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PRELIMINARY EXAMINATION

SEPTEMBER 2024

MATHEMATICS: PAPER 2

EXAMINATION NUMBER

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TIME: 3 HOURS

150 MARKS

MEMO

EXAMINER: SCHOLEFIELD

MODERATOR: SOMIAH-NAIDOO

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 28 pages, including one blank page for extra working, and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions. Write your answers in the spaces provided.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. A correct answer only will not necessarily be awarded full marks.
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.

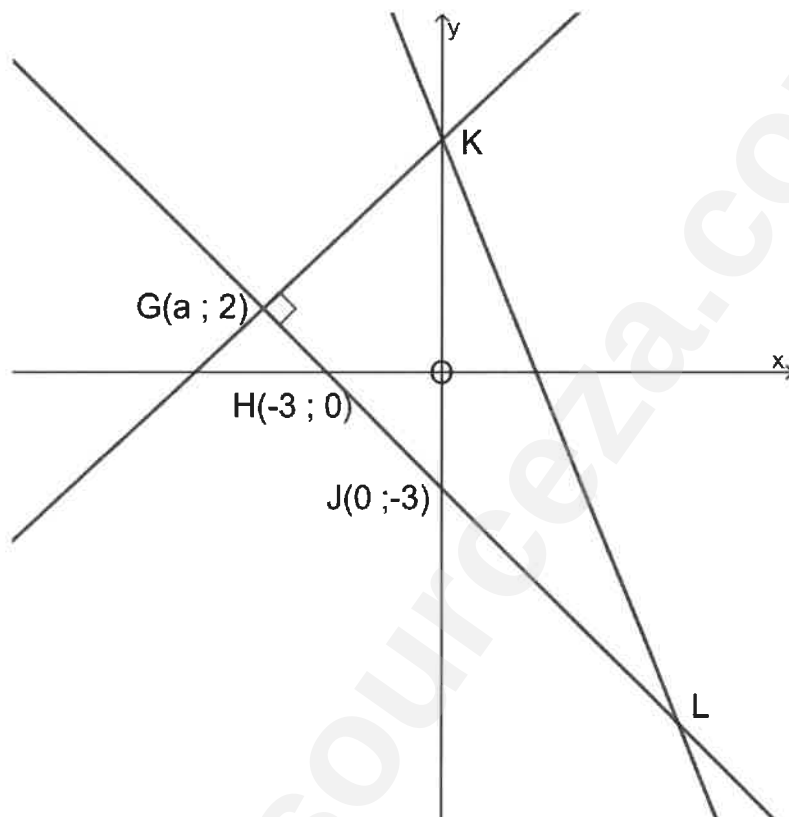
FOR OFFICIAL USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
20	6	8	13	13	8	8	18	19	10	17	10	/150

SECTION A

QUESTION 1

In the diagram below, $\triangle GKL$ has $GK \perp GL$ with $G(a; 2)$. The points $H(-3; 0)$, and $J(0; -3)$ lie on the line GL and K lies on the y -axis.



- (a) Determine the equation of line GL .

$$y = -x - 3$$

(2)

- (b) Show that $a = -5$.

$$\begin{aligned} 2 &= -a - 3 \\ a &= -5 \end{aligned}$$

(2)

- (c) Determine the equation of line GK .

$$\begin{aligned}y &= x + c \\2 &= -5 + c \checkmark \\c &= 7 \\y &= x + 7 \checkmark\end{aligned}$$

(3)

- (d) If the equation of line KL is $y = -2x + 7$, determine the coordinates of L .

$$\begin{aligned}-x - 3 &= -2x + 7 \checkmark \\x &= 10 \\y &= -13 \\L & (10; -13) \checkmark\end{aligned}$$

(3)

- (e) Determine the area of $\triangle GKJ$

$$\begin{aligned}KJ &= 7 + 3 \checkmark = 10 \checkmark \\ \therefore \text{Area} &= \frac{1}{2} (10)(5) \checkmark \\ &= 25 \text{ u}^2 \checkmark\end{aligned}$$

(3)

(f) Determine the size of $G\hat{L}K$

$$\tan \theta = -2$$

$$\theta = 116,6^\circ \checkmark$$

$$\tan B = -1$$

$$\beta = 135^\circ \quad \checkmark$$

$$\hat{L} = 180^\circ - (45^\circ + 114,6^\circ)$$

$$= 18,4^\circ \checkmark$$

(3)

(g) The point L is translated vertically to a point L' where $\triangle GKL'$ is isosceles. Determine one possible set of coordinates of L' .

$$L^1(10; y) \checkmark$$

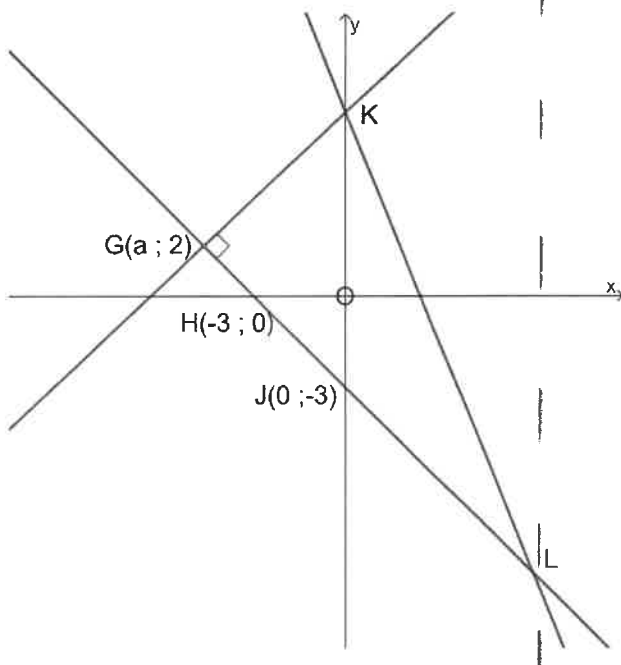
$$(10+5)^2 + (y-2)^2 = (10-0)^2 + (y-7)^2 //$$

$$225 + y^2 - 4y + 4 = 100 + y^2 - 14y + 49.$$

$$10y = -80$$

11-8

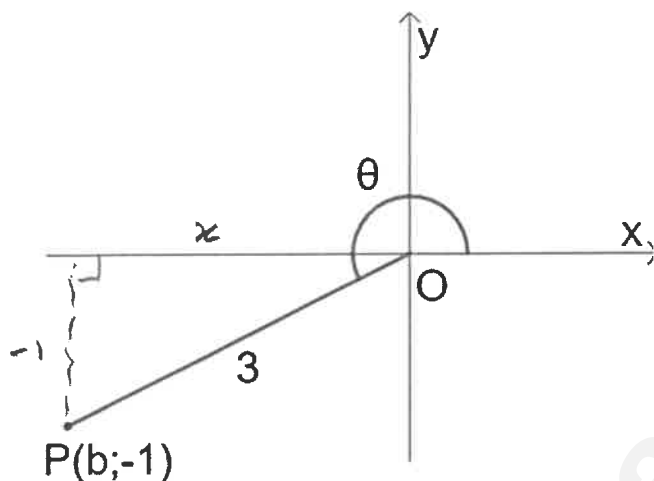
$$L'(10; -8) \quad \checkmark$$



(4)
[20]

QUESTION 2

In the diagram below, $P(b; -1)$ is at the endpoint of OP as shown. $OP = 3$ units and $\angle XOP = \theta$.



Without the use of a calculator, determine the value of:

(a) b

$$x^2 = 3^2 - (-1)^2 \checkmark$$

$$b = -\sqrt{8} \checkmark$$

(2)

(b) $\tan \theta$

$$= \frac{-1}{-\sqrt{8}} \checkmark$$

$$= \frac{1}{\sqrt{8}} \checkmark$$

(1)

(c) $\frac{1}{1 - \cos 2\theta}$

$$= \frac{1}{1 - (2\cos^2 \theta - 1)} \checkmark$$

$$= \frac{1}{2 - 2\left(-\frac{\sqrt{8}}{3}\right)^2} \checkmark$$

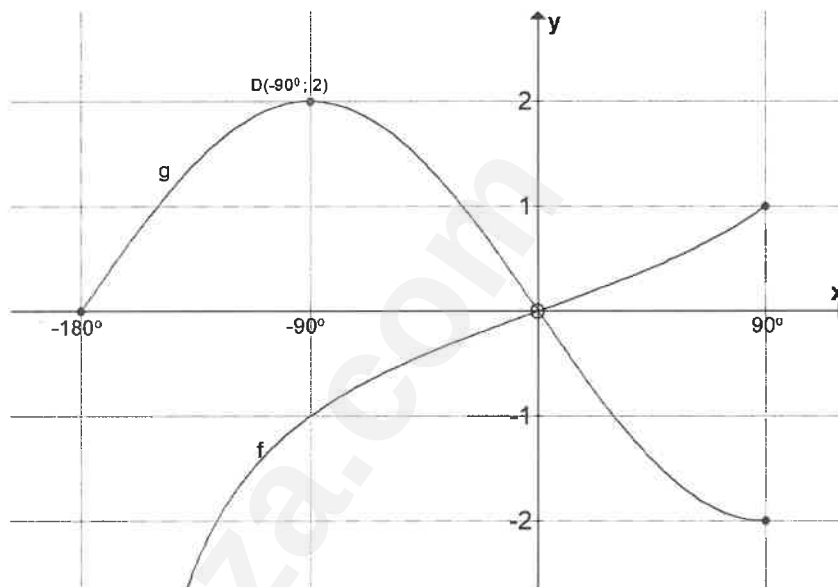
$$= \frac{9}{2} \checkmark$$

(3)
[6]

QUESTION 3

In the diagram, $f(x) = a \tan bx$ and $g(x) = c \sin dx$ have been sketched for $x \in [-180^\circ; 90^\circ]$.

$D(-90^\circ; 2)$ is a local maximum of g .



Determine the value(s) following:

- (a) a, b, c and d .

$$\begin{aligned} a &= 1 & \checkmark \\ b &= \frac{1}{2} & \checkmark \\ c &= -2 & \checkmark \\ d &= 1 & \checkmark \end{aligned}$$

(4)

- (b) x for which $f(x) \leq g(x)$.

$$x \in (-180^\circ; 0^\circ] \quad \checkmark \quad \checkmark$$

(2)

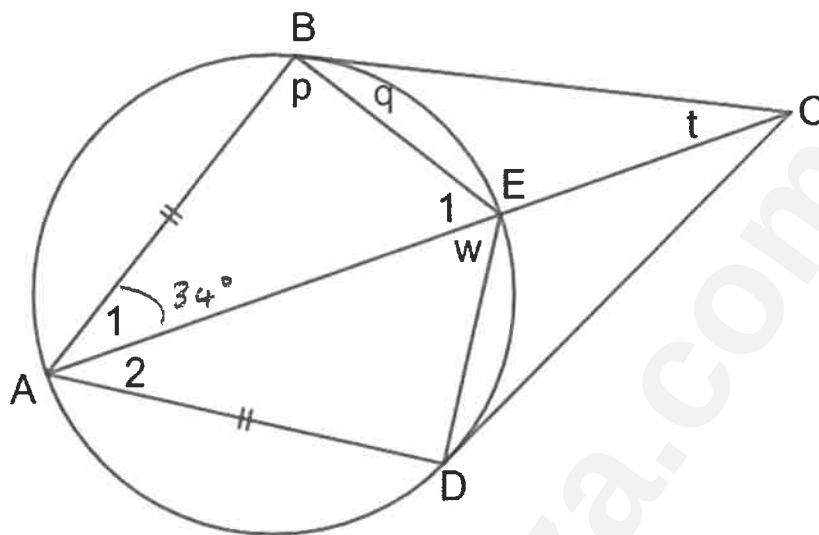
- (c) k for which $h(x) = g(x) + k$ has no real roots for $x \in [-180^\circ; 90^\circ]$.

$$k > 2 \quad \text{or} \quad k < -2 \quad \checkmark \quad \checkmark$$

(2)

[8]

- (b) In the diagram below, the circle passes through points A, B, E and D .
The diameter AE meets the tangents to the circle at B and D , at C , as shown.
 $AB = AD$ and $\hat{A}_1 = 34^\circ$.



Calculate the sizes of the angles marked p, q, t , and w .

$$p = 90^\circ$$

$\angle$ in semi circle. ✓

$$q = 34^\circ$$

tan - chord ✓

$$t = 22^\circ$$

$\angle$'s in Δ . ✓

$$\hat{E}_1 = 56^\circ$$

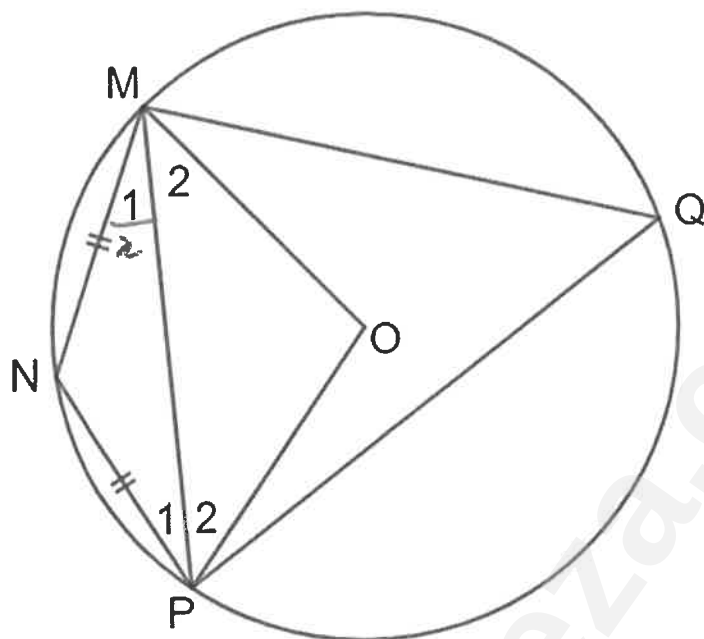
$\angle$'s in Δ .

$$w = 56^\circ$$

= $\angle$'s opp = chords ✓

(4)

- (c) In the diagram below the circle centre O passes through the points M, N, P and Q , and $MN = NP$ as shown.



If $\hat{M}_1 = x$, determine $\hat{MOP}$ in terms of x .

$$\begin{aligned}
 & \text{Radii} \\
 & \hat{M}_1 = 2x \quad \text{Opp = sides} \checkmark \\
 & \hat{N} = 180^\circ - 2x \quad \text{L's in } \Delta \checkmark \\
 & \hat{Q} = 2x \quad \text{Opp L's cyclic quad} \checkmark \\
 & \hat{O} = 4x \quad \text{L at centre} = 2 \times \text{L at circum} \checkmark
 \end{aligned}$$

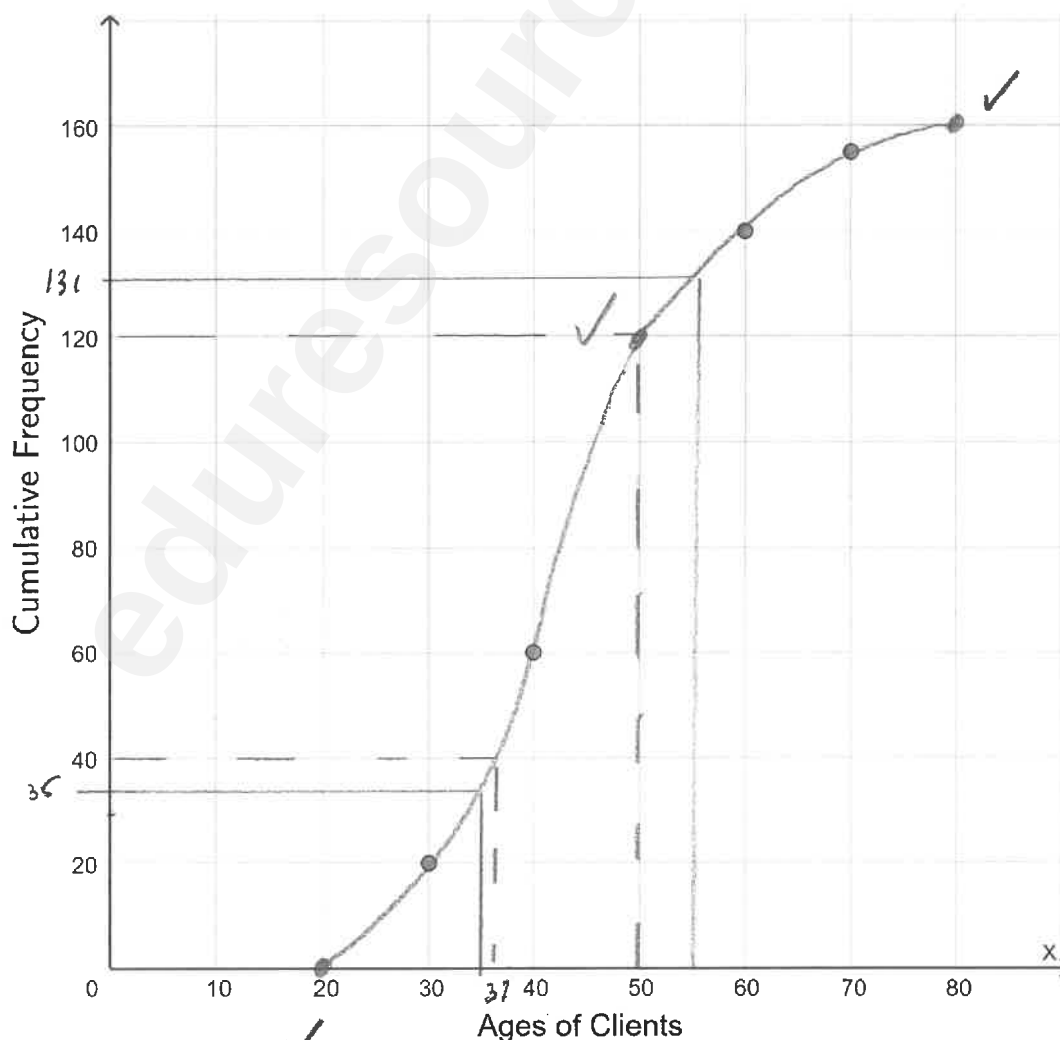
(4)
[13]

QUESTION 5

Trust Bank offers loans to its clients. During December the bank notices an increase in the number of loans being granted. In the table below, the number of loans granted per age of client (in years), are indicated.

Ages of clients	Number of loans	Cumulative number of loans
$20 \leq x < 30$	20	20
$30 \leq x < 40$	40	A = 60 ✓
$40 \leq x < 50$	B = 60 ✓	C = 120 ✓
$50 \leq x < 60$	20	140
$60 \leq x < 70$	15	155
$70 \leq x < 80$	5	160

- (a) Write the values of **A**, **B**, and **C** next to the appropriate letter in the table. (3)
- (b) On the system of axes below, complete the cumulative frequency graph for the data.



- (c) (1) Use your graph to determine the interquartile range for the data.

$$\begin{aligned} IQR &= 50 - 37 = 13 \\ \text{or } 50 - 36 &= 14 \end{aligned}$$

(3)

- (2) Approximately what percentage of those given loans were aged between 35 and 55 years?

$$\begin{aligned} \frac{131 - 35}{160} &= \frac{96}{160} \\ &= 60\% \end{aligned}$$

(2)

- (d) If the bank had made a decision not to grant loans to those over the age of 60, what effect would this have on:

- (1) The median of the data

It is lower.

(1)

- (2) The standard deviation of the data.

It is lower.

(1)

[13]

QUESTION 6

- (a) A kettle is boiled. The temperature of the water is measured after the water has boiled. The results are shown in the table below.

Time in minutes	2	12	7	8	9	4
Temperature in °C	81	23	61	54	50	63

- (1) What is the value of the correlation coefficient for the data, giving your answer correct to four decimal places? What statement accurately describes this value.

$$r = -0,9509 \checkmark$$

Very strong, negative $\checkmark$

(2)

- (2) Write down the equation of the line of best-fit for the data, giving your answers correct to four decimal places.

$$y = 90,8802 - 5,0781x \checkmark \checkmark$$

(2)

- (3) Use your equation to predict the temperature of the water 10 minutes after the water has boiled. Explain whether or not this result is reliable.

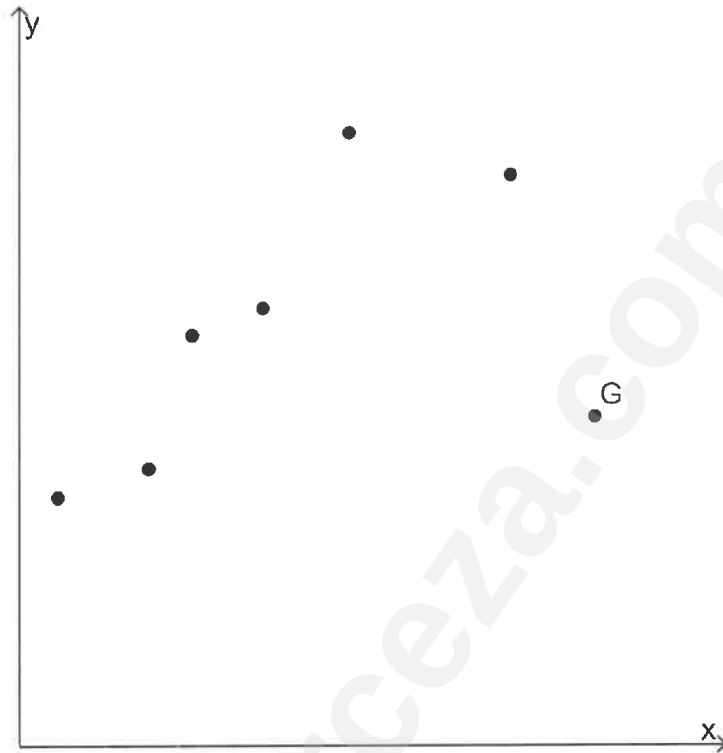
$$y = 90,8802 - 5,0781(10)$$

$$y = 40,1^{\circ}\text{C} \checkmark$$

Reliable, interpolation $\checkmark$

(2)

- (b) In the diagram below, a scatter plot has been created showing the relationship between two variables, x and y .



The data element labelled G is an outlier.

The line of best-fit for the data **including** G is given as: $y = 5,3 + 0,6x$ and the value of the correlation coefficient r is 0,86.

If the data element G is **excluded** from the data, write down new **possible** values for B and r .

$$B = 0,7 \quad (> 0,6, < 1)$$

$$r = 0,9 \quad (> 0,86, < 1)$$

(2)
[8]

QUESTION 7

- (a) Simplify this expression to one trigonometrical function in terms of x .

$$\begin{aligned} & \sin(90^\circ - x) \cdot \cos(-x) + \cos(x - 90^\circ) \cdot \sin(180^\circ + x) \\ &= \cos x \cdot \cos x + \cos(-(90^\circ - x)) \cdot (-\sin x) \\ &= \cos^2 x + \cos(90^\circ - x) \cdot (-\sin x) \quad \checkmark \\ &= \cos^2 x - \sin^2 x \quad \checkmark \\ &= \cos 2x \quad \checkmark \end{aligned}$$

(3)

- (b) It is given that $\cos 18^\circ = t$. Express the following in terms of t , without the use of a calculator.

(1) $\sin 288^\circ$

$$\begin{aligned} &= \sin(360^\circ - 72^\circ) \\ &= -\sin 72^\circ \quad \checkmark \\ &= -\cos 18^\circ \\ &= -t \quad \checkmark \end{aligned}$$

(2)

(3) $\sin 54^\circ$

$$\begin{aligned} &= \cos 36^\circ \\ &= \cos 2(18^\circ) \quad \checkmark \\ &= 2\cos^2 18^\circ - 1 \quad \checkmark \\ &= 2t^2 - 1 \quad \checkmark \end{aligned}$$

(3)

[8]

76 marks

SECTION B

QUESTION 8

(a) Prove the identity:

$$\frac{2\sin 5x \cdot \cos 4x - \sin 9x}{\cos x} = \tan x$$

$$\begin{aligned} \text{LHS} &= \frac{2\sin 5x \cdot \cos 4x - (\sin 5x \cos 4x + \cos 5x \sin 4x)}{\cos x} \\ &= \frac{\sin 5x \cos 4x - \cos 5x \sin 4x}{\cos x} \\ &= \frac{\sin(5x - 4x)}{\cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x = \text{RHS} \end{aligned}$$

(5)

- (b) Determine the general solution for the equation:

$$\cos \beta \cdot \sin \beta = -\frac{2}{5}$$

$$2 \sin \beta \cos \beta = -\frac{4}{5}$$

$$\sin 2\beta = -\frac{4}{5}$$

$$2\beta = -53,1^\circ + k360^\circ$$

$$\beta = -26,6^\circ + k180^\circ$$

$$\text{or } 2\beta = 180 - (-53,1^\circ) + k360^\circ$$

$$2\beta = 233,1^\circ + k360^\circ$$

$$\beta = 116,6^\circ + k180^\circ, \quad k \in \mathbb{Z}$$

(5)

- (c) It is given that: $\cos 12^\circ \cdot \cos \theta = 0,95$ and $\sin 12^\circ \cdot \sin \theta = 0,05$.
Determine, without the use of a calculator, the value of: $\cos(168^\circ - \theta)$.

$$\cos(168^\circ - \theta)$$

$$= \cos 168^\circ \cos \theta + \sin 168^\circ \sin \theta$$

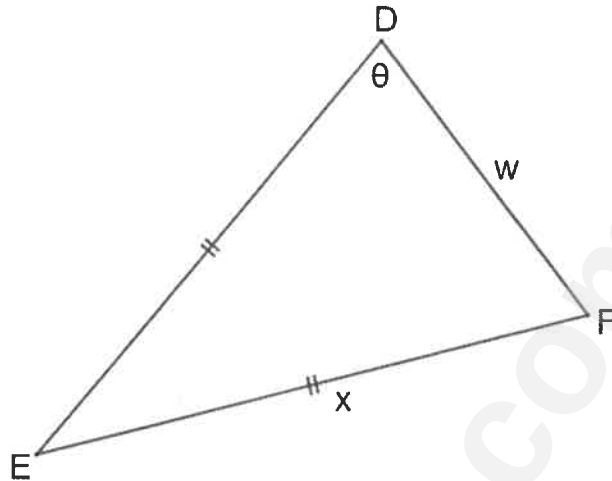
$$= -\cos 12^\circ \cos \theta + \sin 12^\circ \sin \theta$$

$$= -(0,95) + 0,05$$

$$= -0,9$$

(4)

- (c) In the diagram, $\triangle DEF$ has $EF = ED$. $\hat{D} = \theta$, $DF = w$ units and $EF = x$ units as shown.



Prove that $w = 2x \cos \theta$

$$\hat{F} = \theta$$

$$\hat{E} = 180^\circ - 2\theta$$

$$\therefore \frac{w}{\sin(180 - 2\theta)} = \frac{x}{\sin \theta} \quad \checkmark \checkmark$$

$$w = \frac{2x \sin 2\theta}{\sin \theta} \quad \checkmark$$

$$= \frac{2x \sin \theta \cos \theta}{\sin \theta} \quad \checkmark$$

$$w = 2x \cos \theta$$

(4)

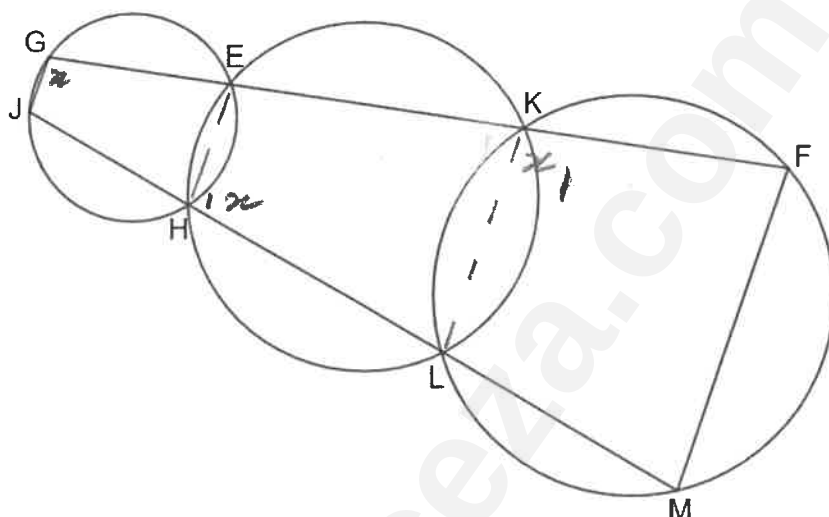
~~11~~

[12]

QUESTION 9

Reasons must be given for all Euclidean Geometry.

- (a) In the diagram below, two circles meet at E and H , and two at K and L . The lines EK and HL are extended to meet the outer circles at G and F , and at J and M , respectively, as shown. G and J are joined and F and M are joined.



- (1) Prove that $GFMJ$ is a cyclic quadrilateral.

Construct EH and KL ✓

$$\hat{G} = x$$

$$\hat{H} = x$$

$$\hat{K} = y$$

$$\hat{M} = 180^\circ - x$$

Ext $\angle$ of cyclic quad

Ext $\angle$ of cyclic quad

Opp $\angle$'s cyclic quad

$\therefore GFMJ$ is a cyclic quad. Con opp $\angle$'s cyclic quad ✓
 (3) quad

- (2) In the diagram above there are a total of three circles. If the pattern was continued so that there were 20 circles, what would be the type of quadrilateral equivalent to $GFMJ$ above, where G and J are points on the first circle, and F and M are points on the 20th circle? Explain your answer.

$$\hat{K} = x$$

$$\therefore GJ \parallel KL$$

$$\text{Constr } \angle = \checkmark$$

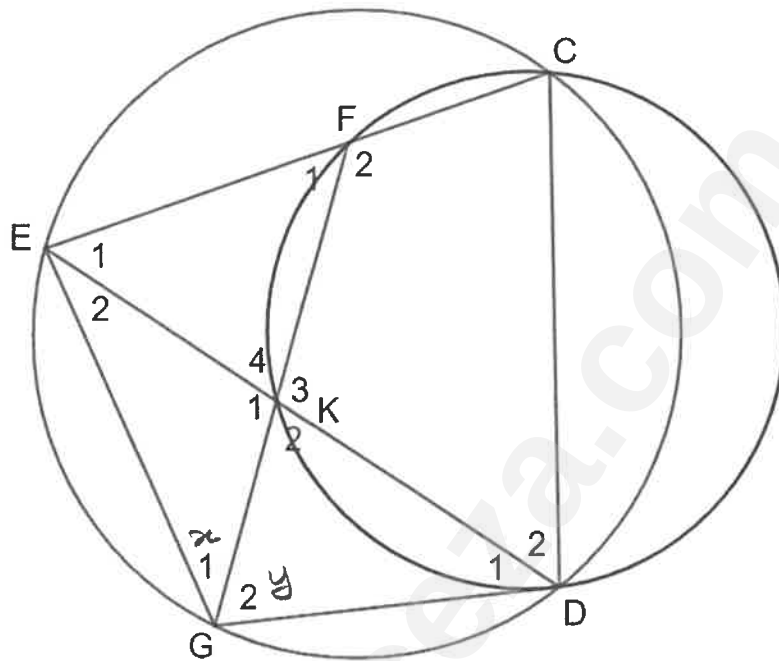
$$\therefore 20 \text{ circles}$$

$GFMJ$ is a trapezium ✓

1 pr opp sides $\parallel$ ✓

(3)

- (b) In the diagram, two circles intersect at C and D . Point E lies on the larger circle and lines CE and DE intersect the smaller circle at F and K respectively. The line FK meets the larger circle at G . EG and GD are drawn, as shown.



Let $\hat{G}_1 = x$ and $\hat{G}_2 = y$.

- (1) Express the following angles in terms of x and/or y .

(i) $\hat{C} = 180^\circ - (x + y)$ opp L's cyclic quad ✓

(ii) $\hat{K}_3 = x + y$ opp L's cyclic quad ✓ (1)

(1)

- (2) Prove that EG is a tangent to the circle DGK .

$\hat{D}_1 = x$

Ex + L Δ ✓

$\hat{G}_1 = \hat{D}_1 = x$ ✓

$\therefore EG$ is a tangent conv tan-chord ✓

(3)

- (3) Prove that $\triangle EGK$ and $\triangle EDG$ are similar.

$$\begin{aligned} \hat{E}_2 & \text{ is common } \checkmark \\ \hat{G}_1 & = \hat{D}_1 & \text{Above } \checkmark \\ \hat{K}_1 & = \hat{G}_1 + \hat{G}_2 & \text{3rd L} \\ \therefore \triangle EGK & \parallel \triangle EDG & \text{AAA } \checkmark \end{aligned}$$

(3)

- (4) Hence, or otherwise, prove that $EG^2 = CE \cdot FE$

$$\frac{EK}{ED} = \frac{EG}{EG} \quad \text{Sim } \Delta's \checkmark$$

$$EG^2 = ED \cdot EK \quad \checkmark$$

In $\triangle EFK$ and $\triangle EDC$.

$\hat{E}_1$ is common.

$$\hat{K}_4 = \hat{C}$$

Ext L cyclic quad.

$$\hat{F}_1 = \hat{D}_1$$

Ext L cyclic quad

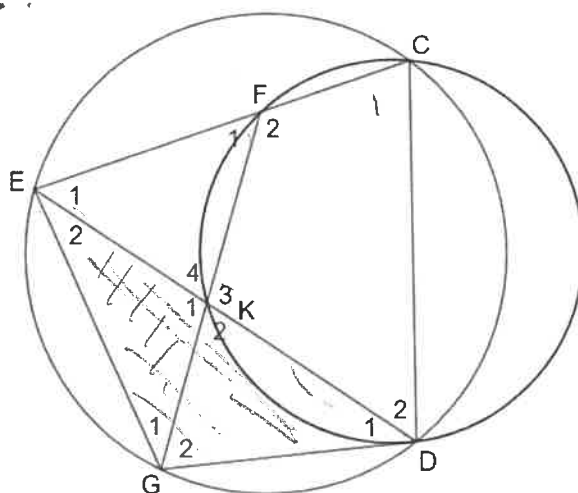
$$\triangle EFK \parallel \triangle EDC \quad \text{AAA } \checkmark$$

$$\frac{EK}{CE} = \frac{EF}{ED}$$

Sim $\Delta's \checkmark$

$$ED \cdot EK = CE \cdot FE$$

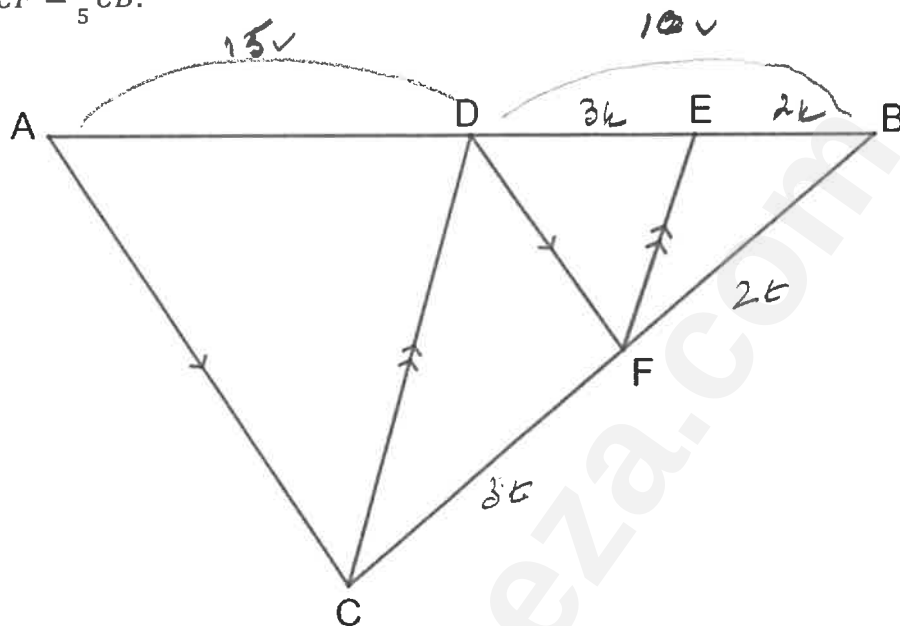
$$\therefore EG^2 = CE \cdot FE.$$



(5)
[19]

QUESTION 10

In the diagram $\triangle ABC$ has F on BC and D and E on AB such that $EF \parallel DC$, and $FD \parallel CA$.
It is given that $CF = \frac{3}{5}CB$.



Determine the following:

(a) $BE : ED$

$$CF : FB = 3 : 2$$

$$\therefore BE : ED = 3 : 2 \quad \checkmark \quad \text{since } \parallel \text{ to side } AC \quad \checkmark$$

(2)

(b) $AD : DE$

$$AD : DB = 3 : 2 \quad \text{since } \parallel \text{ to side } AC \quad \checkmark$$

$$= 15 : 10$$

$$\therefore AD : DE = 15 : 6 \quad \checkmark$$

$$= 5 : 2 \quad \checkmark$$

(3)

(c) $\frac{\text{Area } \triangle DEF}{\text{Area } \triangle BCD}$

$$\frac{\text{Area } \triangle DEF}{\text{Area } \triangle BEF} = \frac{3k}{2k} \checkmark \quad \text{Common height } \checkmark$$

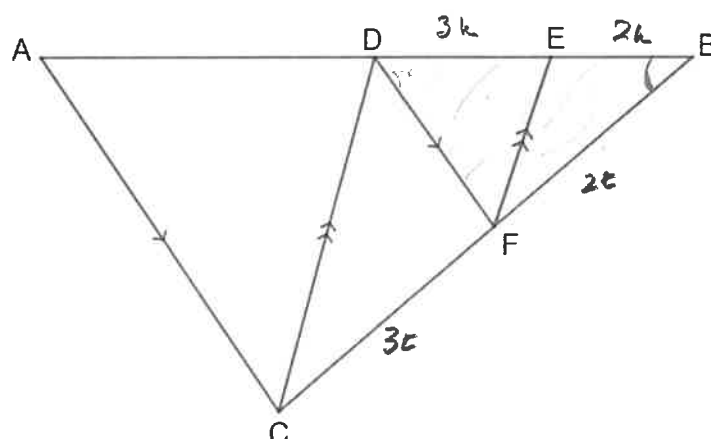
$$\text{Area } \triangle BEF = \frac{2 \text{ Area } \triangle DEF}{3}$$

$$\frac{\text{Area } \triangle BEF}{\text{Area } \triangle BCD} = \frac{\frac{1}{2}(2k)(2c) \sin B}{\frac{1}{2}(5k)(5c) \sin B} \checkmark \quad \text{Common } \angle \checkmark$$

$$\frac{2 \text{ Area } \triangle DEF}{3} = \frac{4}{25}$$

$$\text{Area } \triangle BCD$$

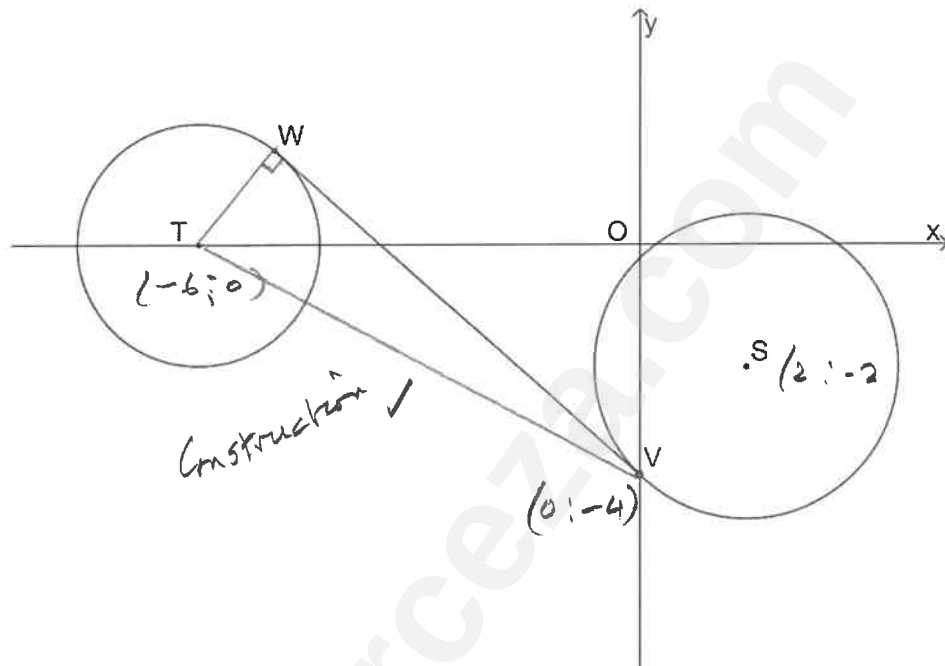
$$\frac{\text{Area } \triangle DEF}{\text{Area } \triangle BCD} = \frac{3}{2} \times \frac{4}{25} = \frac{6}{25} \checkmark$$



(5)
[10]

- (b) In the diagram, circle centre S has the equation $(x - 2)^2 + (y + 2)^2 = 8$ and circle centre T has the equation $(x + 6)^2 + y^2 = 2$.

- Point V is a y -intercept of the circle centre S .
- VW is a common tangent to both circles as shown.



- (1) Calculate the coordinates of V .

$$\begin{aligned} (0 - 2)^2 + (y + 2)^2 &= 8 \quad // \\ (y + 2)^2 &= 4 \quad . \\ y + 2 &= \pm 2 \\ y &= -4 \end{aligned}$$

$$V(0, -4) \quad \checkmark$$

(3)

- (2) Calculate the distance VW .

$$\begin{aligned} \hat{W} &= 90^\circ \\ TW &= \sqrt{2} \quad \checkmark \\ VW^2 &= \end{aligned}$$

$$(-6 - 0)^2 + (0 - (-4))^2 - (\sqrt{2})^2 \quad \checkmark$$

$$= 52 - 2 = 50$$

$$VW = \sqrt{50} = 5\sqrt{2} \quad \checkmark$$

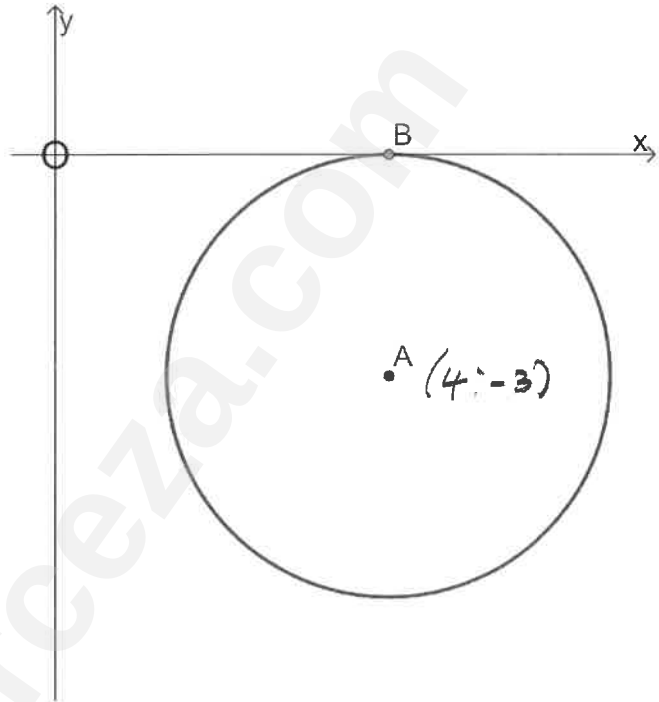
(4)

QUESTION 11

- (a) In the diagram, circle centre A has the equation:

$$x^2 - 8x + y^2 + 6y = p$$

The circle touches the x -axis at B.



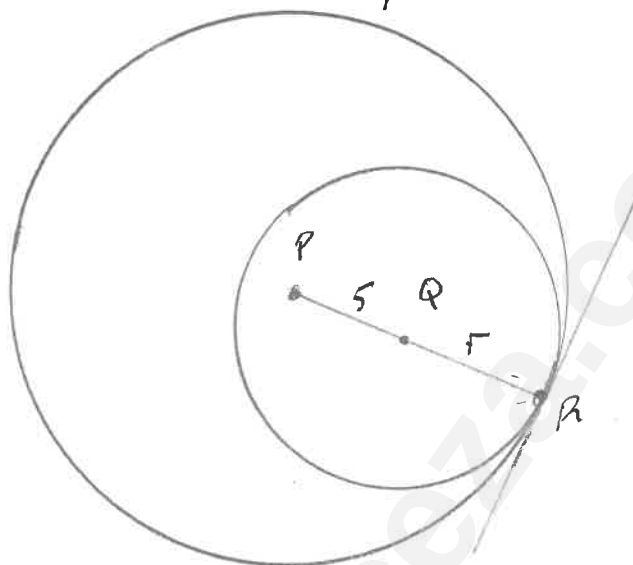
Determine the value of p .

$$\begin{aligned} x^2 - 8x + 16 + y^2 + 6y + 9 &= 25 + p \\ (x - 4)^2 + (y + 3)^2 &= 25 + p \\ \therefore AB = 3 &= \text{radius} \\ \therefore \sqrt{25 + p} &= 3 \\ p &= 9 - 25 \\ p &= -16 \end{aligned}$$

(5)

- (c) $P(-3; 4)$ and $Q(1; 7)$ are the centres of two circles which **touch internally**, (i.e. the circles share a common tangent.)
The ratio of the radius of circle P to the radius of circle Q is $k:1$, where $k > 1$.
If the radius of the smaller circle is r units, determine the value of A such that:

$$k = \frac{A}{r} + 1.$$



$$PQ = \sqrt{(-3-1)^2 + (4-7)^2} \quad \checkmark$$

$$= 5 \quad \checkmark$$

$$\frac{r+5}{r} = \frac{k}{1} \quad \checkmark$$

$$r+5 = kr$$

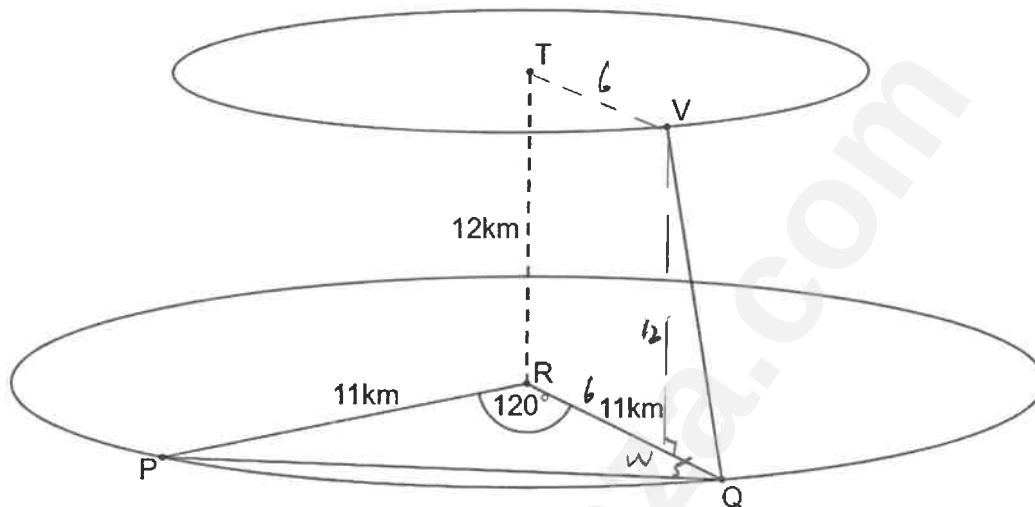
$$k = \frac{5}{r} + 1 \quad \checkmark$$

$$\therefore A = 5 \quad \checkmark$$

(5)
[17]

QUESTION 12

In the diagram, points P and Q lie on a horizontal plane, and are also on the circumference of a circle, centre R , with a radius of 11 km. The angle subtended by the chord PQ at the centre of the circle is 120° as shown.

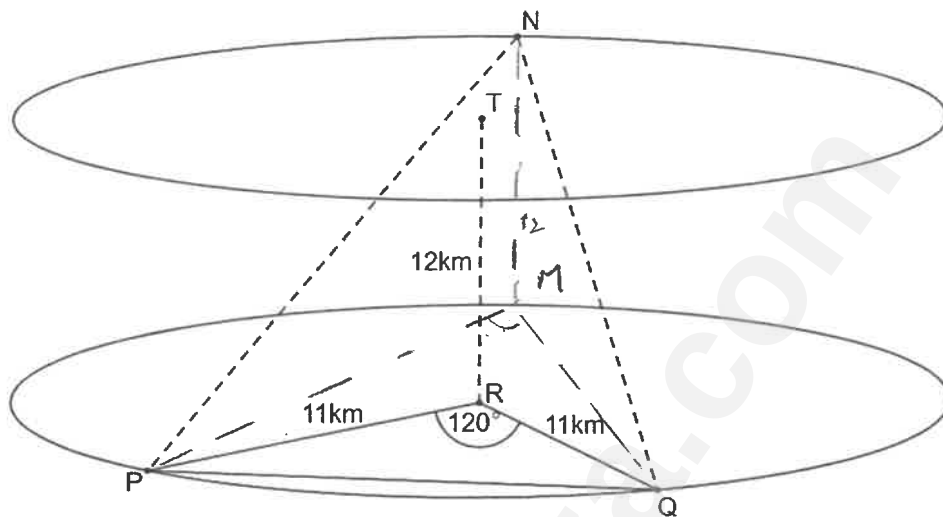


- (a) A drone flies a horizontal circular path of radius 6 km. The centre T of the drone's circular path is 12 km vertically above R . The drone is closest to point Q when it is at point V in the diagram. Determine the distance QV .

$$\begin{aligned} VW &= 12 \\ RW &= 6 \quad \checkmark \\ \therefore QW &= 5 \quad \checkmark \\ \therefore QV &= 13 \text{ km} \quad \checkmark \end{aligned}$$

(3)

- The radius of the path of the drone is now **increased** to 11 km
- The centre of the path (T) remains 12 km vertically above R .
- N is a point on the new path of the drone, **furthest** from the line PQ .



Determine the angle $P\hat{N}Q$.

$$\hat{M} = 60^\circ \checkmark$$

$$PQ^2 = 11^2 + 11^2 - 2(11)(11) \cos 120^\circ \checkmark$$

$$PQ = \sqrt{363} = 11\sqrt{3}.$$

$$PM = QM$$

$$\therefore M\hat{P}Q = M\hat{Q}P = 60^\circ \checkmark$$

$$\therefore p_m = 1153$$

$$\therefore NP^2 = (11\sqrt{3})^2 + 12^2 \checkmark$$

$$NP = 1353$$

$$\therefore \cos Z = \frac{(13\sqrt{3})^2 + (13\sqrt{3})^2 - (11\sqrt{3})^2}{2(13\sqrt{3})(13\sqrt{3})} \quad \checkmark \checkmark$$

$$\hat{N} = 50,1^\circ \checkmark$$

(7)

[10]

71 marks

Total 150 marks

PAGE FOR EXTRA WORKING

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