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PRELIMINARY EXAMINATION

SEPTEMBER 2024

MATHEMATICS: PAPER 1**EXAMINATION NUMBER**

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Time: 3 hours

150 marks

Examiner: J Ruiz-Mesa

Moderators: M Somiah-Naidoo

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 26 pages and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions. Write your answers in the spaces provided.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. If you run out of space for a question, use the additional space provided at the end. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
16	16	12	20	11	16	9	16	11	7	16	/150
SECTION A = 75					SECTION B = 75						

SECTION A**QUESTION 1**(a) Solve for x

$$(1) \ 3x^2 = 4(x + 2) + 4$$

$$(2) \ \log_{16} x = \frac{3}{4}$$

(3)

(2)

(b) Given the equation $p - 8 = \frac{20}{p}$

(1) Solve for p

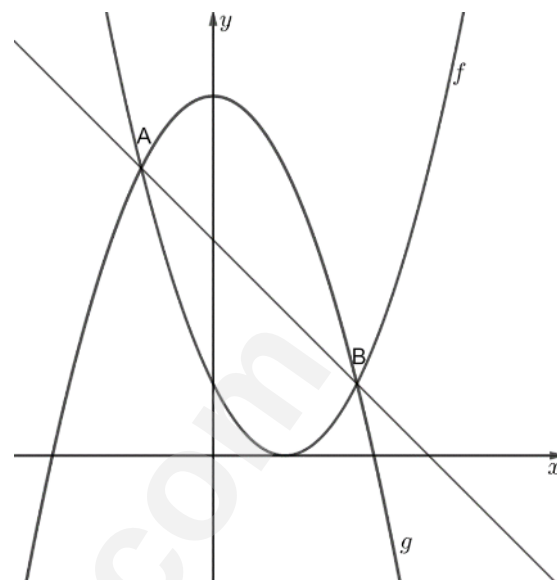
(2) Hence, or otherwise, solve for x if $\sqrt{x+3} = \frac{20}{\sqrt{x+3}} + 8$

(3)

(3)

- (c) The diagram alongside shows the graphs of $f(x) = x^2 - 2x + 1$ and $g(x) = 5 - x^2$. The graphs intersect at the points A and B.

Determine the equation of the line AB.



QUESTION 2

(a) The following information is given about a 6-speed manual car.

- Maximum speed in 1st gear is 28 km/h
- Maximum speed in 6th gear is 115 km/h.

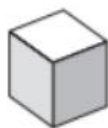
(1) Given that the fastest speed of the car in successive gears is modelled by an Arithmetic Sequence, determine fastest speed of the car in 3rd gear.

(3)

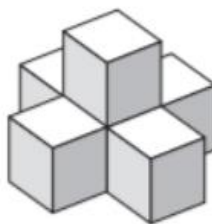
(2) Given that the fastest speed of the car in successive gears is modelled by a Geometric Sequence, determine fastest speed of the car in 5th gear.

(3)

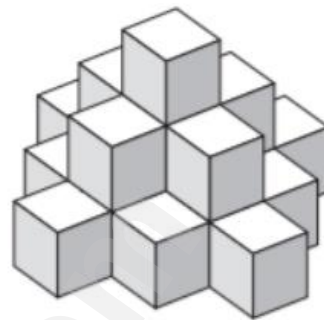
- (b) The diagram below show a picture of three towers built using cubes. The number of cubes is modelled by a quadratic sequence.



Tower 1



Tower 2



Tower 3

Tower 1 uses 1 cube.

Tower 2 uses 6 cubes (*please note there are 5 cubes in the bottom layer*).

- (1) Write down the number of cubes in Tower 3.

(1)

- (2) Find an expression for the number of cubes used to build Tower n (n th term).

(5)

- (3) Showing all your working explain why a Tower which follows this pattern cannot be built using 47 cubes.

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(4)

[16]

QUESTION 3

Mr Scholefield buys a new vehicle. He puts down a deposit of R50 000 and takes out a loan for the balance. Mr Scholefield makes monthly payments of R1 500 for a period of 6 years, starting in one month's time. The interest on the loan is 11,9% compounded monthly.

- (a) Determine the total value of the vehicle, correct to the nearest Rand.

(5)

- (b) Mr Scholefield decides to settle the loan after making 30 payments. Determine the outstanding balance of the loan immediately after the 30th payment.

(3)

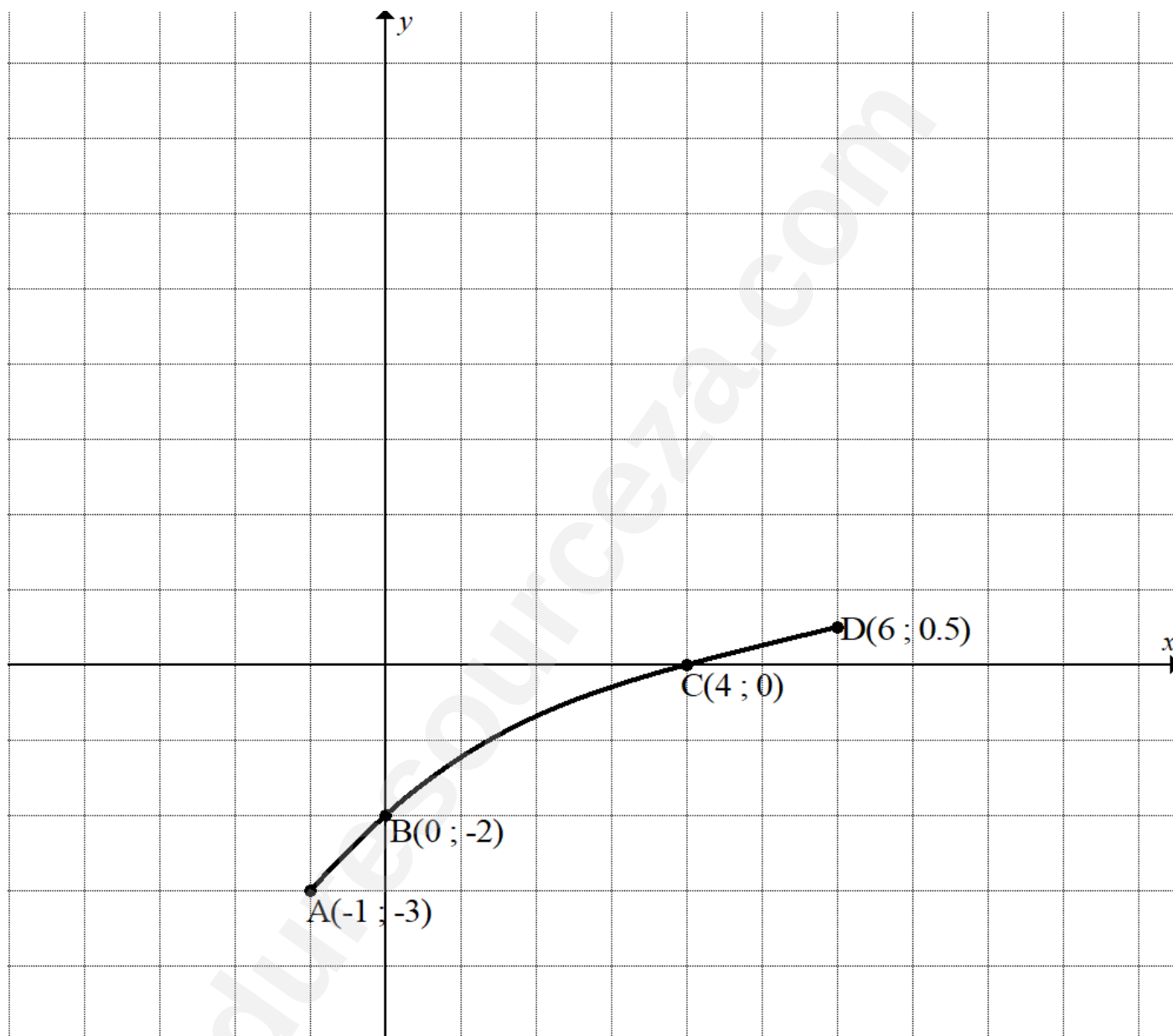
- (c) What percentage of the 30 payments made by Mr Scholefield have gone to service the interest of the original loan.

(4)

[12]

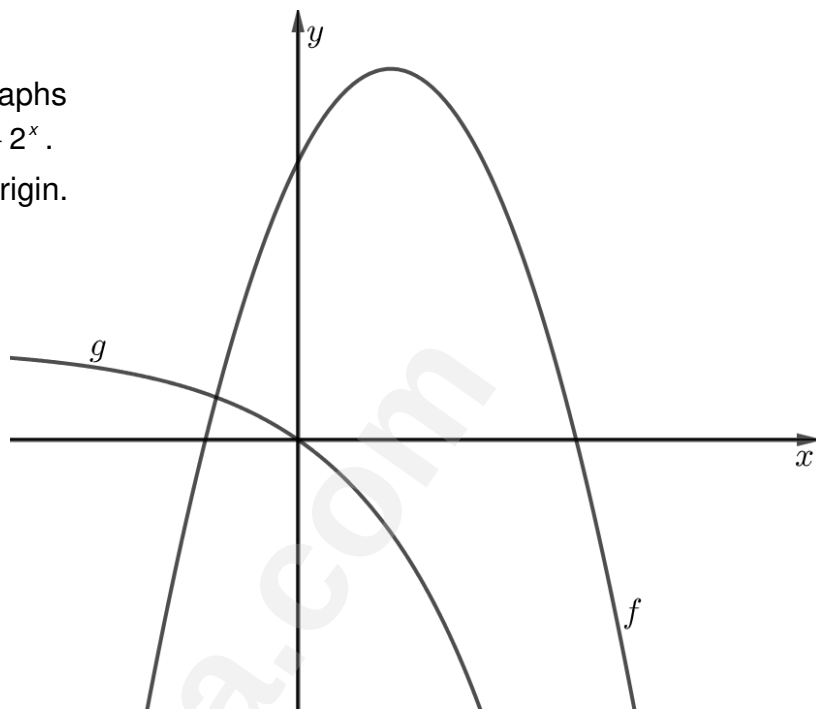
QUESTION 4

(a) The diagram below shows the graph of the function $f(x)$, defined for $-1 \leq x \leq 6$.



On the same diagram sketch the graph of $f^{-1}(x)$ clearly indicating the coordinates of the end points as well as any intercept with the axes.

- (b) The diagram alongside shows the graphs of $f(x) = -x^2 + 2x + 3$ and $g(x) = 1 - 2^x$. The graph of g passes through the origin.



- (1) For which value(s) of x is $f(x) \cdot g(x) \geq 0$?

(4)

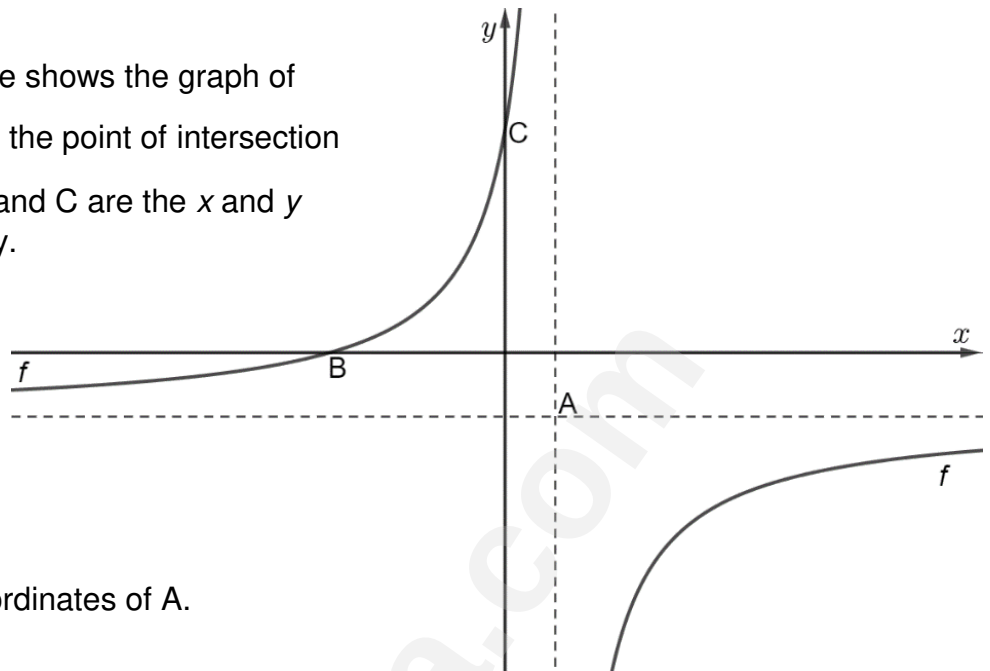
- (2) Determine the value of k for which the function $h(x) = f(x) + k$ will have only one real root.

(2)

- (3) The graph of m is a reflection of g about the y -axis. Write down the equation of m in the form $y = \dots\dots$

(2)

- (c) The diagram alongside shows the graph of $f(x) = -\frac{9}{x-1} - 2$. A is the point of intersection of the asymptotes. B and C are the x and y intercepts respectively.



- (1) Write down the coordinates of A.

(2)

- (2) Determine the coordinates of B and C, the intercept of f with the axes.

(4)

- (3) Write down the equation of the axis of symmetry of f which has a negative gradient, in the form $y = \dots\dots$

(2)

- (4) The graph of f is reflected about the x – axis to obtain the graph of g . Write down the equation of g in the form $y = \dots\dots$

(1)

[20]

QUESTION 5

(a) Given the function $f(x) = 3x - 5x^2$. Determine $f'(x)$ using First Principles.

(5)

(b) Determine $D_x \left(3x^2 - \frac{3}{2x^2} + x^{\frac{2}{3}} \right)$

(4)

(c) Given $y = \sqrt[4]{x^{\pi}} - 2a$. Determine $\frac{dy}{dx}$

(2)
[11]

SECTION B
QUESTION 6

(a) Given the function $f(x) = \left(\frac{1}{3}\right)^x$

(1) Determine $f^{-1}(x)$ in the form $y = \dots\dots$

(2)

(2) Write down the equation of the asymptote of $f(-x) - 5$

(2)

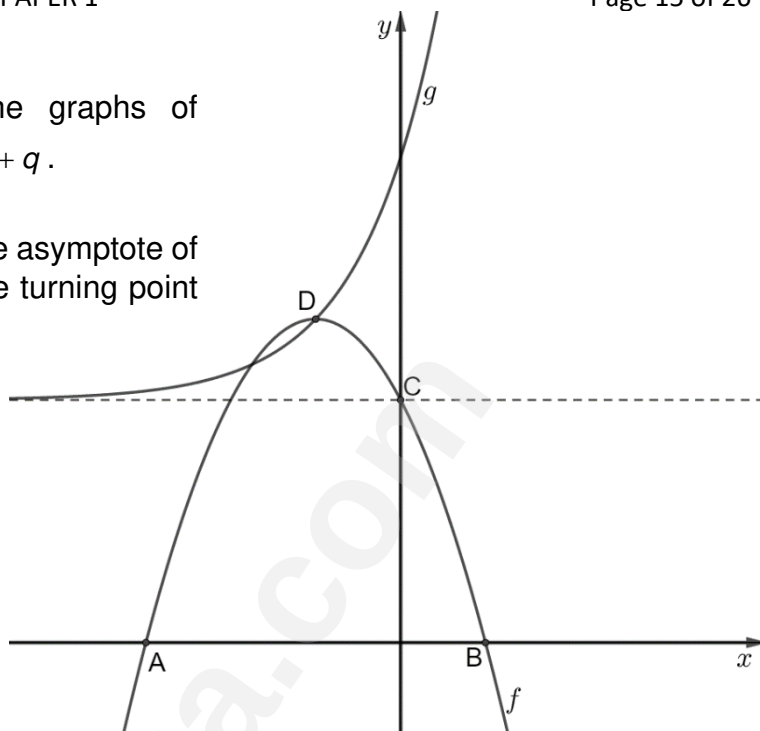
(3) Describe the transformation from f to g if $g(x) = \log_3 x$

(2)

- (b) The diagram alongside shows the graphs of $f(x) = -(x+1)^2 + 4$ and $g(x) = a \cdot 3^x + q$.

A and B are the x-intercepts of f .

C is the y-intercept of f and lies on the asymptote of g . The two graphs intersect at D, the turning point of f .



- (1) Write down the range of f .

(1)

- (2) Determine the equation of g .

(3)

- (3) Determine the coordinates of the point on f for which the tangent to f will have a gradient of 1.

(4)

- (4) For which value of k will $g(x) - k$ have the same y -intercept as $f(x)$?

(2)
[16]

QUESTION 7

(a) A sequence of numbers $T_1; T_2; T_3; \dots$ is defined as follows:

$$T_{k+1} = \frac{1}{T_k} \text{ and } T_1 = \frac{2}{3}$$

Determine the value of $\sum_{k=1}^{100} T_k$

(4)

(b) Show that $\sum_{n=1}^{16} (3 + 5n + 2^n) = 131\,798$

(5)
[9]

QUESTION 8

- (a) A cell phone store receives a shipment containing 8 iPhones and 7 Android Phones. In how many different ways can the phones be displayed in a line if all the iPhones are identical and so are all the Android phones?

(2)

- (b) At a sports capping ceremony the players of a team are to be placed in a row for a photograph. The team comprises of a Captain, a Vice-Captain and eight other players. In how many distinct ways can the players be arranged if the Captain and Vice-Captain may not stand next to each other?

(3)

- (c) In a class of 25 students, each student is assigned a number from 1 to 25. Two students are chosen at random, one after the other, without replacement. What is the probability that the student number for both pupils is either a multiple of 5 or of 7?

(d) If A and B are events such that:

- $P(A) = 0,5$; $P(A \cup B) = 0,6$
- $P(B) = p$.

Determine the value of p if A and B are independent events.

(4)

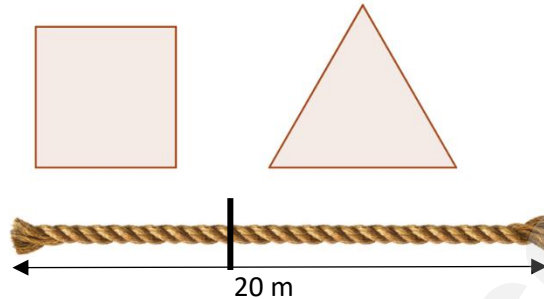
- (e) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0,01 for scooter drivers, 0,03 for car drivers and 0,15 for truck drivers. One of the insured drivers was involved in an accident. Determine the probability that it was a scooter driver. Write your answer correct to 4 decimal places.

(4)

[16]

QUESTION 9

- (a) A 20 metre long rope is to be cut off into two pieces. One piece will be used to make a square and the other one to make an equilateral triangle as shown in the diagram below. Let the piece used to make the square to have a length x .



- (1) Write down the dimensions of the square and the triangle in terms of x .

(2)

- (2) Show that the total area enclosed by the square and the triangle together using the 20 m rope is given by:

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(20 - x)^2$$

(5)

- (3) Hence, or otherwise, determine the value of x for which the area enclosed by the square and square together will be a maximum.

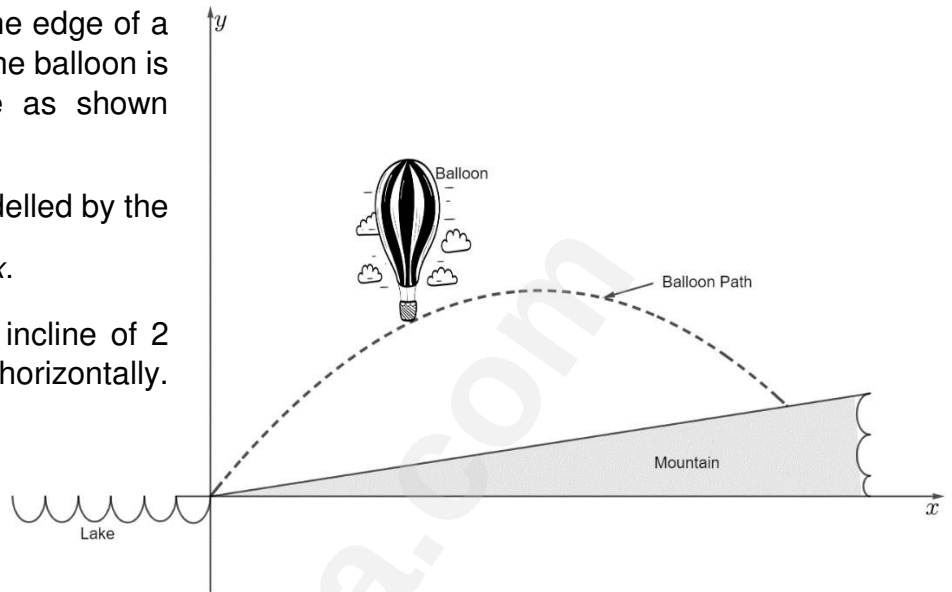
(4)

[11]

QUESTION 10

A hot air balloon takes off from the edge of a mountain lake. The trajectory of the balloon is modelled on a Cartesian plane as shown alongside.

- The path of the balloon is modelled by the function $f(x) = -\frac{2}{2500}x^2 + \frac{4}{5}x$.
- The land rises at a constant incline of 2 metres up for every 20 metres horizontally.



Determine the height of the balloon above the lake when the balloon's height above the ground is at its maximum.

(7)

QUESTION 11

- (a) It is given that the tangent to the function $h(x) = -x^3 + ax^2 + bx$ at $x = -2$ is parallel to the line $g(x) = -12x$ and that h changes concavity at $x = \frac{1}{2}$. Determine the values of a and b .

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- (b) The local government in a town contracted an actuarial firm to conduct a study that modelled the rate of a change of the town's population. The study found that the town's populations (measured in thousands of people per year from the year 2024) can be modelled by the function $P(t) = -\frac{1}{3}t^3 + 60t^2 + 8t + 9500$ where P is the population and t is the number of years from 2024.

(1) Determine the town's initial population.

(1)

(2) In what year will the town's population be a maximum?

(4)

(3) In what year will the growth rate of the population be a maximum?

(3)

(4) At what rate will the population be growing in the year 2124?

(2)
[16]

ADDITIONAL SPACE FOR ALL QUESTIONS

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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