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INTERNAL EXAMINATIONS

INDEPENDENT EXAMINATIONS BOARD

Name: MEMORANDUM

Examiner: S Stevens & S Nyabadza
Moderators: V Peche & R Kluge

26 August 2024
Time: 3 Hours
Total: 150 Marks

MATHEMATICS GRADE 12

Prelim Examination

Paper One

Please read these instructions carefully:

1. This paper consists of 25 pages, 10 questions and an information sheet of 2 pages. Check that your paper is complete.
2. Answer ALL the questions on the question paper and hand it in at the end of the examination session.
3. Clearly show ALL calculations, diagrams, graphs, et cetera you have used in determining the answers. **Answers only will NOT necessarily be awarded full marks.**
4. Two blank pages (22 - 23) have been included at the end of the exam paper. If you run out of space for a question, use these pages.
5. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise. Ensure that your calculator is in **DEGREE** mode.
6. Round off to **ONE** decimal digits, unless otherwise stated.
7. Note that diagrams and graphs are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present your work neatly.

Mark:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10		TOTAL
11	18	23	19	14	13	14	14	9	15		150

SECTION A

QUESTION 1

[11]

Solve for x leave your answer correct to one decimal digit where necessary.

(a) $(2x + 3)(x + 1) = 3$

(3)

$$\begin{aligned}
 2x^2 + 5x + 3 - 3 &= 0 \checkmark \\
 2x^2 + 5x &= 0 \\
 x(2x + 5) &= 0 \\
 x = 0 \quad | \quad x = -5/2 \checkmark
 \end{aligned}$$

(3)

(b) $\sqrt{7 - 2x} + 1 = 2x$

(4)

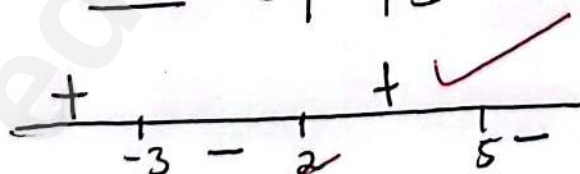
$$\begin{aligned}
 (\sqrt{7 - 2x})^2 &= (2x - 1)^2 \checkmark \\
 7 - 2x &= 4x^2 - 4x + 1 \\
 4x^2 - 2x - 6 &= 0 \\
 2x^2 - x - 3 &= 0 \checkmark \\
 (2x - 3)(x + 1) &= 0 \\
 x = 3/2 \quad | \quad x = -1 \checkmark
 \end{aligned}$$

(4)

(c) $(5 - x)(x^2 + x - 6) \geq 0$

(4)

$$\begin{aligned}
 (5 - x)(x + 3)(x - 2) &\geq 0 \checkmark \\
 \text{cvs} \quad -3 \quad | \quad 2 \quad | \quad 5
 \end{aligned}$$



$$x \leq -3 \quad | \quad \text{or} \quad 2 \leq x \leq 5$$

(4)

QUESTION 2

[18]

a) Given: $\sum_{n=1}^k (5x-3)^n$

1) If $k = \infty$, for which value(s) of x will the given sum exist? (3)

$-1 < r < 1$
 $-1 < 5x-3 < 1$
 $2 < 5x < 4$
 $\frac{2}{5} < x < \frac{4}{5}$

$5x-3, (5x-3)^2, (5x-3)^3$
 $r = 5x-3$

$\frac{2}{5} < x < \frac{4}{5}$

3

2) If $k = 10$ and $x = 1$, evaluate the given expression. (4)

$\sum_{n=1}^{10} (5(1)-3)^n, (5(1)-3)^2, (5(1)-3)^3$
 $2, 4, 8$
 $r = 2$

$S_{10} = \frac{2(2^{10}-1)}{2-1} = 2046$

4

b) Given the quadratic sequence: 34; 42; 48; 52; 54; ...

1) Determine an expression for the general term T_n , of the quadratic sequence. (4)

34 42 48 52 54
 8 6 4 2
 -2 -2 -2

$2a = -2$
 $a = -1$

$3(-1) + b = 8$
 $b = 11$

$-1 + 11 + c = 34$
 $c = 24$

$T_n = -n^2 + 11n + 24$

4

- 2) Jack calculates the 20th term to be -155. Without doing any calculations, explain why this answer has to be incorrect. (1)

The sequence is increasing therefore it can not be negative. (1)

- c) The 5th term of an arithmetic sequence has a value of -13 and the 13th term has a value of -45. Determine the value of the 18th term in this sequence. (6)

$$a + 4d = -13 \checkmark$$

$$a = -13 - 4d$$

$$a + 12d = -45 \checkmark$$

$$-13 - 4d + 12d = -45 \checkmark$$

$$\frac{8d}{8} = \frac{-32}{8}$$

$$d = -4 \checkmark$$

$$a = -13 - 4(-4)$$

$$= -13 + 16$$

$$= 3 \checkmark$$

$$T_{18} = a + 17d$$

$$= 3 + 17(-4)$$

$$= -65 \checkmark$$

QUESTION 3

[23]

a) Given: $f(x) = \frac{-3}{x+2} + 3$

- 1) Sketch the graph of f showing the asymptotes and the intercepts with the axes

$$f(0) = \frac{-3}{2} + 3$$

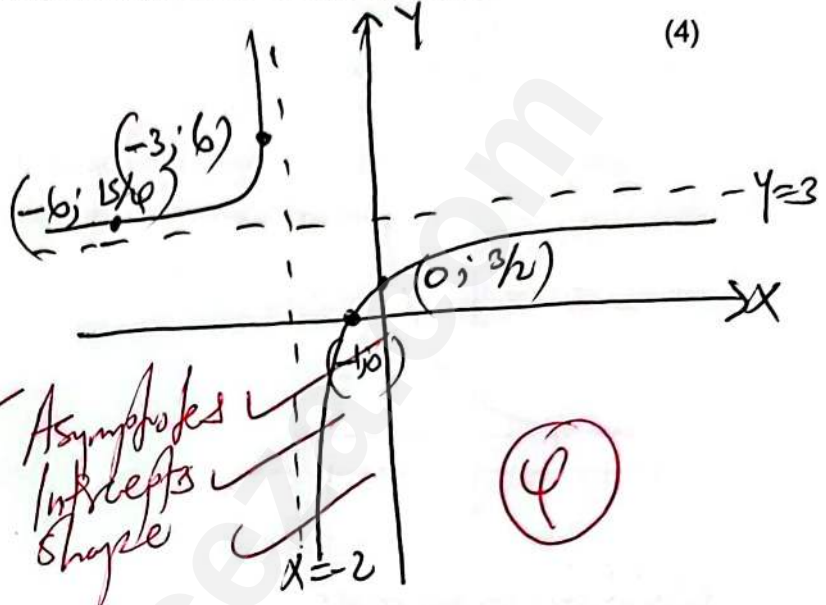
$$= \frac{3}{2}$$

$$0 = \frac{-3}{x+2} + 3$$

$$-3x - 6 = -3$$

$$-3x = 3$$

$$x = -1$$



- 2) Determine the equation of the positive axis of symmetry of f .

$$y = x + 2 + 3$$

$$y = x + 5$$

$$y = x + c$$

$$3 = -2 + c$$

$$5 = c$$

$$y = x + 5$$

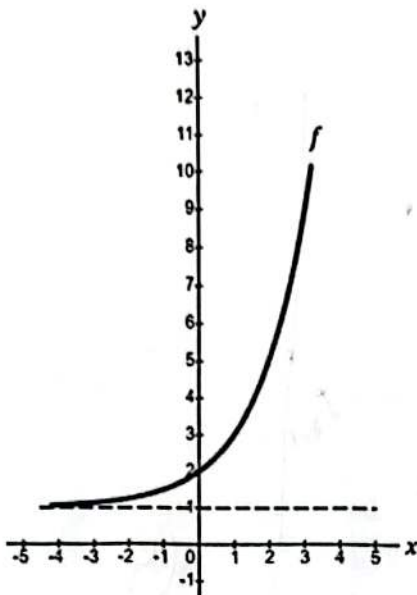
- 3) Write down the equations of the asymptotes of f^{-1} .

$$x = 3$$

$$y = -2$$

(2)

- b) On the Cartesian Plane below the graph of $f(x) = 2^x + 1$ is drawn to scale. Study the function and answer the questions that follow.



- 1) Write down the domain of f

$x \in \mathbb{R};$ or $x \in (-\infty; \infty)$ ✓

①

(1)

- 2) Write down the range of f

$y > 1;$ or $y \in (1; \infty)$ ✓

①

(1)

- 3) Find the equation of $f^{-1}(x)$ in the form $y = \dots$

(3)

$y = 2^x + 1$
 $x = 2^y + 1$ ✓
 $x - 1 = 2^y$ ✓
 $\log(x - 1) = y \log 2$ ✓
 $y = \frac{\log(x - 1)}{\log 2}$ ✓
 $y = \log_2(x - 1)$ ✓

③

- 4) $f^{-1}(x)$ is translated to become a new function $h(x) = f^{-1}(x - p)$.

Determine the value(s) of p so that h will have a y -intercept.

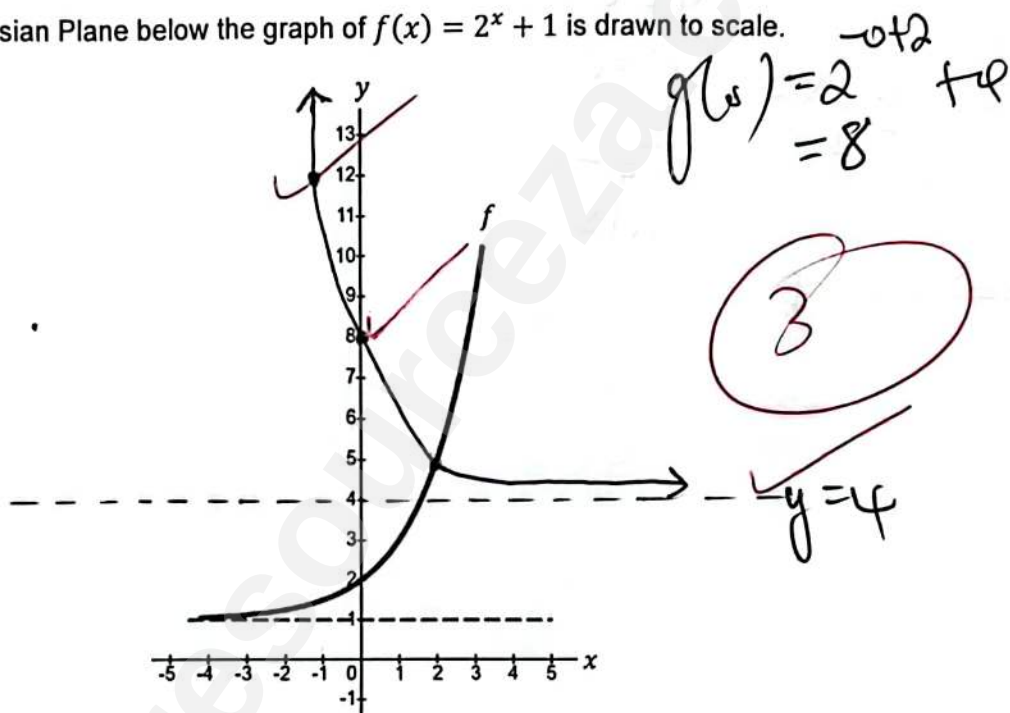
(2)

$$h(x) = \log_2(x - 1 - p)$$

$p < -1$

Note: This is the same diagram as on page 6

On the Cartesian Plane below the graph of $f(x) = 2^x + 1$ is drawn to scale.



- 5) ON THE CARTESIAN PLANE ABOVE: Sketch the graph of $g(x) = 2^{-x+2} + 4$.

Clearly show all intercept(s) with the axis/es and/or asymptote(s), where applicable

(3)

6) Show all calculations in solving algebraically for x where $f(x) = g(x)$.

(5)

$$2^x + 1 = 2^{-x+2} + 4 \checkmark$$

$$2^x = 2^{-x} \cdot 2^2 + 3$$

$$2^x = \frac{4}{2^x} + 3$$

$$2^{2x} = 4 + 3 \cdot 2^x \checkmark$$

$$2^{2x} - 3 \cdot 2^x - 4 = 0 \checkmark$$

$$(2^x - 4)(2^x + 1) = 0 \checkmark$$

$$2^x = 2^2 \quad \text{or} \quad 2^x = -1$$

$$\underline{\underline{x = 2}} \rightarrow$$

n/s

5

QUESTION 4

[19]

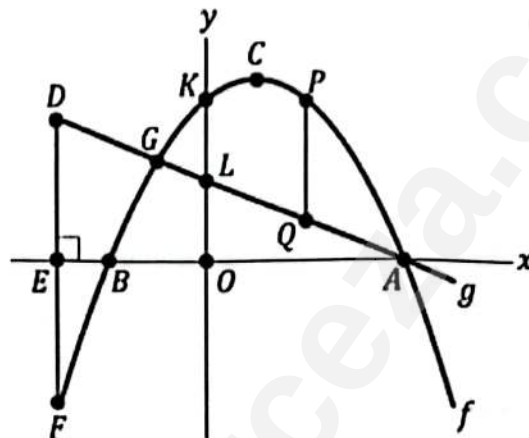
In the diagram below: $f(x) = -x^2 + 2x + 8$ and $g(x) = -x + 4$.

O is the origin. K and L are y -intercepts of f and g respectively.

A is a shared x -intercept of f and g . G is a second point of intersection.

B is the other x -intercept of f . C is the turning point of f .

Line DEF is perpendicular to the x -axis. PQ is a vertical line that can move between the functions where $f(x) \geq g(x)$.



- a) Determine the length of KL .

(2)

$$K(0, 8) \quad L(0, 4)$$

$$KL = 8 - 4$$

$$= 4 \text{ units}$$

2

- b) If $OE = 3$ units, determine the length of DF .

(4)

$$f(-3) = -(-3)^2 + 2(-3) + 8$$

$$= -7$$

$$g(-3) = -(-3) + 4$$

$$= 7$$

$$DF = 14 \text{ units}$$

$$\text{OR}$$

$$DF = g(x) - f(x)$$

$$= -x + 4 - (-x^2 + 2x + 8)$$

$$= x^2 - 3x - 4$$

$$= (-3)^2 - 3(-3) - 4$$

$$= 14 \text{ units}$$

4

- c) If $OB = 2$ units and $OA = 4$ units, use this information to find the co-ordinates of C . (4)

$$x = \frac{-2+4}{2} \checkmark$$

$$f(1) = -1^2 + 2(1) + 8 \checkmark$$

$$= 9 \checkmark$$

$$\therefore C(1, 9) \rightarrow \textcircled{4}$$

- d) For which value(s) of x will $f(x) \leq g(x)$? (5)

$$-x^2 + 2x + 8 = -x + 4 \checkmark$$

$$0 = x^2 - 3x - 4 \checkmark$$

$$(x-4)(x+1) = 0 \checkmark$$

$$x = -1 \text{ or } x = 4 \checkmark$$

$$x \leq -1 \text{ or } x \geq 4 \rightarrow \textcircled{5}$$

- e) Determine the maximum distance of PQ .

$$PQ = f(x) - g(x)$$

$$= -x^2 + 2x + 8 - (-x + 4)$$

$$= -x^2 + 3x + 4 \checkmark$$

$$= -1(x^2 - 3x - 4)$$

$$= -\left[(x - \frac{3}{2})^2 - 4 - (\frac{9}{4})\right]$$

$$= -\left[(x - \frac{3}{2})^2 - \frac{25}{4}\right]$$

$$= -(x - \frac{3}{2})^2 + \frac{25}{4} \checkmark$$

$$PQ = \frac{25}{4} \text{ units} \rightarrow \textcircled{4}$$

$$f'(x) - g'(x) = 0 \quad (4)$$

$$-2x + 2 - (-1) = 0 \checkmark$$

$$-2x + 3 = 0 \checkmark$$

$$x = \frac{3}{2} \checkmark$$

$$PQ_{\text{Max}} = f(\frac{3}{2}) - g(\frac{3}{2})$$

$$= -(\frac{3}{2})^2 + 2(\frac{3}{2}) + 8 - (-\frac{3}{2} + 4)$$

$$= \frac{25}{4} \quad | \quad 6.25 \text{ units} \checkmark$$

QUESTION 5

[14]

- a) If $f(x) = 2x^2 - 3x + 1$, determine $f'(x)$ USING FIRST PRINCIPLES.

(5)

$$\begin{aligned}
 f(x+h) &= 2(x+h)^2 - 3(x+h) + 1 \\
 &= 2(x^2 + 2xh + h^2) - 3x - 3h + 1 \\
 &= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1 \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - (2x^2 - 3x + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h \rightarrow 0} (4x + 2h - 3) = 4x - 3
 \end{aligned}$$

- b) Determine $f'(x)$ if $f(x) = \frac{x^4 + 3x^2 - 28}{x^2 - 4}$

(4)

$$\begin{aligned}
 f(x) &= \frac{(x^2 + 7)(x^2 - 4)}{(x^2 - 4)} \\
 f(x) &= x^2 + 7 \\
 f'(x) &= 2x
 \end{aligned}$$

$$4x - 3$$

- c) Find the equation of the tangent to the curve $f(x) = x^2 - 4x$ which is parallel to $y - 2x = 3$

(5)

$$\begin{aligned}
 f'(x) &= 2x - 4 \\
 2x - 4 &= 2 \\
 2x &= 6 \\
 x &= 3 \\
 f(3) &= (3)^2 - 4(3) \\
 &= 9 - 12 \\
 &= -3
 \end{aligned}$$

$$\begin{aligned}
 y &= 2x + 3 \\
 m &= 2 \\
 -3 &= 2(3) + c \\
 -9 &= c \\
 y &= 2x - 9
 \end{aligned}$$

SECTION A: 85 MARKS

Page | 11

SECTION B

QUESTION 6

[13]

A green energy company buys a Tesla Cybertruck as a marketing campaign for R2 000 000.

Due to the Cybertruck's components it depreciates at 30% p.a. on the reducing balance method.

Due to supply and demand, the inflation rate on a new Cybertruck is 27% per annum.

The company plans on setting up a sinking fund over 5 years (calculated at 15% p.a. compounded monthly) and will use the trade in value of the old Cybertruck towards purchasing a new one.

- a) Considering the scenario above, list the four processes the company needs to follow to set up a sinking fund. (4)

→ Depreciate asset ✓
 → Inflation on asset ✓
 → Subtract old asset value from new value ✓
 → Solve for monthly payments using Future Value ✓

- b) Apply the steps listed above and determine the monthly payment into the sinking fund (5)

$$\begin{aligned}
 A &= P(1 - i)^n \\
 &= 2\,000\,000(1 - 30\%)^5 \\
 &= R\,336\,140 \quad \checkmark \\
 A &= P(1 + i)^n \\
 &= 2\,000\,000(1 + 27\%)^5 \\
 &= R\,6\,607\,673,88 \quad \checkmark \\
 SF &= 6\,607\,673,88 - R\,336\,140 \\
 &= R\,6\,271\,533,88 \quad \checkmark \\
 6\,271\,533,88 &= x \left[\left(1 + \frac{15\%}{12}\right)^{60} - 1 \right] \\
 x &= R\,70\,805,18 \quad \checkmark
 \end{aligned}$$

- c) John retired at the end of 2022 and received his pension of R 1 400 000.
How many full months can he live off the fund if he withdraws R 13 500 every month? Interest is calculated at 10,7% p.a. compounded monthly. (4)

$$1\,400\,000 = \frac{13\,500 \left[1 - \left(1 + \frac{10,7\%}{12} \right)^{-n} \right]}{\frac{10,7\%}{12}}$$

$$\frac{37\,450}{3} = 13\,500 \left[1 - \left(1 + \frac{10,7\%}{12} \right)^{-n} \right]$$

$$\frac{749}{810} = 1 - \left(1 + \frac{10,7\%}{12} \right)^{-n}$$

$$\left(1 + \frac{10,7\%}{12} \right)^{-n} = 1 - \frac{749}{810}$$

$$\left(1 + \frac{10,7\%}{12} \right)^{-n} = \frac{61}{810}$$

$$-n \log \left(1 + \frac{10,7\%}{12} \right) = \log \frac{61}{810}$$

$$-n = \log \frac{61}{810} \div \log \left(1 + \frac{10,7\%}{12} \right)$$

$$-n = -291,3278456$$

$$\underline{\underline{n = 291 \text{ months}}}$$

QUESTION 7

[14]

- a) A recruiting agency received multiple applications for a lead position in a data-mining company. The applicants need one or more of the following skills to apply:

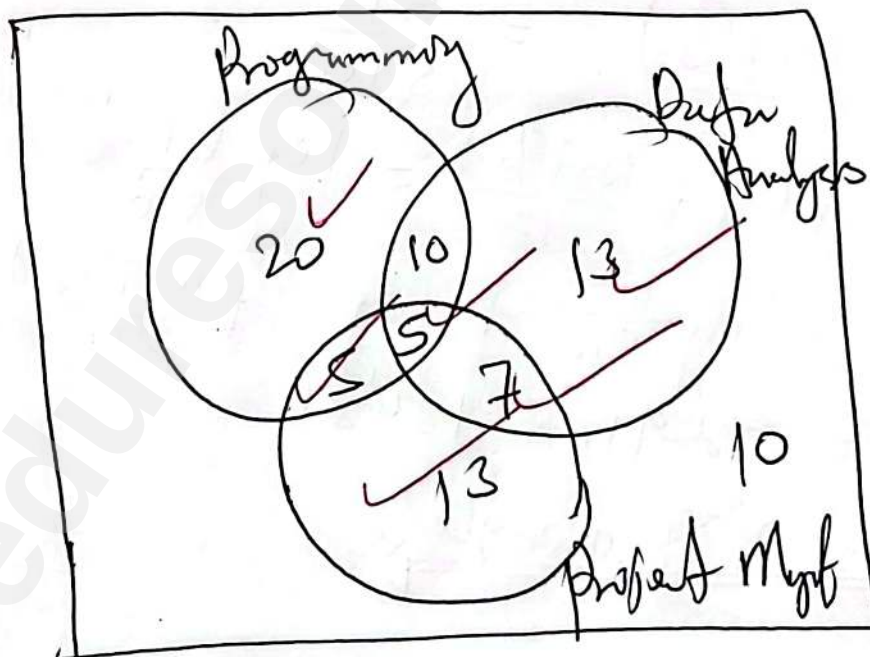
Programming, Data Analysis and/or Project Management.

The recruiting agency organised the applicants as follows:

- 40 applicants are skilled in Programming.
- 35 applicants are skilled in Data Analysis.
- 30 applicants are skilled in Project Management.
- 15 applicants are skilled in both Programming and Data Analysis.
- 10 applicants are skilled in both Programming and Project Management.
- 12 applicants are skilled in both Data Analysis and Project Management.
- 5 applicants are skilled in all three areas.
- 10 applicants do not have any of these skills.

Use the above information to determine the probability of finding the best candidate for the position. Show all methods and/or calculations used in determining your answer.

(7)



$$\therefore \text{Best Candidate} = \frac{5}{83}$$

- b) Each of the digits 1; 1; 2; 3; 4; 7 is written on a separate card. The cards are then placed next to each other to make a 6 - digit number.

1) How many different 6 - digit numbers can be formed from these digits? (2)

$$\frac{6!}{2!} = 360 \rightarrow \textcircled{2}$$

2) How many numbers start and end with the same digit? (2)

$$\frac{1}{4!} = 24 \rightarrow \textcircled{2}$$

3) What is the probability of getting a number that starts and ends with the same digit? (1)

$$\frac{24}{360} = \frac{1}{15} \rightarrow \textcircled{1}$$

4) Find the probability that a number is 112347 or 743211. (2)

$$\frac{2}{360} = \frac{1}{180} \rightarrow \textcircled{2}$$

QUESTION 8

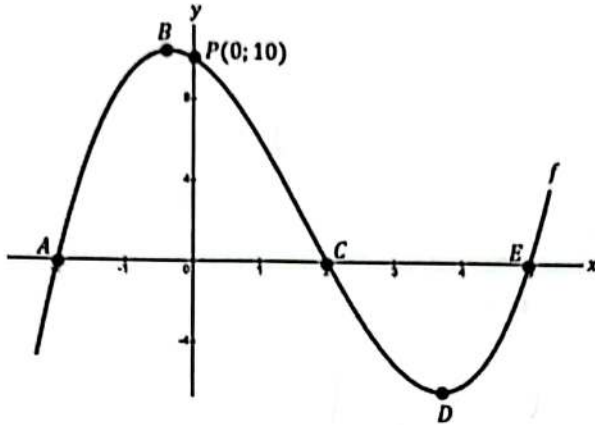
[14]

In the diagram below is the graph of $f(x) = ax^3 + bx^2 + cx + 10$.

The x -intercepts are at: $A(-2; 0)$, $C(2; 0)$ and $E(5; 0)$.

B and D are the local maximum and local minimum respectively.

Point $P(0; 10)$ the y -intercept of f .



a) Show that $a = \frac{1}{2}$, $b = \frac{-5}{2}$ and $c = -2$.

(6)

$$f(x) = a(x+2)(x-2)(x-5)$$

$$10 = a(2)(-2)(-5)$$

$$\frac{10}{20} = \frac{20a}{20}$$

$$\frac{1}{2} = a$$

$$f(x) = \frac{1}{2}(x+2)(x-2)(x-5)$$

$$= \frac{1}{2}(x^2 - 4)(x-5)$$

$$= \frac{1}{2}(x^3 - 5x^2 - 4x + 20)$$

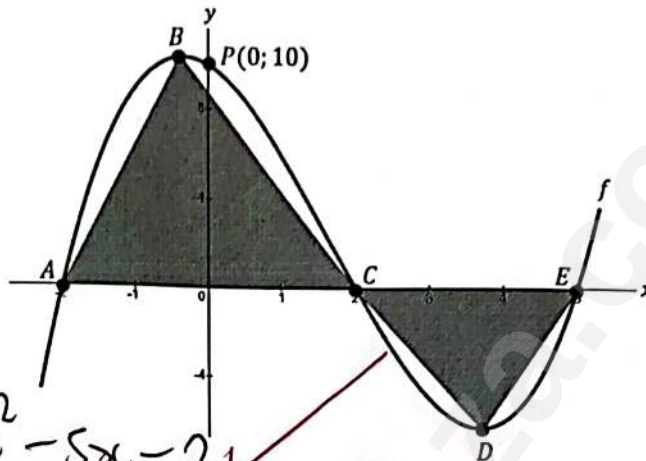
$$= \frac{1}{2}x^3 - \frac{5}{2}x^2 - 2x + 10$$

$$a = \frac{1}{2}; b = \frac{-5}{2}; c = -2$$

b) If $f(x) = \frac{1}{2}x^3 - \frac{5}{2}x^2 - 2x + 10$ and $\triangle ABC$ and $\triangle CDE$ are constructed,

determine the numerical value of $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle CDE}$

Show all calculations and round off your final answer to ONE decimal place. (8)



$$f'(x) = \frac{3}{2}x^2 - 5x - 2$$

$$\frac{3}{2}x^2 - 5x - 2 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(\frac{3}{2})(-2)}}{2(\frac{3}{2})}$$

$$= 3,694254177 \quad / \quad -0,3609208434$$

$$\approx 3,7 \quad / \quad -0,4$$

$$f(-0,4) = 10,4 \quad \therefore B(-0,4; 10,4)$$

$$f(3,7) = -6,3 \quad \therefore D(3,7; -6,3)$$

$$\text{Area of } \triangle ABC = \frac{1}{2}bh$$

$$= \frac{1}{2}(4)(10,4)$$

$$= 20,8$$

$$\text{Area of } \triangle CDE = \frac{1}{2}bh$$

$$= \frac{1}{2}(3)(6,3)$$

$$= 9,5$$

$$\therefore \frac{20,8}{9,5} = 2,2$$

QUESTION 9

[9]

In the diagram below is an aluminium forged seamless ring. These rings are used in a wide range of industries including aerospace, automotive, and oil and gas. They are used in applications where strength, durability, and reliability are critical, such as in high-pressure valves, gears, and bearings. Forged rings come in a variety of materials and sizes.



The manufacturing company have the following build specifications:

The outer radius is R meters.

The inner radius, r , is 90% of the length of R . The height must be $(6 - R)$ meters

- a) Express the interior radius, r , in terms of R (1)

$$r = 0,9R \quad \text{①}$$

- b) Show that the volume of aluminium used can be expressed as
 $V = -0,19\pi R^3 + 1,14\pi R^2$. (4)

$$\begin{aligned} V &= \pi R^2 h - \pi r^2 h \\ &= \pi R^2 (6 - R) - \pi (0,9R)^2 (6 - R) \\ &= 6\pi R^2 - \pi R^3 - 4,86\pi R^2 + 0,81\pi R^3 \\ &= -0,19\pi R^3 + 1,14\pi R^2 \quad \text{Shown} \quad \text{④} \end{aligned}$$

c) Determine height of the ring when the value of R is at a maximum.

(4)

$$\frac{dN}{dr} = -0,57 \pi R^2 + 2,28 \pi R$$

$$-0,57 \pi R^2 + 2,28 \pi R = 0$$

$$R = 4m \text{ or } R \neq 0m$$

$$h = 6 - 4 = 2m$$



QUESTION 10

[15]

Given: $f(x) = x^2 - 2x$ The minimum values of $f(x) + 2f(x) + 4f(x) + 8f(x) + \dots$ form a geometric series.

- a) Determine the first four terms of the series

(4)

$$f(1) = 1^2 - 2 \cdot 1 = -1$$

$$f(2) = 2^2 - 2 \cdot 2 = 0$$

$$f(3) = 3^2 - 2 \cdot 3 = 3$$

$$f(4) = 4^2 - 2 \cdot 4 = 8$$

$$-1; -2, -4, -8$$



- b) If you were to find the sum of the minimum values to n terms, between which two consecutive terms in the series will the difference of their sums be $-536\,870\,912$?

(6)

$$S_{n+1} - S_n = -536\,870\,912$$

$$\frac{a(r^{n+1} - 1)}{r - 1} - \frac{a(r^n - 1)}{r - 1} = -536\,870\,912$$

$$\frac{-1(2^{n+1} - 1)}{2 - 1} - \frac{-1(2^n - 1)}{2 - 1} = -536\,870\,912$$

$$-2^{n+1} + 1 + 2^n - 1 = -536\,870\,912$$

$$2^n - 2^{n+1} = -536\,870\,912$$

$$2^n(1 - 2) = -536\,870\,912$$

$$2^n = 536\,870\,912$$

$$n = \log_2 536\,870\,912$$

$$n = 29$$

$\therefore$ between S_{29} and S_{30} terms

c) $\sum_{n=2}^m 32(2)^{5-n} < 500$

Determine the value of m for which the above-mentioned statement will be true, by using the correct sum formula. (5)

$$32(2)^{5-2}; 32(2)^{5-3}; 32(2)^{5-4}$$

$$256; 128; 64; 32$$

$$a = 256$$

$$r = \frac{1}{2}$$

$$\frac{a(1-r^m)}{1-r} < 500$$

$$\frac{256(1-0,5^m)}{1-0,5} < 500$$

$$1-0,5^m < \frac{500 \times 0,5}{256}$$

$$1-0,5^m < \frac{125}{128}$$

$$-0,5^m < -\frac{3}{128}$$

$$0,5^m > \frac{3}{128}$$

$$m > \log_{0,5} \frac{3}{128}$$

$$m > 5,4150$$

$$\therefore m = 6$$

5

SECTION B: 65 MARKS

TOTAL: 150 MARKS

ADDITIONAL SPACE (ALL questions)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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MATHEMATICS
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^n 1 = n$$

$$T_n = a + (n-1)d$$

$$T_n = ar^{n-1}; \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad r \neq 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni)$$

$$A = P(1 + i)^n$$

$$F = x \left[\frac{(1+i)^n - 1}{i} \right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$P = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$M \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$y - y_1 = m(x - x_1)$$

$$m = \tan \theta$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum f x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$