



eduresourceza.com



ST STITHIAN
BOYS' COLLEGE

GR 12
P1

INSPIRING EXCELLENCE. MAKING A WORLD OF DIFFERENCE

GRADE 12
PRELIMINARY EXAMINATION PAPER 1
MATHEMATICS CORE
MARKING GUIDELINE

Departments of Mathematics

GRADE 12	
PRELIMINARY EXAMINATION PAPER 1	
DATE: September 2023	TIME: 3 hours
TOTAL MARKS: 150	

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 11 questions and 24 pages.
2. A separate information (formula) sheet has been provided to you.
3. Answer ALL the questions and clearly show the calculations you have used to determine your answers.
4. Answers only may not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless specified otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Please note that diagrams are not drawn to scale.
8. It is in your own interest to write legibly and to present your work neatly.
9. PLEASE FILL IN YOUR NAME AND PUT A CROSS IN YOUR TEACHER'S BLOCK ON THE BACK PAGE.

Question 1

Solve each of the following for x , leaving your answers correct to two decimal places if necessary:

(a) $(2x - 5)(x + 4) = -6$ (4)

$$2x^2 + 3x - 20 + 6 = 0 \quad \checkmark \text{ distribute}$$

$$2x^2 + 3x - 14 = 0 \quad \checkmark \text{ trinomial}$$

$$(2x + 7)(x - 2) = 0$$

$$x = -\frac{7}{2} \quad \checkmark \quad \text{or} \quad x = 2 \quad \checkmark$$

(b) $3^{2x+1} = 11$ (2)

$$2x + 1 = \log_3 11 \quad \checkmark \text{ log form}$$

$$2x = 2,183 - 1$$

$$2x = 1,183$$

$$x = 0,59 \quad \checkmark$$

(c) $x(x + 6) \leq 16$ (4)

$$x^2 + 6x - 16 \leq 0 \quad \checkmark \text{ distribute}$$

$$\text{CV: } x^2 + 6x - 16 = 0$$

$$\checkmark (x + 8)(x - 2) = 0$$

$$x = -8 \text{ or } x = 2$$

$$x \in [-8; 2] \quad \text{or} \quad -8 \leq x \leq 2$$

$\checkmark \quad \checkmark$

(d) $(x-7)^{\frac{1}{3}} = -3$ (2)

$$x-7 = (-3)^3 \quad \checkmark$$

$$x-7 = -27$$

$$x = -20 \quad \checkmark$$

(e) $2 + \sqrt{8-x} = x$ (4)

$$\sqrt{8-x} = x-2 \quad \checkmark$$

$$8-x = (x-2)^2$$

$$8-x = x^2 - 4x + 4 \quad \checkmark$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \text{ or } x = -1$$

$\checkmark \quad \checkmark$ excluded answer

[14]

Question 2

- (a) Determine the value(s) of p for which the roots of $x^2 + 2(4-p)x + p^2 = 0$ are non-real (imaginary). (3)

For imaginary roots,

$$b^2 - 4ac < 0$$

$$[2(4-p)]^2 - 4(1)(p^2) < 0$$

$$4(4-p)^2 - 4p^2 < 0$$

$$64 - 32p + 4p^2 - 4p^2 < 0$$

$$64 - 32p < 0$$

$$32p > 64$$

$$p > 2$$

- (b) Prove $\sum_{k=1}^4 \log_5 \left(\frac{k+1}{k} \right) = 1$ (without use of a calculator; show working) (3)

Substituting k values,

$$\text{LHS} = \log_5 \left(\frac{1+1}{1} \right) + \log_5 \left(\frac{2+1}{2} \right) + \log_5 \left(\frac{3+1}{3} \right) + \log_5 \left(\frac{4+1}{4} \right)$$

$$= \log_5 \left(\frac{2}{1} \right) + \log_5 \left(\frac{3}{2} \right) + \log_5 \left(\frac{4}{3} \right) + \log_5 \left(\frac{5}{4} \right)$$

$$= \log_5 \left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \right)$$

$$= \log_5 5$$

$$= 1 = \text{RHS}$$

(c) $3\log_8 x = 12\log_x 8$

(4)

$$3\log_8 x = \frac{12}{\log_8 x}$$

Let $k = \log_8 x$ ✓

$$3k^2 = 12$$

$$k^2 = 4$$

$$k = \pm 2$$
 ✓

$$\log_x 8 = -2 \text{ or } \log_x 8 = 2$$

$$= \frac{1}{64}$$
 ✓

$$x = 64$$
 ✓

[10]

Question 3

- (a) Given $f(x) = 5x - x^2$, find $f'(x)$ from first principles (using the definition of derivative). (5)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5(x+h) - (x+h)^2 - (5x - x^2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5x + 5h - x^2 - 2xh - h^2 - 5x + x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5h - 2xh - h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(5 - 2x - h)}{h} \\
 &= \lim_{h \rightarrow 0} (5 - 2x - h) \\
 &= 5 - 2x - (0) \\
 &= 5 - 2x
 \end{aligned}$$

- (b) Determine $\frac{dy}{dx}$ if $y = \frac{6}{\sqrt[3]{x}} - \frac{x^2 + 5}{x}$ (5)

$$\begin{aligned}
 y &= 6x^{-\frac{1}{3}} - x - 5x^{-1} \\
 \frac{dy}{dx} &= -2x^{-\frac{4}{3}} - 1 + 5x^{-2}
 \end{aligned}$$

- (c) Find the coordinates of the point on the graph of $g(x) = -4x^2 - 6x + 1$, where the gradient of the tangent to the graph is equal to 2. (5)

$$g'(x) = -8x - 6 \quad \checkmark$$

$$-8x - 6 = 2 \quad \checkmark$$

$$-8x = 8$$

$$x = -1 \quad \checkmark$$

$$g(-1) = -4(-1)^2 - 6(-1) + 1 \quad \checkmark$$

$$y = 3 \quad \checkmark$$

$$(-1; 3)$$

[16]

Question 4

Consider the quadratic sequence: $-4 ; 10 ; 28 ; 50 ; 76 ; \dots$

- (a) Find the general (n^{th}) term for the sequence. (4)

$$\begin{array}{ccccccccc} -4 & 10 & 28 & 50 & 76 & & & & \\ & +14 & +18 & +22 & +26 & & & & \\ & & +4 & +4 & +4 & & & & \end{array}$$

$$2a = 4$$

$$a = 2 \quad \checkmark$$

$$3a + b = 14$$

$$3(2) + b = 14$$

$$b = 8 \quad \checkmark$$

$$a + b + c = -4$$

$$(2) + (8) + c = -4$$

$$c = -14 \quad \checkmark$$

$$T_n = 2n^2 + 8n - 14 \quad \checkmark$$

- (b) Calculate the 40th term of the sequence. (2)

$$T_n = 2n^2 + 8n - 14$$

$$T_{40} = 2(40)^2 + 8(40) - 14 \quad \checkmark$$

$$T_{40} = 3506 \quad \checkmark$$

- (c) Determine the first term of the sequence that is greater than 500. (4)

$$2n^2 + 8n - 14 > 500 \quad \checkmark$$

$$2n^2 + 8n - 514 > 0$$

$$n^2 + 4n - 257 > 0 \quad \checkmark$$

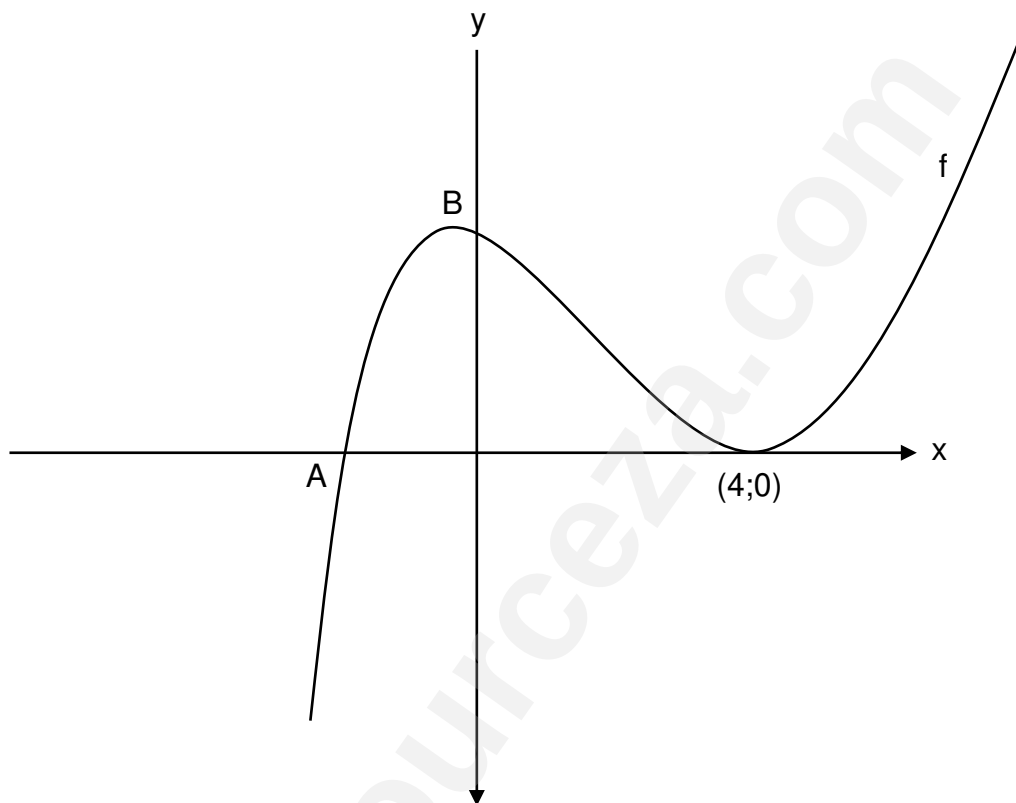
$$n < -18,16 \text{ or } n > 14,1 \quad \checkmark$$

$$n = 15 \quad \checkmark$$

[10]

Question 5

The graph of $f(x) = x^3 - 7x^2 + 8x + d$ is sketched below, with an x-intercept at A, and stationary points at $(4;0)$ and B.



- (a) Calculate the value of d. (2)

$$f(x) = x^3 - 7x^2 + 8x + d$$

$$f(4) = (4)^3 - 7(4)^2 + 8(4) + d = 0 \quad \checkmark$$

$$-16 + d = 0$$

$$d = 16 \quad \checkmark$$

- (b) Determine the x-coordinate of A. (2)

$$x^3 - 7x^2 + 8x + 16 = 0 \quad \checkmark$$

$$x = -1 \text{ or } x = 4$$

$\checkmark$

Answer only: 1 out of 2

- (c) Calculate the x-coordinate of B. (3)

$$f(x) = x^3 - 7x^2 + 8x + 16$$

$$f'(x) = 3x^2 - 14x + 8 = 0 \quad \checkmark$$

$$(3x - 2)(x - 4) = 0$$

$$x = \frac{2}{3} \text{ or } x = 4 \quad \checkmark$$

$$y = f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 7\left(\frac{2}{3}\right)^2 + 8\left(\frac{2}{3}\right) + 16$$

$$y = \frac{500}{27}$$

$$B\left(\frac{2}{3}, \frac{500}{27}\right) \quad \checkmark$$

- (d) Determine the equation of the tangent to f at $x = 5$? (4)

$$f(x) = x^3 - 7x^2 + 8x + 16$$

$$f(5) = (5)^3 - 7(5)^2 + 8(5) + 16$$

$$f(5) = 6 \quad \checkmark$$

$$f'(x) = 3x^2 - 14x + 8$$

$$f'(5) = 3(5)^2 - 14(5) + 8$$

$$f'(5) = 13 \quad \checkmark$$

$$y - 6 = 13(x - 5) \quad \checkmark$$

$$y = 13x - 59 \quad \checkmark$$

- (e) For what values of x is the graph of f concave up? (3)

$$f'(x) = 3x^2 - 14x + 8$$

$$f''(x) = 6x - 14 = 0 \quad \checkmark$$

$$6x = 14$$

$$x = \frac{7}{3} \quad \checkmark$$

$$x \in \left(\frac{7}{3}; \infty\right) \quad \text{or} \quad x > \frac{7}{3} \quad \checkmark$$

- (f) Write down the coordinates of the turning points of h(x) if $h(x) = -f(x - 1)$. (4)

$$f(x) = x^3 - 7x^2 + 8x + 16$$

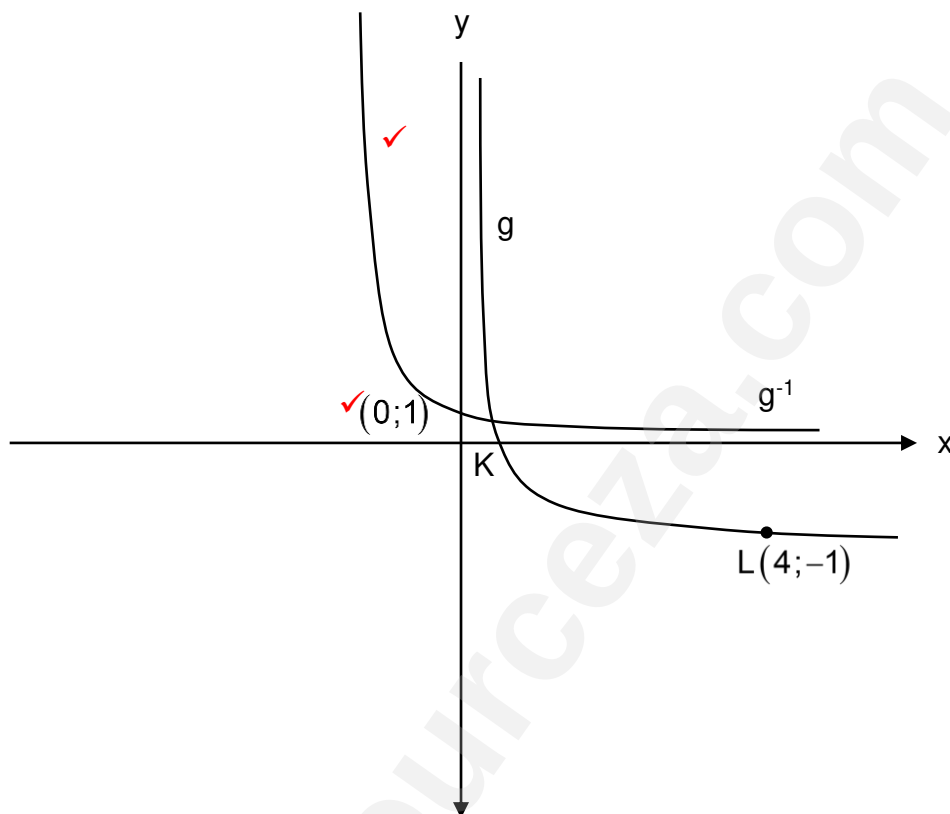
$$-f(x - 1) = -(x - 1)^3 + 7(x - 1)^2 - 8(x - 1) - 16 \quad \checkmark \quad \checkmark$$

$$\text{Turning points will be at } (5; 0) \text{ and } \left(\frac{5}{3}; -\frac{500}{27}\right) \quad \checkmark \quad \checkmark$$

[18]

Question 6

- (a) The graph of the function $g(x) = \log_a x$ is sketched below. Point K is the x-intercept of g and point L(4; -1) lies on g:



- (1) Write down the coordinates of K. (1)

$$\log_a x = 0$$

$$x = 1$$

$$K(1; 0) \quad \checkmark$$

- (2) Calculate the value of a. (2)

$$g(x) = \log_a x$$

$$-1 = \log_a 4 \quad \checkmark$$

$$a^{-1} = 4$$

$$a = \frac{1}{4} \quad \checkmark$$

- (3) Determine the equation of $g^{-1}(x)$ in the form $y = \dots$. (2)

$$g(x) = \log_a x$$

For the inverse, let $x = \log_{\frac{1}{4}} y$ ✓

$$y = \left(\frac{1}{4}\right)^x \quad \checkmark$$

- (4) On the set of axes given on the previous page, sketch the graph of $g^{-1}(x)$, labelling any intercept(s) with the axes. (2)

- (5) Write down the equation of the asymptote of $g^{-1}(x)$. (1)

$$y = 0 \quad \checkmark$$

- (6) Write down the new coordinates of point L if $g(x)$ is transformed onto $j(x)$ by $j(x) = g(x+1) + 2$. (2)

$$L(4; -1) \rightarrow L'(3; 1) \quad \checkmark \checkmark$$

- (b) Given $f(x) = bx^2$, calculate the value of b if $f^{-1}(1) = f'(4)$. (6)

$$f(x) = bx^2$$

$$f'(x) = 2bx \quad \checkmark$$

$$f'(4) = 2b(4) = 8b \quad \checkmark$$

For the inverse, let $x = by^2$

$$y = f^{-1}(x) = \sqrt{\frac{x}{b}} \quad \checkmark$$

$$f^{-1}(1) = \sqrt{\frac{1}{b}} \quad \checkmark$$

$$8b = \sqrt{\frac{1}{b}}$$

$$64b^2 = \frac{1}{b}$$

$$b^3 = \frac{1}{64} \quad \checkmark$$

$$b = \sqrt[3]{\frac{1}{64}} = \frac{1}{4} \quad \checkmark$$

[16]

Question 7

- (a) A farmer buys a new tractor for R1,35 million. The value of the tractor depreciates annually at a rate of 5,85% on the reducing balance method. Calculate the book value of the tractor after 8 years. (2)

$$\begin{aligned} A &= 1\,350\,000(1 - 5,85\%)^8 \quad \checkmark \\ &= R833\,482,27 \quad \checkmark \end{aligned}$$

- (b) How many **years** will it take for an amount of R54 000 to accrue (grow) to a value of R137 683,56 in an investment account earning interest at a rate of 8,60% compounded quarterly? (4)

$$137\,683,56 = 54\,000 \left(1 + \frac{8,6\%}{4} \right)^{4n} \quad \checkmark$$

$$2,55 = \left(1 + \frac{8,6\%}{4} \right)^{4n} \quad \checkmark$$

$$4n = \log_{\left(1 + \frac{8,6\%}{4} \right)} 2,55 \quad \checkmark$$

$$4n = 44$$

$$n = 11 \text{ years} \quad \checkmark$$

- (c) Thembi buys a house costing R1,75 million using her savings of R560 000 as a deposit. She obtains a home loan for the balance of the purchase price at an interest rate of 5,25% compounded monthly over 20 years. She pays her first monthly instalment of R8 018,75 on 31 March 2023.

- (1) Calculate what the balance outstanding on the home loan will be on 28 February 2039, 16 years after the loan is granted. (4)

$$BO = 1\,190\,000 \left(1 + \frac{5,25\%}{12}\right)^{16 \times 12} - 8\,018,75 \left[\frac{\left(1 + \frac{5,25\%}{12}\right)^{16 \times 12} - 1}{\frac{5,25\%}{12}} \right]$$

$$= R346\,489,84$$

PV METHOD WILL HAVE MAX $\frac{3}{4}$ AS PER IEB FINALS MARKING

- (2) Hence, determine the total amount of interest that Thembi will have paid over the first 16 years of the home loan. (4)

$$\begin{aligned} \text{Capital paid} &= 1\,190\,000 - 346\,489,84 \\ &= 843\,510,16 \end{aligned}$$

$$\begin{aligned} \text{Total instalments paid} &= 8\,018,75 \times 16 \times 12 \\ &= 1\,539\,600 \end{aligned}$$

$$\begin{aligned} \text{Total interest paid} &= 1\,539\,600 - 843\,510,16 \\ &= R696\,089,84 \end{aligned}$$

- (3) Calculate what the monthly instalment would be if Thembi obtained a home loan for the whole purchase price of R1,75 million (no deposit) at an interest rate of 5,25% compounded monthly over 15 years. (3)

$$1\,750\,000 = x \left[\frac{1 - \left(1 + \frac{5,25\%}{12}\right)^{15 \times 12}}{\frac{5,25\%}{12}} \right]$$

$$x = \text{R}14\,067,86$$

[17]

Question 8

The second term of an arithmetic series is -6 and the seventh term is 19 .

- (a) Find the general (n^{th}) term for the series. (4)

$$T_2 = -6 \text{ and } T_7 = 19$$

$$5d = 19 - (-6) \quad \checkmark$$

$$5d = 25$$

$$d = 5 \quad \checkmark$$

$$a = -11 \quad \checkmark$$

$$T_n = -11 + (n-1)(5)$$

$$T_n = 5n - 16 \quad \checkmark$$

- (b) How many terms are there in the series if the last term is 544 ? (2)

$$T_n = 5n - 16$$

$$544 = 5n - 16 \quad \checkmark$$

$$5n = 560$$

$$n = 112 \quad \checkmark$$

- (c) Calculate the sum of the **last 100 terms** of the sequence. (5)

$$T_{13} = 5(13) - 16 \quad \checkmark$$

or

$$S_{12} = \frac{12}{2} [2(-11) + (12-1)(5)] \quad \checkmark$$

$$T_{13} = 65 - 16$$

$$S_{12} = 198 \quad \checkmark$$

$$T_{13} = 49 \quad \checkmark$$

$$S_{112} = \frac{112}{2} [2(-11) + (112-1)(5)] \quad \checkmark$$

$$S_{100} = \frac{100}{2} [2(49) + (100-1)(5)] \quad \checkmark$$

$$S_{112} = 29848 \quad \checkmark$$

$$= 29650 \quad \checkmark$$

$$S_{100} = 29848 - 198$$

$$S_{100} = 29650 \quad \checkmark$$

[11]

Question 9

The sum of the first 3 terms of a geometric series is 30 and the first term is 40.

- (a) Prove that the series converges. (4)

$$T_n = ar^{n-1}$$

$$T_1 = a = 40 \text{ and } T_2 = ar = 40r \text{ and } T_3 = ar^2 = 40r^2$$

$$40 + 40r + 40r^2 = 30 \quad \checkmark$$

$$40(r^2 + r + 1) = 30$$

$$r^2 + r + 1 = 0,75$$

$$r^2 + r + 0,25 = 0$$

$$(r + 0,5)^2 = 0 \quad \checkmark$$

$$r = -\frac{1}{2} \quad \checkmark$$

$$-1 < \left(r = -\frac{1}{2}\right) < 1 \text{ and the series converges} \quad \checkmark$$

- (b) Calculate the sum to infinity for the series. (2)

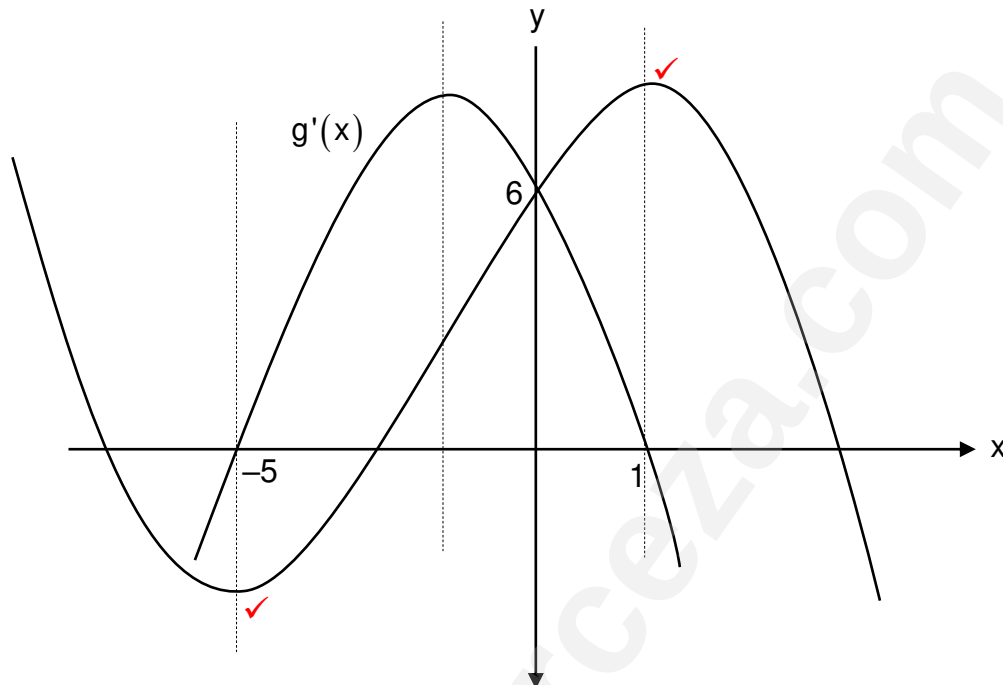
$$S_{\infty} = \frac{40}{1 - \frac{1}{2}} \quad \checkmark$$

$$S_{\infty} = \frac{80}{3} = 26,67 \quad \checkmark$$

[6]

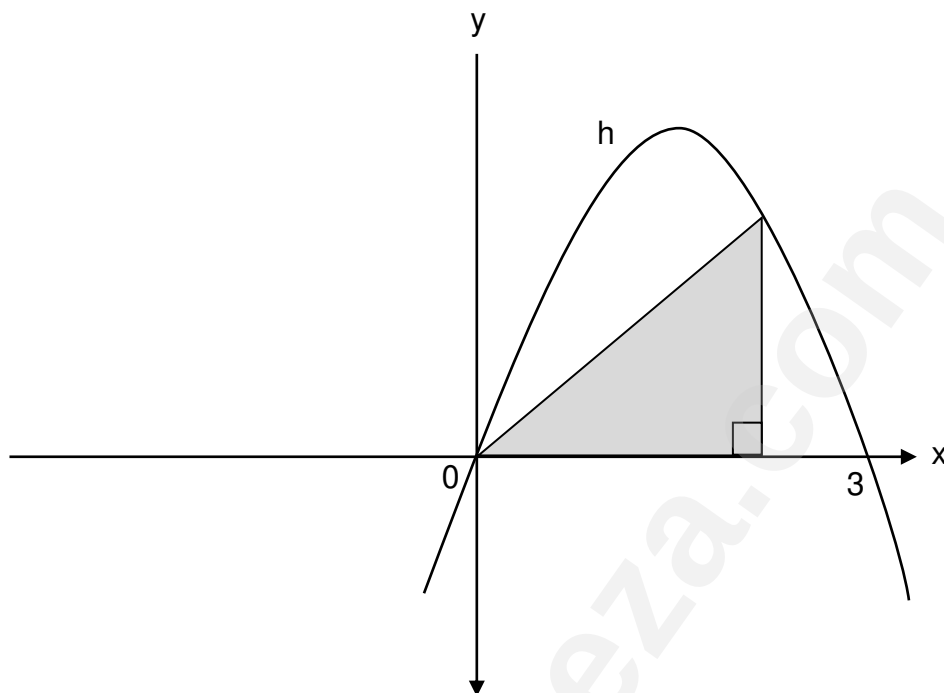
Question 10

- (a) Sketched below, is the graph of the function $g'(x)$, with x-intercepts at -5 and 1 , and a y-intercept at 6 :



- (1) Write down the x-coordinate(s) of the turning point(s) of $g(x)$. (2)
 $x = -5$ and $x = 1$
- (2) Write down the x-coordinate of the point of inflection of $g(x)$. (1)
 $x = -2$
- (3) For what values of x is the graph of $g(x)$ decreasing? (2)
 $x \in (-\infty; -5) \cup (1; \infty)$ or $x < -5$ and $x > 1$
- (4) For what values of x is the graph of $g(x)$ concave up? (2)
 $x \in (-\infty; -2)$ or $x < -2$
- (5) On the set of axes given above, sketch the graph of $g(x)$. (2)

- (b) Sketched below is the graph of $h(x) = -2x^2 + 6x$:



What are the dimensions and area of the largest shaded right-angled triangle with **maximum area**, that can be drawn from the origin, between h , the positive x -axis and the positive y -axis? (7)

$$\begin{aligned}
 A &= \frac{1}{2}(\text{base})(\text{height}) \\
 &= \frac{1}{2}(x)(-2x^2 + 6x) \\
 &= -x^3 + 3x^2
 \end{aligned}$$

$$\begin{aligned}
 A' &= -3x^2 + 6x = 0 \\
 &= -3x(x - 2) = 0 \\
 x &\neq 0 \text{ or } x = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{base} &= 2 \text{ units} \\
 \text{height} &= 4 \text{ units} \\
 A_{\max} &= 4 \text{ units}^2
 \end{aligned}$$

[16]

TRIAL AND ERROR 6/7 MAXIMUM AS PER IEB FINALS MARKING

Question 11

- (a) Events A and B occur as follows:

$$P(A) = 0,45$$

$$P(B) = 0,26$$

If events A and B are independent, calculate $P(A \cup B)$. (4)

For independent events, $P(A) \times P(B) = P(A \cap B)$

$$P(A \cap B) = 0,45 \times 0,26$$

$$P(A \cap B) = 0,117 \quad \checkmark$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0,45 + 0,26 - 0,117 \quad \checkmark \checkmark$$

$$P(A \cup B) = 0,593 \quad \checkmark$$

- (b) Consider the letters of the word: P O S S I B I L I T Y

- (1) If the letters are arranged in any order to form different words, determine in how many unique (different) ways the letters of the word can be arranged? (You do not need to simplify your answer) (3)

$$N(\text{arrangements}) = \frac{11!}{2! \times 3!} = 3326400 \quad \checkmark$$

- (2) If the letters are arranged in any order to form different words, calculate the probability that the word formed starts with an I and ends with a S? (4)

$$P(I \dots S) = \frac{\frac{9!}{2!}}{\frac{11!}{2! \times 3!}} = \frac{181440}{3326400} = \frac{3}{55} = 0,0545$$

- (c) n people, numbered 1, 2, 3, 4, etc. are arranged randomly in a line ($n \in \mathbb{N}, n > 3$).

Find the probability, in terms of n , that the people numbered 1, 2 and 3 are standing next to each other.

(4)

(You do not need to simplify your answer)

$$P(1+2+3 \text{ together}) = \frac{\frac{(n-3+1)!}{3!}}{n!} = \frac{\frac{(n-2)!}{3!}}{n!} = \frac{(n-2)!}{n!}$$

OR

$$P(1+2+3 \text{ any order}) = \frac{(n-2)! \times 3!}{n!}$$

[15]

END OF PAPER

THIS PAGE IS INTENTIONALLY BLANK

eduresourceza.com

FOR OFFICIAL USE ONLY
MARK RECORD SHEET

Full Name: _____

Teacher (mark with an X):

☐ Mr Stolk ☐ Mr Lombard ☐ Mrs M^cLachlan
☐ Mrs Wilde ☐ Mr Lebabo ☐ Mrs Frankiskos

Question	Algebra & Equations	Patterns & Sequences	Finance	Functions & Graphs	Calculus	Probability
1	/16					
2	/10					
3					/15	
4		/10				
5				/18		
6				/16		
7			/17			
8		/11				
9		/6				
10					/16	
11						/15
TOTAL	/26	/27	/17	/34	/31	/15
					/150	%