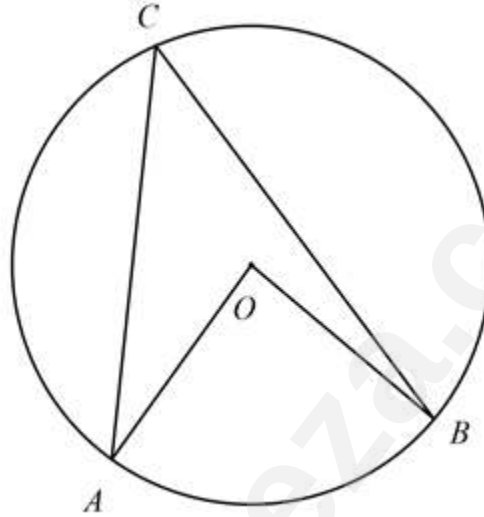
ii) For which values of x is $f(x) - g(x) = 1$ (1)

iii) Write down the new equation of $g(x)$ if the y axis is moved 45° to the right, in simplified form? (2)

TOTAL SECTION A: 71 MARKS

SECTION B**QUESTION 6****[12]**

- a) Prove the theorem which states: "The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle."



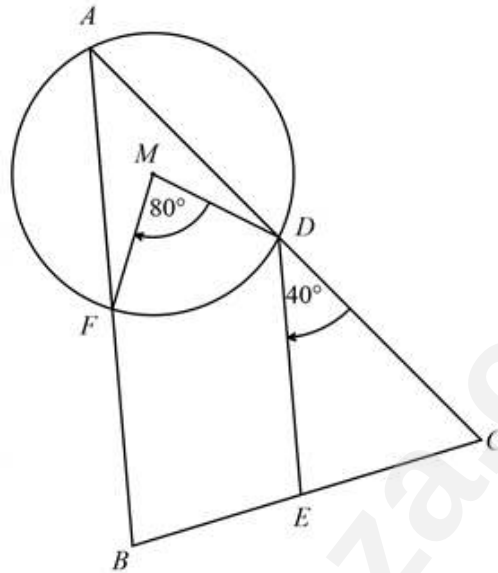
Required to prove: $\angle AOB = 2\angle ACB$

(6)

Construction:

Proof:

- b) In the given figure, M is the centre of circle AFD. Chords AF and AD are produced to B and C respectively and BC is joined. $\widehat{EDC} = 40^\circ$ and $\widehat{FMD} = 80^\circ$.



- i) Prove that $AB \parallel DE$ (3)

- ii) If $\frac{BE}{EC} = \frac{7}{8}$ and $AC = 22,5\text{cm}$, calculate the length of AD. (3)

QUESTION 7

[20]

- a) If $\cos 55^\circ = A$ and $\sin(90^\circ - k) \cdot \sin 35^\circ - \sin 215^\circ \cdot \cos 35^\circ = B$,
find the values of k if $A = B$ and $0^\circ \leq k \leq 90^\circ$.

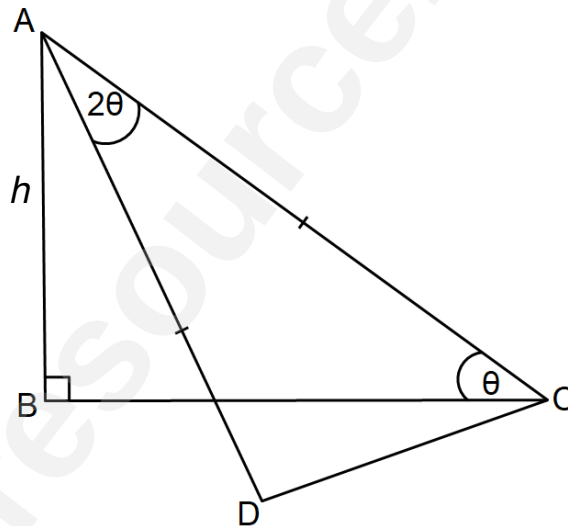
(5)

- b) i) Prove that $4 \sin \alpha \cdot \cos^3 \alpha - 4 \cos \alpha \cdot \sin^3 \alpha = \sin 4 \alpha$

(4)

- ii) Hence, solve for α , if $1 + 4 \sin \alpha \cdot \cos^3 \alpha = 4 \cos \alpha \cdot \sin^3 \alpha$ if $\alpha \in [-90^\circ; 90^\circ]$
Give answer correct to one decimal place. (5)

- c) In the diagram below, AB is a pole anchored by two cables at C and D. B, C and D are in the same horizontal plane. The height of the pole is h and the angle of elevation from C to the top of the pole, A, is θ . $\hat{CAD} = 2\theta$ and $AC = AD$.

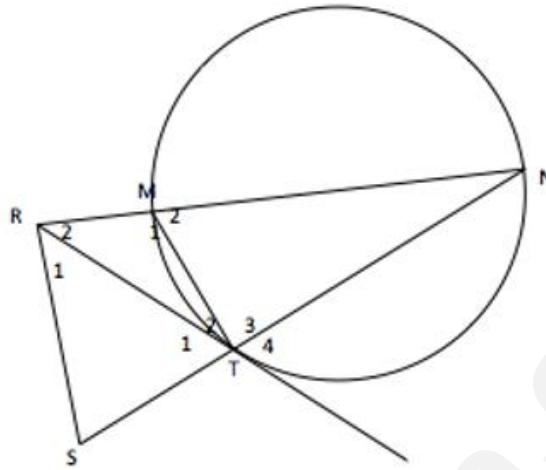


Determine CD, the distance between the two anchors, in terms of h . (6)

QUESTION 8

[13]

RT is a tangent to the circle at T. Lines MN and NT are produced to R and S such that $RT = RS$.



Let $\widehat{T_4} = x$.

a) Prove that RMTS is a cyclic quadrilateral. (5)

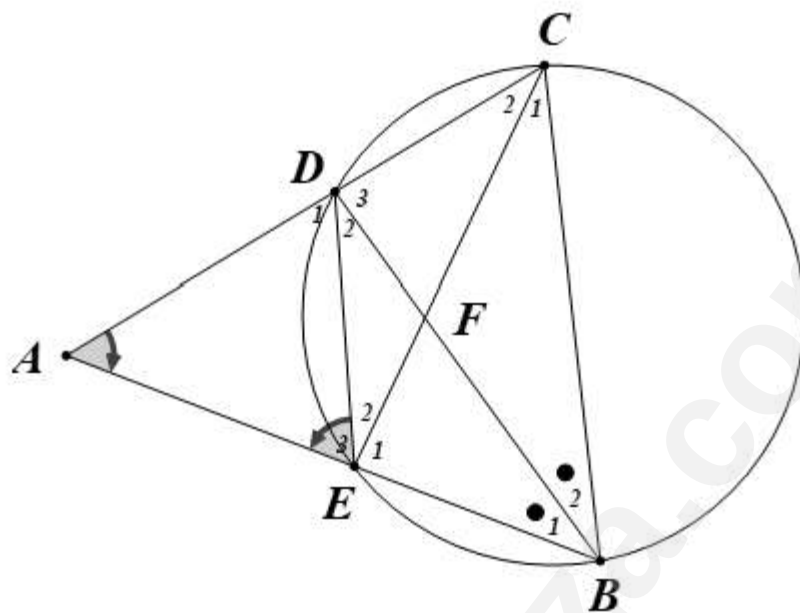
b) Prove that $\triangle RTM \sim \triangle RNT$ (5)

- c) Hence, find the length of RT if $RM = 2$ units and $MN = 6$ units (3)

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QUESTION 9

[5]



In the diagram, BC is the diameter of the circle BCDE. BD bisects $\angle ABC$, and $\angle BAC = \angle BEC$

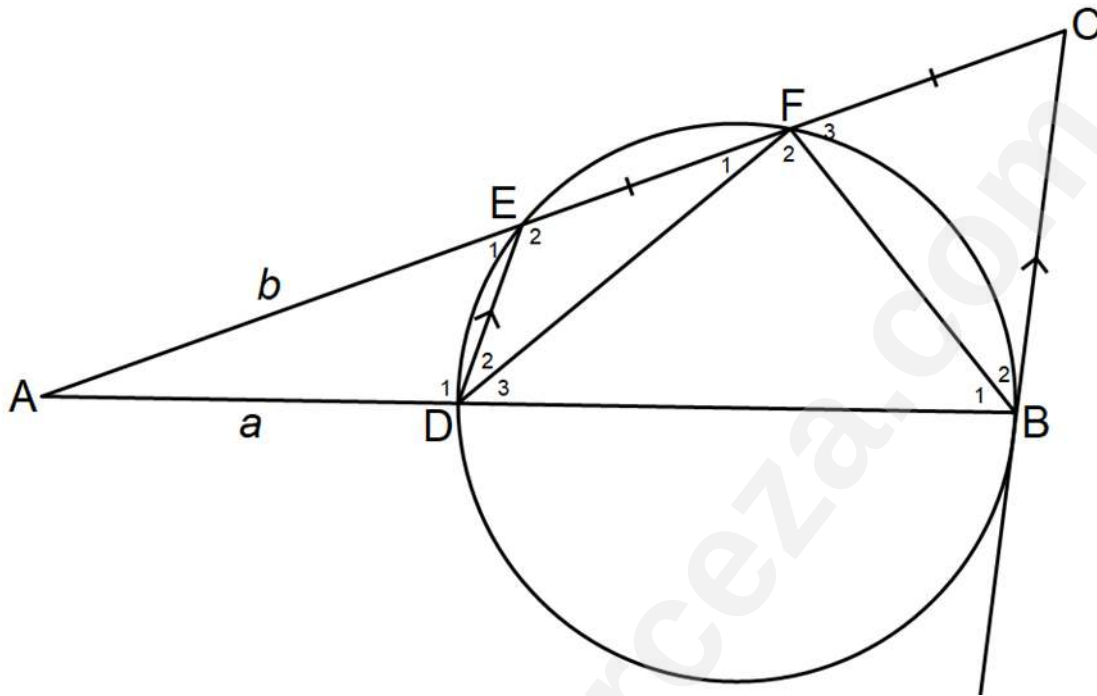
Prove $AD^2 = DF \cdot DB$

(5)

QUESTION 10

[10]

In the figure D is a point on AB and E a point on AC of $\triangle ABC$. $DE \parallel BC$. $AD = a$, $AE = b$ and $AD : DB = 1 : 2$. F lies on AC and on the circle DBE and F is the midpoint of EC.



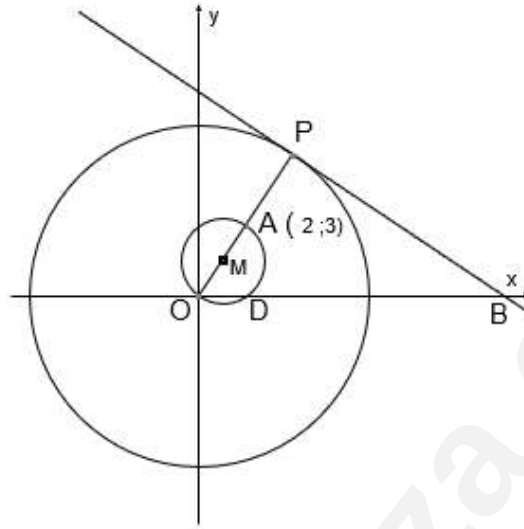
- a) Prove that $\frac{BC}{BF} = \frac{3a}{2b}$ (6)

- b) If it is further given that $\triangle DBF \parallel \triangle BCF$ deduce that $BC = \sqrt{3} a$. (4)

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QUESTION 11**[12]**

A (2 ; 3), the midpoint of radius OP, lies on the circumference of the smaller circle with diameter OA.



Write down:

a) The equation of the larger circle. (3)

b) The equation of the smaller circle. (3)

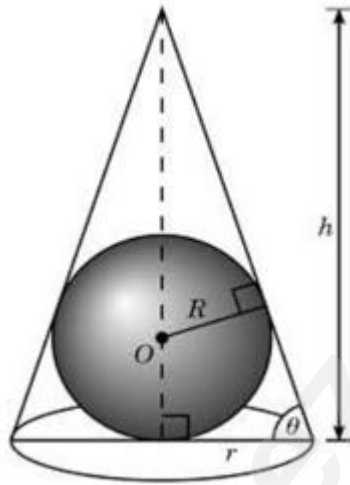
- c) Determine the equation of the tangent to the large circle at P. (4)

- d) Hence, determine the equation of a tangent to the large circle which is parallel to the tangent in (c). (2)

QUESTION 12

[7]

- a) The diagram below shows a sphere placed inside a cone which has a radius (r) of 7 cm. The total surface area of the sphere is $100\pi \text{ cm}^2$ and the height (h) of the cone is three and a half times the length of the radius of the sphere (R).



Use the following formulae

$$SA = \pi r^2 + \pi rs \qquad V = \frac{1}{3} \pi r^2 h$$

$$SA = 4\pi R^2 \qquad V = \frac{4}{3} \pi R^3$$

Calculate the volume of the empty space in the cone, to one decimal.

(5)

- b) Nandi has two spheres in her office, a small marble and a large bowling ball. If the bowling ball has a volume which is 2744 times the volume of the marble, how many times larger is the radius of the bowling ball than the radius of the marble? Show all working. (2)

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TOTAL SECTION B: 79 MARKS

TOTAL: 150 MARKS

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE, TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^n 1 = n$$

$$T_n = a + (n-1)d$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni)$$

$$A = P(1 + i)^n$$

$$F = x \left[\frac{(1+i)^n - 1}{i} \right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_\infty = \frac{a}{1-r}; -1 < r < 1$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$P = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$M \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$y - y_1 = m(x - x_1)$$

$$m = \tan \theta$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$