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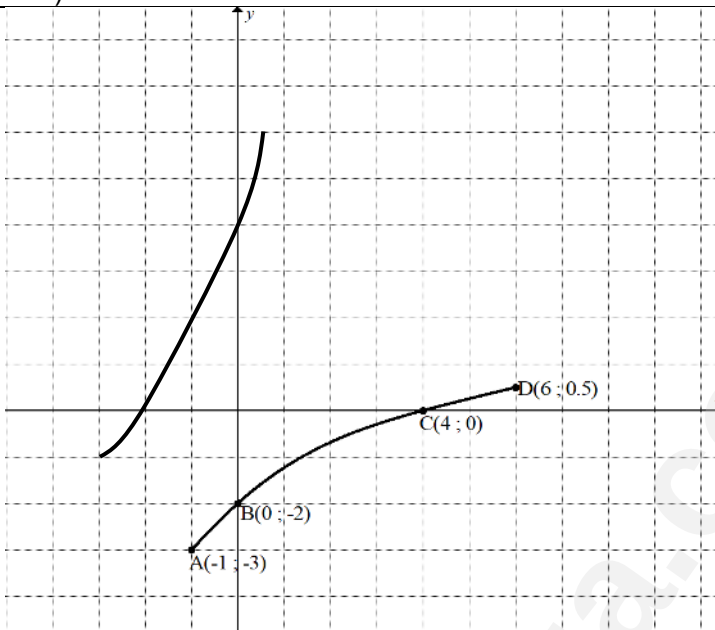
BEAULIEU COLLEGE
PRELIMINARY EXAMINATION 2024
MATHEMATICS PAPER 1

MARKING GUIDELINES

Question 1 (16 marks)		
a) 1	$3x^2 = 4(x+2) + 4$ $3x^2 - 4x - 12 = 0$ $x = \frac{2 \pm 2\sqrt{10}}{3} \text{ or } x = \frac{4 \pm \sqrt{160}}{6}$	Standard form : 1 Use of formula : 1 Answers : 1 (3)
a) 2	$\log_{16} x = \frac{3}{4}$ $x = 16^{\frac{3}{4}}$ $x = 8$	Exponential form : 1 Answer : 1 (2)
b) 1	$p - 8 = \frac{20}{p}$ $p^2 - 8p = 20$ $p^2 - 8p - 20 = 0$ $p = 10 \text{ or } p = -2$	Standard form : 1 Answers : 2 (1 each) (3)
b) 2	$\text{let } \sqrt{x+3} = p$ $\sqrt{x+3} = 10$ $x+3 = 100$ $x = 97$ $\sqrt{x+3} = -2$ N/A	Sub: 1 Solve: 2 (1 each) (3)
c)	$x^2 - 2x + 1 = 5 - x^2$ $2x^2 - 2x - 4 = 0$ $x^2 - x - 2 = 0$ $x = 2 \text{ or } x = -1$ $y = 1 \text{ or } y = 4$ $m_{AB} = \frac{4-1}{-1-2} = -1$ $y - 1 = -1(x - 2)$ $\therefore x + y = 3 \text{ or } y = 3 - x$	Equating functions: 1 Solving quadratic: 2 Calculating gradient: 1 Writing equation: 1 (5)

Question 2 (16 marks)														
a) 1	$28 + 5d = 115$ $d = \frac{87}{5}$ $\therefore T_3 = 28 + 2\left(\frac{87}{5}\right)$ $T_3 = 62,8 \text{ km} / h$	Formulating to find d : 1 Solving for d : 1 Finding T_3 : 1 (3)												
2	$28r^5 = 115$ $r = 1,3$ $\therefore T_5 = 28(1,3)^4$ $T_5 = 87,6 \text{ km} / h$	Formulating to find r : 1 Solving for r : 1 Finding T_5 : 1 (3)												
b) 1	19	No of Cubes: 1 (1)												
2	<table border="0"> <tr> <td>T_1</td> <td>T_2</td> <td>T_3</td> </tr> <tr> <td>1</td> <td>6</td> <td>19</td> </tr> <tr> <td></td> <td>5</td> <td>13</td> </tr> <tr> <td></td> <td>8</td> <td></td> </tr> </table> $2a = 8$ $3(4) + b = 5$ $4 - 7 + c = 1$ $a = 4$ $b = -7$ $c = 4$	T_1	T_2	T_3	1	6	19		5	13		8		Getting to the 1 st and 2 nd differences: 1 $a = 4$: 2 $b = -7$: 1 $c = 4$: 1 (5)
T_1	T_2	T_3												
1	6	19												
	5	13												
	8													
3	$4n^2 - 4n + 4 = 47$ $4n^2 - 4n - 43 = 0$ $n = \frac{1 \pm 2\sqrt{11}}{2}$ $n \notin \mathbb{N}$ $\therefore \text{not possible to build}$ Or $T_3 = 19$ $T_4 = 52$ $\therefore \text{not possible with 47 cubes}$	Equating to 47: 1 Solving for n : 2 Conclusion: 1 Calculating T_4 : 2 Considering T_3 : 1 Conclusion: 1 (4)												

Question 3 (12 marks)								
a)	$V = 50\,000 + 1500 \left[\frac{1 - \left(1 + \frac{0,119}{12}\right)^{-72}}{\frac{0,119}{12}} \right]$ $V = 126\,930$ NB: IF USE Fv: 1/5	P_v formula: 1 Correct Sub (i and n) : 2 Adding R50 000: 1 Answer: 1 (5)						
b)	$OB = 76\,930 \left(1 + \frac{0,119}{12}\right)^{30} - 1\,500 \left(\frac{\left(1 + \frac{0,119}{12}\right)^{30} - 1}{\frac{0,119}{12}} \right)$ $OB = 51\,321,90$	Correct formula OB: 1 Correct $n = 30$: 1 Answer OB: 1 (3)						
c)	<table><tr><td>$Loan : 76\,930$</td><td>$\frac{45\,000 - 25\,608,10}{45\,000}$</td></tr><tr><td>$OB : 51\,321,90$</td><td>$= 43\%$</td></tr><tr><td>$Capital : 25\,608,10$</td><td></td></tr></table>	$Loan : 76\,930$	$\frac{45\,000 - 25\,608,10}{45\,000}$	$OB : 51\,321,90$	$= 43\%$	$Capital : 25\,608,10$		Determine Capital: 1 Determine Paid: 1 Calculating %: 2 (4)
$Loan : 76\,930$	$\frac{45\,000 - 25\,608,10}{45\,000}$							
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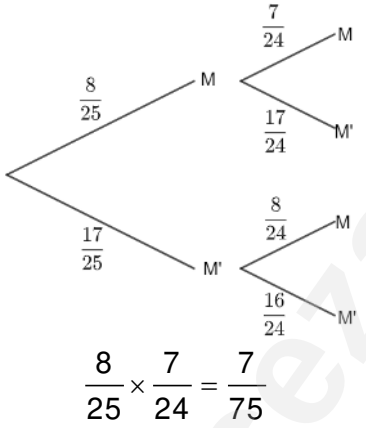
Question 4 (20 marks)		
a)		Coordinate of endpoints: 1 Intercept with axis: 2 (3)
b) 1	$x^2 - 2x - 3 = 0$ $x = 3 \quad x = -1$ $\therefore x \in [-1; 0] \cup [3; \infty)$	Find roots of f: 2 Answer: 2 (4)
2	$x = \frac{-2}{2(-1)} = 1$ $y = 4$ $\therefore k = -4$	Finding coord. of TP: 1 Value of k: 1 (2)
3	$y = g(-x) = 1 - 2^{-x}$ or $y = 1 - \left(\frac{1}{2}\right)^x$	Correct equation : 2 (2)
c) 1	$A(1; -2)$	2 (1 each coordinate) (2)
2	B: $0 = -\frac{9}{x-1} - 2$ $x = -\frac{7}{2}$ $\therefore B\left(-\frac{7}{2}; 0\right)$ C: $y = \frac{9}{0-1} - 2$ $y = 7$ $\therefore C(0; 7)$	Calculating coordinates of B: 2 Calculating coordinates of C: 2 (4)

3	$y = -(x-1) - 2$ $y = -x - 1$	Gradient: 1 y-int: 1 (2)
4	$y = -f(x)$ $\therefore y = \frac{9}{x-1} + 2$	(1)

Question 5 (11 marks)		
a)	$f(x) = 3x^2 - 5x^2$ $f(x+h) = 3x^2 + 3h^2 - 10xh - 5h^2$ $\therefore f(x+h) - f(x) = 3h^2 - 10xh - 5h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(3 - 10x - 5h)}{h} = 3 - 10x$	$f(x+h)$: 2 Set up limit : 1 Factorise & simplify: 1 Correct notation: 1 (5)
b)	$D_x \left(3x^2 - \frac{3}{2x^2} + x^{\frac{2}{3}} \right)$ $D_x \left(3x^2 - \frac{3}{2} x^{-2} + x^{\frac{2}{3}} \right)$ $= 6x + 3x^{-3} + \frac{2}{3} x^{-\frac{1}{3}}$	Transform polynomial: 1 Derivation: 3 (1 each) (4)
c)	$y = \sqrt[4]{x^\pi} - 2a$ $y = x^{\frac{\pi}{4}} - 2a$ $\therefore \frac{dy}{dx} = \frac{\pi}{4} x^{\frac{\pi}{4}-1}$	Convert root to exponential: 1 Derivative: 1 (2)

Question 6 (16 marks)		
a) 1	$y = \log_{\frac{1}{3}} x$	Using log: 1 Correct base: 1 (2)
2	$y = -5$	(2)
3	Reflection on y – axis Reflection on line $y = x$	1 1 (2)
b) 1	$y \in (-\infty; 4]$	(1)
2	$y = a \cdot 3^x + 3$ $q = 3$ $4 = a \cdot 3^{-1} + 3$ $a = 3$	$\therefore g(x) = 3 \cdot 3^x + 3$ or $g(x) = 3^{x+1} + 3$ Value of q : 1 Value of a : 2 (3)
3	$f(x) = -x^2 - 2x + 3$ $f'(x) = -2x - 2$ $-2x - 2 = 1$ $x = -\frac{3}{2}$ $y = \frac{15}{4}$ $\left(-\frac{3}{2}, \frac{15}{4}\right)$	Derivative: 1 Equating to 1: 1 Solving for x : 1 Solving for y : 1 (4)
4	y – int $g : (0; 6)$ y – int $f : (0; 3)$ $\therefore k = 3$	Value of k : 2 (2)

Question 7 (9 marks)		
a)	$T_1 = \frac{2}{3}$ $T_2 = \frac{3}{2}$ $T_3 = \frac{2}{3} \dots$ $\therefore \sum_{k=1}^{100} T_k = 50\left(\frac{2}{3}\right) + 50\left(\frac{3}{2}\right) = \frac{325}{3}$	Determine first three terms at least: 1 Expand n notation: 2 Answer: 1 (4)
b)	$\sum_{n=1}^{16} (3 + 5n + 2^n) = \sum_{n=1}^{16} (3 + 5n) + \sum_{n=1}^{16} (2^n)$ $= [8 + 13 + 18 + \dots] + [2 + 4 + 8 + \dots]$ $= \frac{16}{2}(2(8) + 5(16 - 1)) + \frac{2(2^{16} - 1)}{2 - 1}$ $= 131\,798$	Splitting n: 2 n arithmetic: 1 n geometric: 1 Answer: 1 (5)

Question 8 (16 marks)		
a)	$\frac{15!}{8! 7!} = 6\,435$	Formulate: 1 Answer : 1 (2)
b)	$10! - (2! \times 9!) = 2\,903\,040$ 1 c/a for the final mark provide that subtracted from 10!	Formulate: 2 Answer : 1 (3)
c)	 $\frac{8}{25} \times \frac{7}{24} = \frac{7}{75}$	$\frac{8}{25} : 1$ $\frac{7}{24} : 1$ Answer: 1 (3)
d)	$P(A \text{ or } B) = P(A) + P(B) - P(A) \times P(B)$ $0,6 = 0,5 + p - 0,5p$ $p = 0,2$	Formulating: 1 Substituting: 1 Solving for p: 2 (4)
e)	$P(\text{Scooter} \cap \text{Accident}) = P(\text{Scooter}) \times P(\text{Accident})$ $= \frac{2000}{12000} \times 0,01$ $= 0,0017$	Formulating: 2 $P(\text{Scooter}) : 1$ Answer: 1 (4)

Question 9 (11 marks)		
a) 1	Square: $\frac{x}{4}$ Triangle: $\frac{20-x}{3}$	1 1 (2)
2	$A_{\text{square}} = \frac{x^2}{16}$ $A_{\text{Triangle}} = \frac{1}{2} \left(\frac{20-x}{3} \right)^2 \times \sin 60^\circ$ $= \frac{\sqrt{3}}{4} \left(\frac{20-x}{3} \right)^2$ <i>If used other formula then allocate 1 mark to Pythag's</i> $A = \frac{x^2}{16} + \frac{\sqrt{3}}{4} \left(\frac{20-x}{3} \right)^2$ $= \frac{x^2}{16} + \frac{\sqrt{3}}{36} (20-x)^2$	Area square: 1 Area Triangle: 2 Add areas: 1 Simplify: 1 (5)
3	$A = \frac{x^2}{16} + \frac{\sqrt{3}}{36} (400 - 40x + x^2)$ $= \frac{x^2}{16} + \frac{100\sqrt{3}}{9} - \frac{10\sqrt{3}}{9}x + \frac{\sqrt{3}}{36}x^2$ $\frac{dA}{dx} = \frac{1}{8}x - \frac{10\sqrt{3}}{9} + \frac{\sqrt{3}}{18}x$ $\frac{1}{8}x - \frac{10\sqrt{3}}{9} + \frac{\sqrt{3}}{18}x = 0$ $x = 8,7 \text{ m}$	Expand Polynomial: 1 Derivate: 1 Equate to 0: 1 Solve for x: 1 (4)

Question 10 (7 marks)

Equation of mountain: $y = \frac{1}{10}x$

Height above mountain:

$$h = -\frac{2}{2500}x^2 + \frac{4}{5}x - \frac{1}{10}x$$

$$h = -\frac{2}{2500}x^2 + \frac{7}{10}x$$

$$\frac{dh}{dx} = -\frac{4}{2500}x + \frac{7}{10}$$

$$-\frac{4}{2500}x + \frac{7}{10} = 0$$

$$x = 437,5 \text{ or } x = \frac{875}{2}$$

$$f(437,5) = 196,9 \text{ or } \frac{1575}{8}$$

The balloon will 196,9 m above the lake

- If only calculate TP of $f = BD$: $\left(\frac{2}{7}\right)$

Equation of line: 1

Equation of height: 2

Finding x coord. of maximum: 2

Subst into f : 1

Answer: 1

(7)

Question 11 (16 marks)		
a)	$h'(x) = -3x^2 + 2ax + b$ $-3(-2)^2 + 2a(-2) + b = -12$ $-4a + b = 0$ $h''(x) = -6x + 2a$ $-6\left(\frac{1}{2}\right) + 2a = 0$ $a = \frac{3}{2}$ $b = 6$	Derivate h : 1 Sub -2 and equate to -12: 1 Equation in term of a and b : 1 Second derivative: 1 Equate $h'' = 0$ and find a : 1 Value of b : 1 (6)
b) 1	$P(t) = -\frac{1}{3}t^3 + 60t^2 + 8t + 9500$ $P(0) = 9\,500$	(1)
2	$P'(t) = -t^2 + 120t + 8$ $-t^2 + 120t + 8 = 0$ $t = 120,1 \text{ or } t = -0,1$ The population will reach the maximum in the year 2145.	Derivative = 0: 2 Solve t : 1 Answer: 1 (4)
3	$P''(t) = -2t + 120$ $-2t + 120 = 0$ $t = 60$	Second Derivative: 1 Equate to 0: 1 Answer: 1 (3)
4	$P'(100) = -(100)^2 + 120(100) + 8$ $= 2008$ Answer can also be given as 2008 thousand people per year.	Evaluate P' : 1 Answer: 1 (2)