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St Mary's School
Waverley
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MATHEMATICS PAPER II
PRELIMINARY EXAMINATION

21 September 2023

Form V

Marks: 150

Examiner: Miss N. Sehannie

Time: 3 Hours

Moderators: Miss L. Makhalanyane

Mrs S. Vermeulen

Mrs I. Zlobinsky-Roux

Examination number:

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PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 27 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Write your name on the cover page and on the next page.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical scientific calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. There is additional space included at the end of the paper. If you run out of space for a question, use the additional space. Clearly indicate the number of your answer should you use this extra space.

FORM V MATHEMATICS PRELIMINARY EXAMINATION – PAPER II
DATE
ANALYSIS SHEET

<u>Question</u>	<u>Topic</u>	<u>Mark obtained</u>
1	Statistics [10]	
2	Analytical Geometry [16]	
3	Trigonometry [15]	
4	Euclidean Geometry [7]	
5	Euclidean Geometry [8]	
6	Euclidean Geometry [12]	
7	Statistics [7]	
TOTAL SECTION A	[75]	
8	Statistics [4]	
9	Analytical Geometry [8]	
10	Trigonometry [18]	
11	Trigonometry [7]	
12	Euclidean Geometry [13]	
13	Analytical Geometry [18]	
14	Measurement [7]	
TOTAL SECTION B	[75]	

Scale of Achievement:

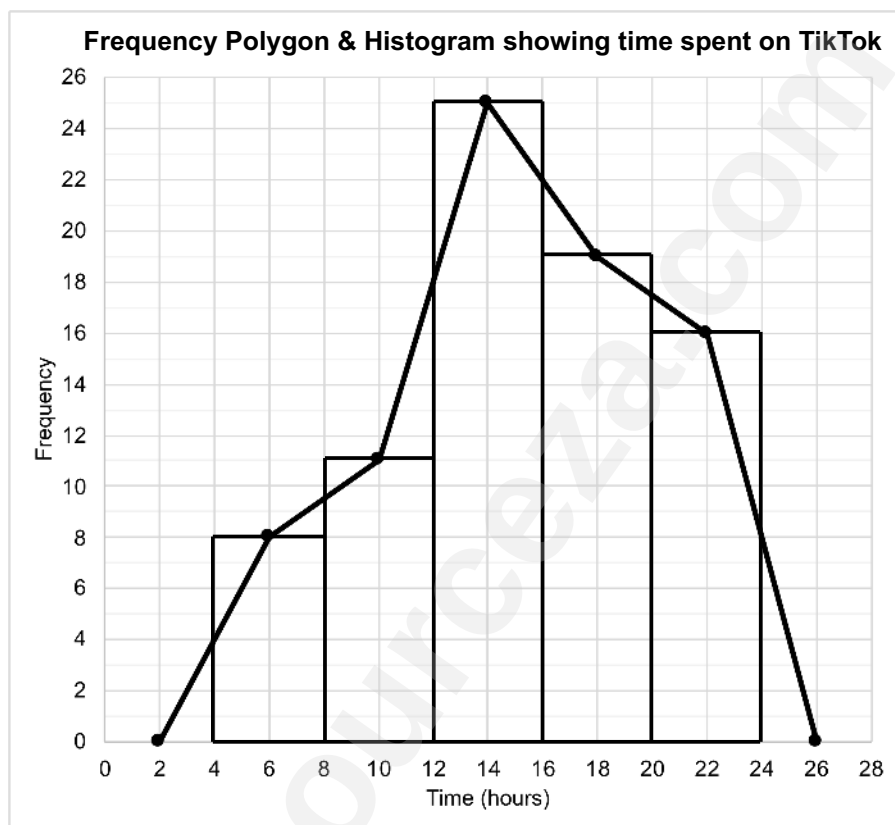
Rating Code	Description of Competence	Marks (%)
7	Outstanding	80 - 100
6	Meritorious	70 – 79
5	Substantial	60 – 69
4	Adequate	50 – 59
3	Moderate	40 – 49
2	Elementary	30 – 39
1	Not achieved	0 – 29

TOTAL:

150 = %

SECTION A**QUESTION 1**

A survey was done on how many hours the Form V students at a particular school spent on TikTok in a particular week. The data collected is represented in the frequency polygon and histogram below:



- (a) Use your calculator to calculate the estimated mean time the Form V students spent on TikTok in the week surveyed.

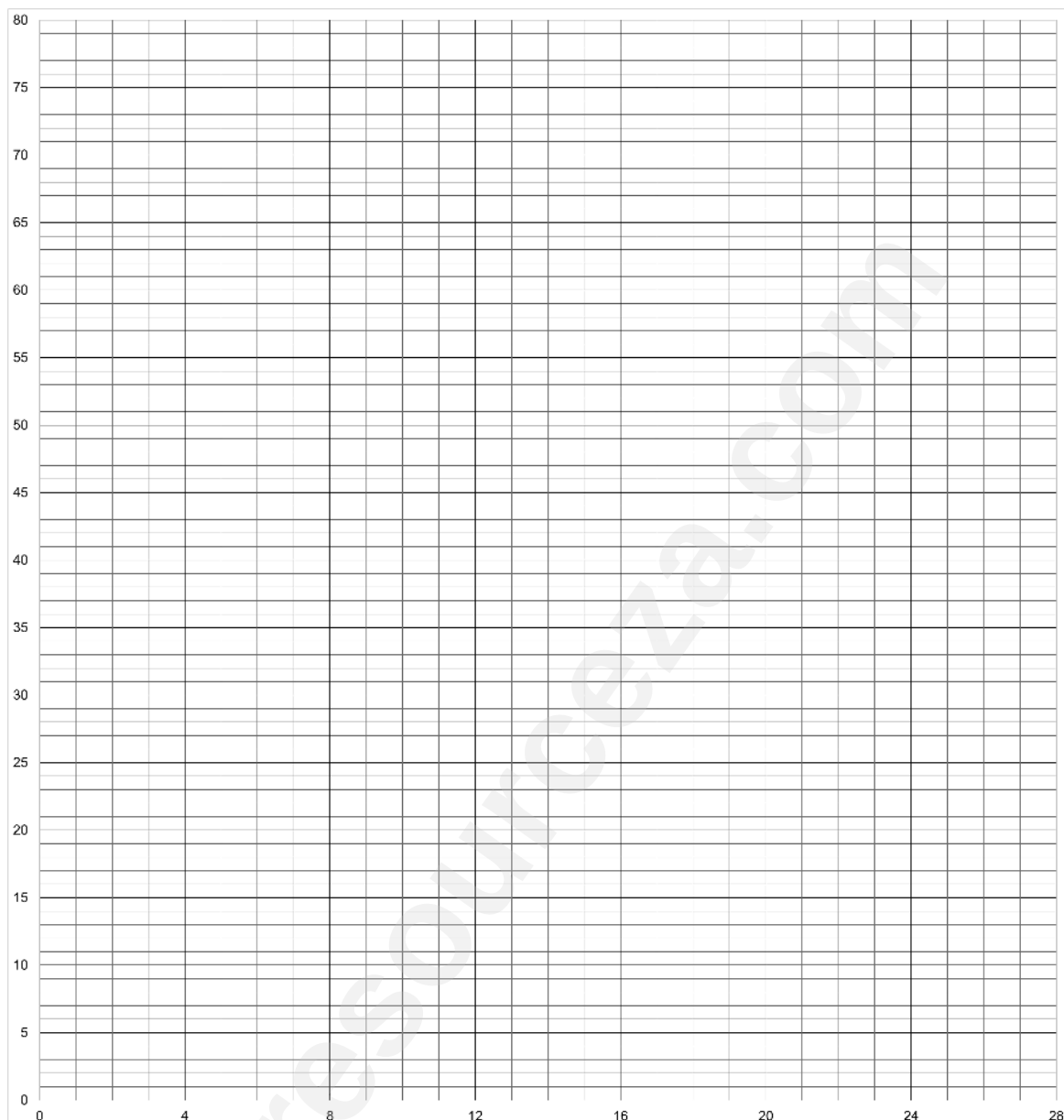
(2)

- (b) Use the frequency polygon and histogram to complete the table below:

	Frequency	Cumulative Frequency
$4 \leq t < 8$	8	
$8 \leq t < 12$		19
$12 \leq t < 16$		
$16 \leq t < 20$		
$20 \leq t < 24$		

(2)

- (c) Hence, draw a cumulative frequency curve on the grid below representing the data.



(4)

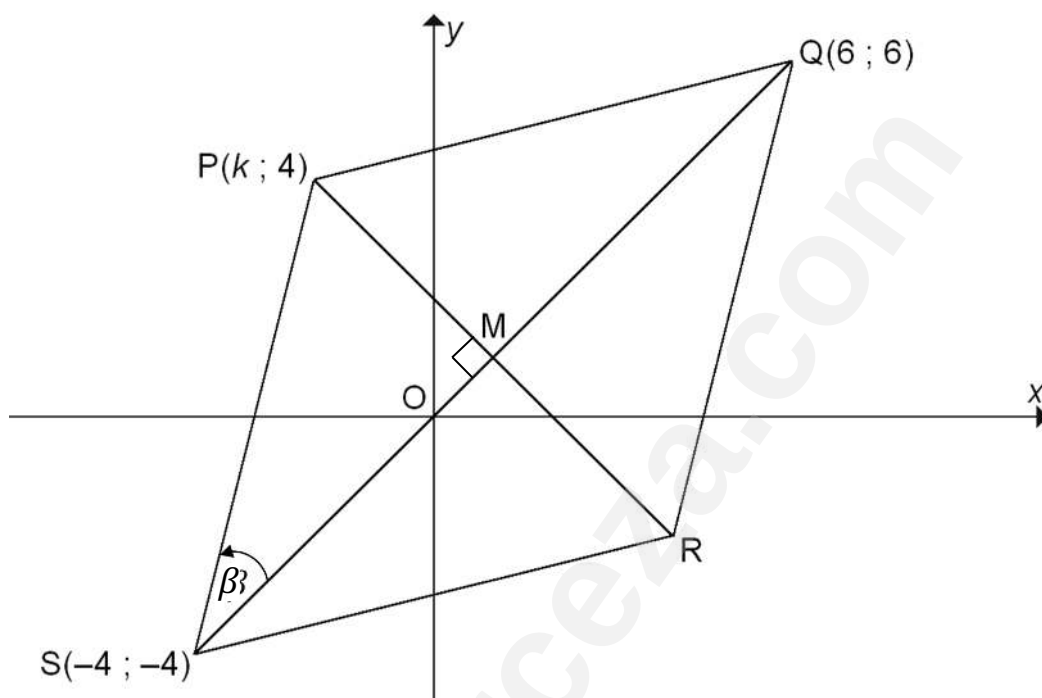
- (d) The World Health Organisation recommends that adolescents are to have no more than 2 hours of screen time **per day**. Using your cumulative frequency curve, determine how many Form V students are spending too much time on TikTok.

(2)

[10]

QUESTION 2

$P(k; 4)$, $Q(6; 6)$, R and $S(-4; -4)$ are the vertices of rhombus PQRS. M is the point of intersection of diagonals PR and QS . $\angle OSP = \beta$.



- (a) Determine the coordinates of M .

(2)

- (b) Show that $k = -2$.

(3)

- (c) Hence, determine the coordinates of R.

(3)

- (d) Determine the equation of SR in the form $y = mx + c$.

(3)

- (e) Calculate the size of β .

(5)

[16]

QUESTION 3

- (a) Simplify without the use of a calculator:

$$\frac{\cos 190^\circ - \sin(90^\circ - \alpha)}{\sin 100^\circ + \cos(-\alpha)}$$

(5)

- (b) Prove that: $\frac{\cos 3\alpha}{\cos \alpha} = 1 - 4\sin^2 \alpha$

(6)

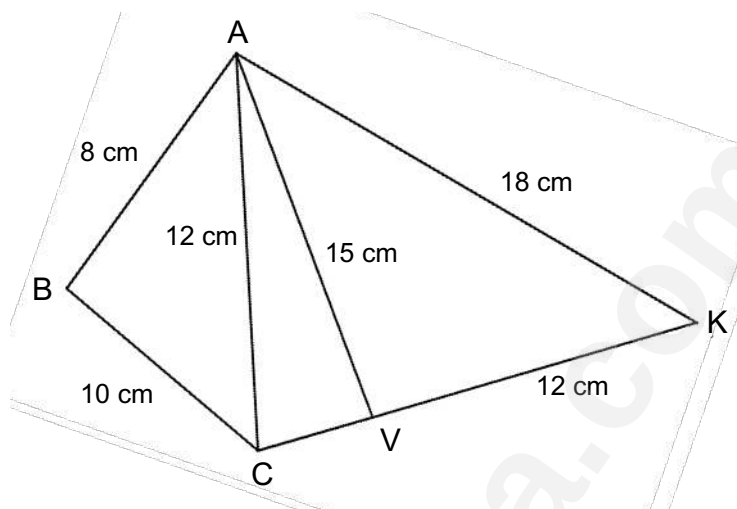
- (c) Solve for x correct to two decimal places:
 $3\tan(2x + 10^\circ) + 5 = 0$ if $0^\circ \leq x \leq 180^\circ$

(4)

[15]

QUESTION 4

In $\triangle ACK$, $AK = 18$ cm. A line is drawn from A to point V on CK such that $AV = 15$ cm and $VK = AC = 12$ cm. A and C are joined to point B such that $AB = 8$ cm and $BC = 10$ cm.



- (a) Prove that $\triangle AKV$ is similar to $\triangle CAB$.

(4)

- (b) Hence, prove that $ABCV$ is a cyclic quadrilateral.

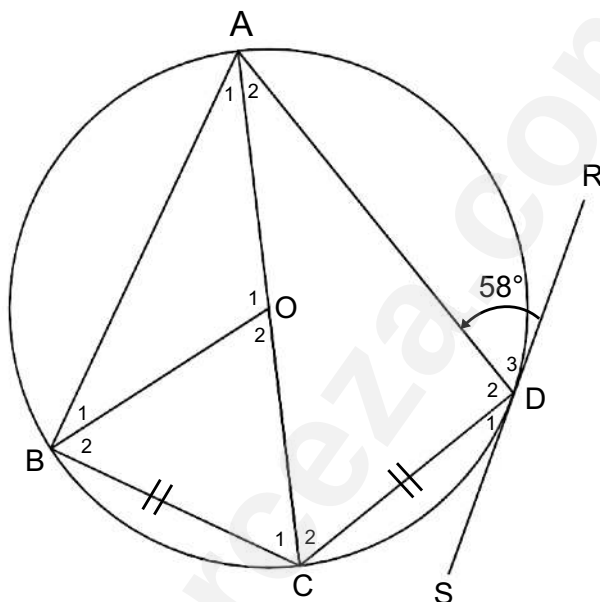
(3)

[7]

QUESTION 5

In the diagram below:

- O is the centre of the circle.
- SDR is a tangent to circle ABCD at D.
- OB and AC are straight lines.
- $BC = CD$ and $\hat{D}_3 = 58^\circ$.



(a) Calculate, with reasons, the size of:

(1) $\hat{A}_1$

(4)

(2) $\hat{O}_2$

(1)

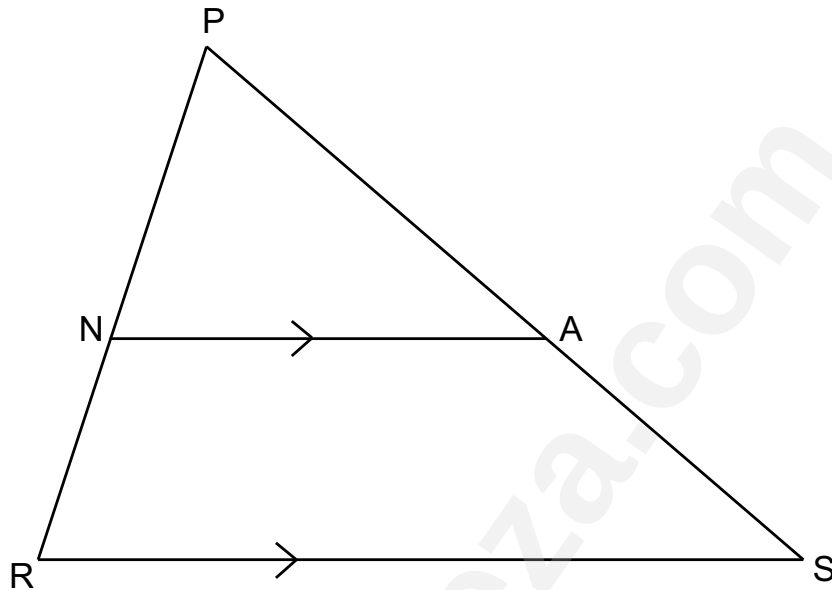
(b) Prove that CD is a tangent to circle OBC at C.

(3)

[8]

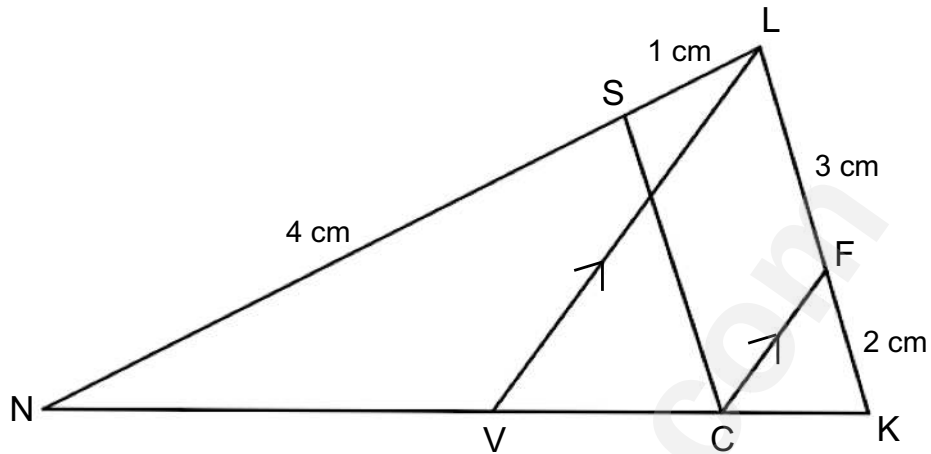
QUESTION 6

- (a) $\triangle PRS$ drawn below has point N on PR and point A on PS such that $NA \parallel RS$.



Prove the theorem which states that: $\frac{PN}{NR} = \frac{PA}{AS}$

- (b) In triangle LNK below, $LV \parallel FC$. $KV = NV$ and $FK = 2\text{ cm}$, $LF = 3\text{ cm}$, $LS = 1\text{ cm}$ and $NS = 4\text{ cm}$



- (1) Determine, with reasons, $KC:CV$.

(2)

- (2) Determine $\frac{CN}{CK}$.

(2)

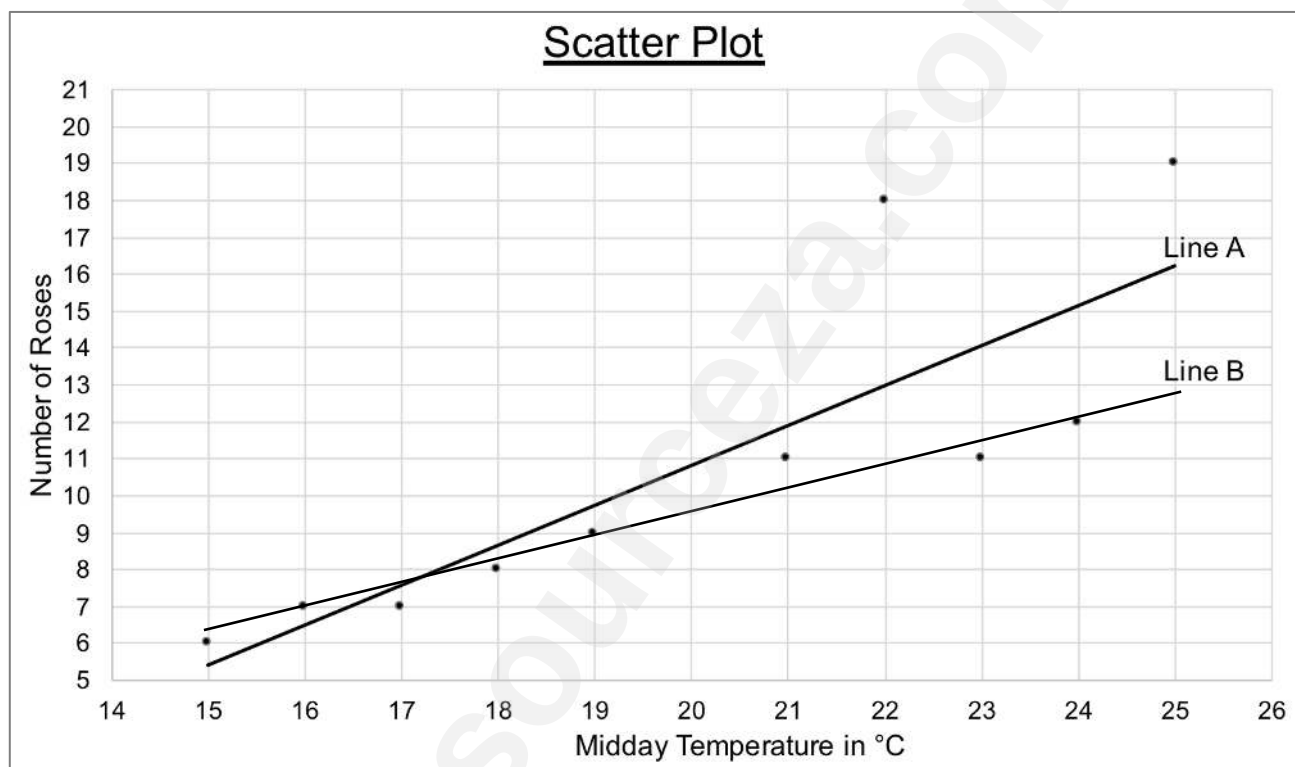
- (3) Prove that $CS \parallel KL$.

(2)

[12]

QUESTION 7

A rose grower keeps a record of the number of roses on a particular rose bush as the midday temperature (in °C) changes over the course of the year. The points (22 ; 18) and (25 ; 19) are considered to be outliers. Two lines of best fit have been drawn on the scatter plot below and each represent the trend of the data depending on whether the outliers are included or not. The data has been summarised in the table below the scatter plot.



Midday Temperature in °C	15	16	17	18	19	21	22	23	24	25
Number of Roses	6	7	7	8	9	11	18	11	12	19

- (a) Use your calculator to determine the equation of the line of best fit in the form $\hat{y} = A + Bx$ if the outliers are excluded. Give your answer correct to two decimal places.

- (b) Calculate the correlation coefficient for the data if the outliers are excluded. Give your answer correct to two decimal places.

(1)

- (c) The equation of the regression line if the outliers are included is $\hat{y} = -10,84 + 1,08x$ and the correlation coefficient is $r = 0,84$. Which line, A or B, represents the line of best fit for the given data if the outliers are included and which line represents the line of best fit for the given data if the outliers are excluded? Your answer must be supported by referring to the gradient of each line as well as the correlation coefficients.

(4)

[7]

75 marks

SECTION B**QUESTION 8**

In a particular data set, the interval of data values which lie within one standard deviation of the mean is given by: $(p^2 + 4p ; 3p^2 - 2p)$

Determine the mean and standard deviation of the data set in terms of p .

[4]**QUESTION 9**

The equation of a circle is $x^2 + 2kx + y^2 - 6y + 4 = 0$.

(a) Determine the radius of the circle in terms of k .

(4)

- (b) If the radius is 3 units and $k < 0$, determine the coordinates of the centre of the circle.

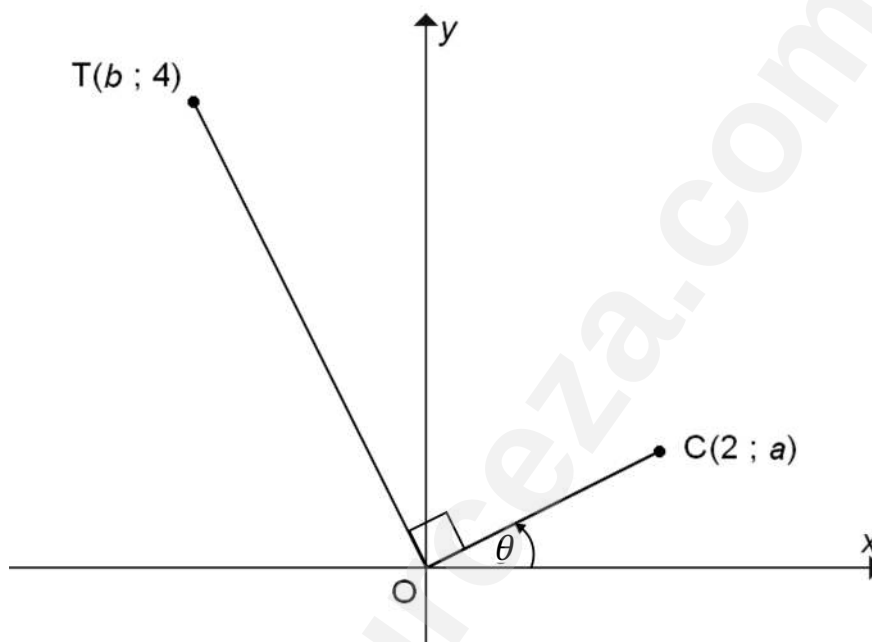
(4)

[8]

QUESTION 10

(a) In the diagram below:

- $T(b ; 4)$ and $C(2 ; a)$ are points in the Cartesian plane
- $\hat{TOC} = 90^\circ$ and the angle which OC makes with the positive x -axis is θ .
- It is further given that $\sqrt{5} \cos \theta - 2 = 0$.



Determine, without the use of a calculator, the numerical value of:

(1) a

(2) b

(4)

(b) Determine the general solution for $4\sin x \cdot \cos x = 3\sin^2 x$.

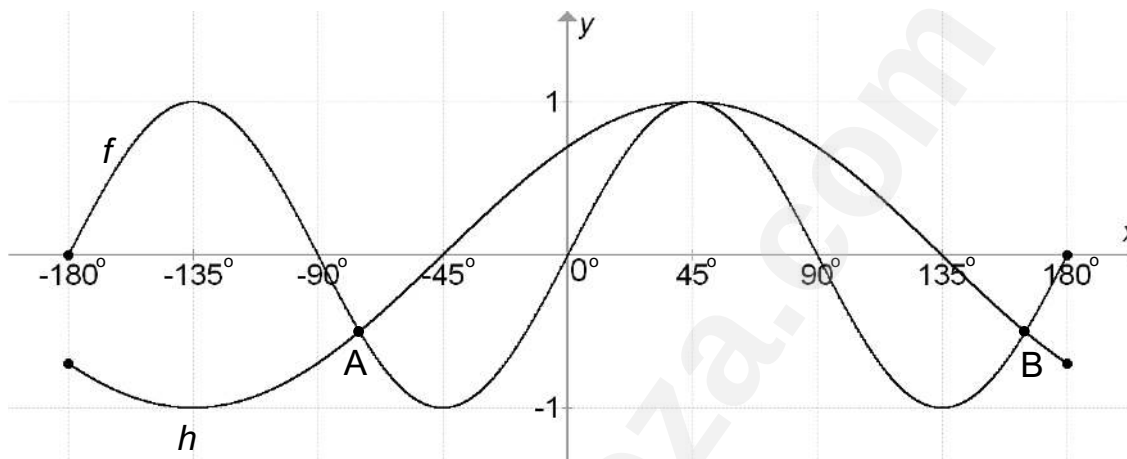
(5)

- (c) If $\cos 40^\circ = m$, determine $\cos 10^\circ + \cos 70^\circ$ in terms of m and without the use of a calculator.

QUESTION 11

In the diagram below:

- The graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45^\circ)$ are sketched for $-180^\circ \leq x \leq 180^\circ$.
- $A\left(-75^\circ; -\frac{1}{2}\right)$ and B are two of the points of intersection between f and h .



- (a) Write down the value of the x -coordinate of B.

(1)

- (b) For which values of x is $f''(x) > 0$?

(2)

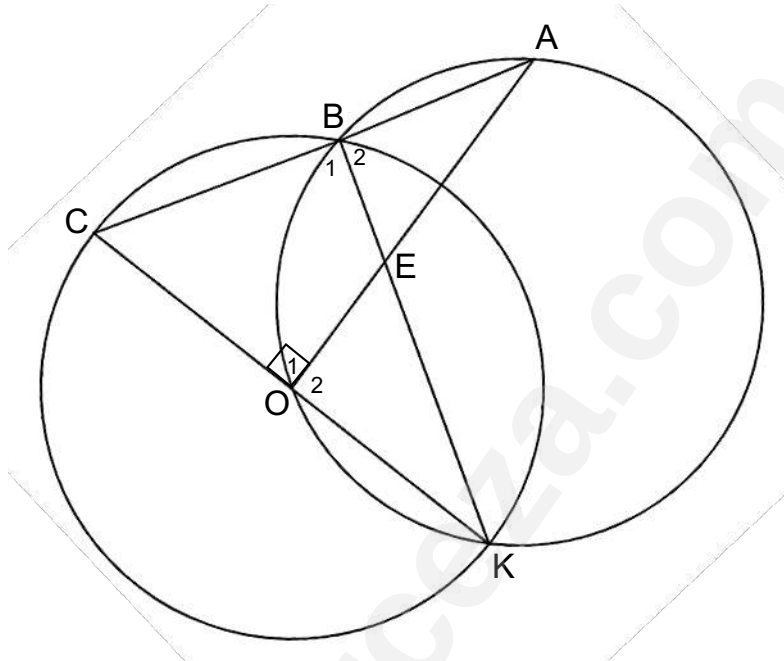
- (c) Use the graphs to solve $4\sin x \cdot \cos x \leq \sqrt{2}(\cos x + \sin x)$. Show all working details.

(4)

[7]

QUESTION 12

BEK is a common chord for two intersecting circles. O is the centre of circle BCK. Chord CB is produced to A on the circumference of the other circle such that $\angle AOB = 90^\circ$. KOC, BEK and AEO are straight lines. KOC, BEK and AEO are straight lines.



Prove, with reasons, that:

- (a) $KC \cdot OC = BC \cdot AC$

(b) $2OC^2 = BC.AC$

(2)

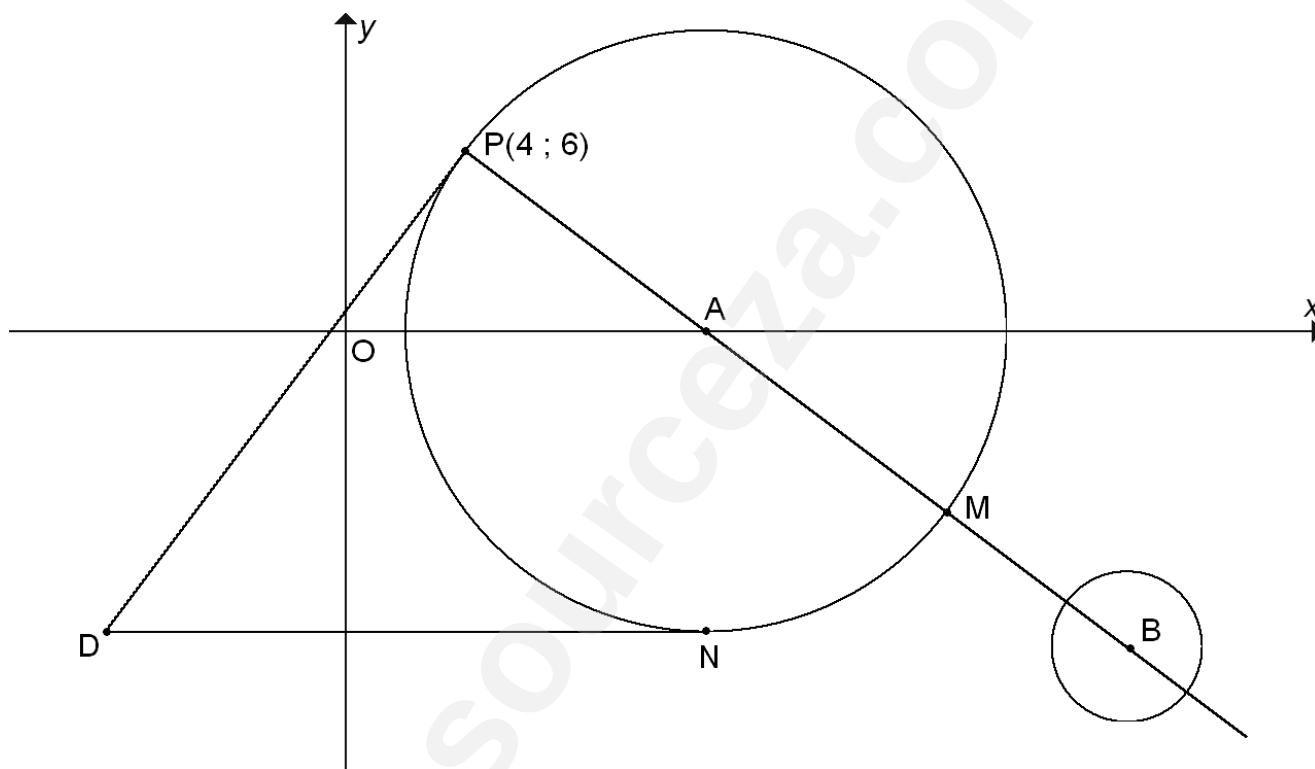
(c) AO bisects $\hat{CAK}$

(6)

[13]

QUESTION 13

- PM is the diameter of circle with centre A where P(4 ; 6).
- DP and DN are tangents to circle with centre A such that DN is parallel to the x-axis.
- The equation of **PM** is **$4y + 3x - 36 = 0$** .
- Circle with centre B has equation $(x - k)^2 + (y - z)^2 = \frac{25}{4}$ and can move to any position provided that its centre remains on PM produced.



- (a) Show that the equation of circle with centre A is $(x - 12)^2 + y^2 = 100$.

(4)

- (b) Determine the equation of tangent PD in the form $y = mx + c$.

(3)

- (c) Determine the coordinates of D.

(3)

- (d) Show that $k = -\frac{4}{3}z + 12$.

(2)

- (e) Hence, for which values of z will circle with centre A and circle with centre B touch internally?

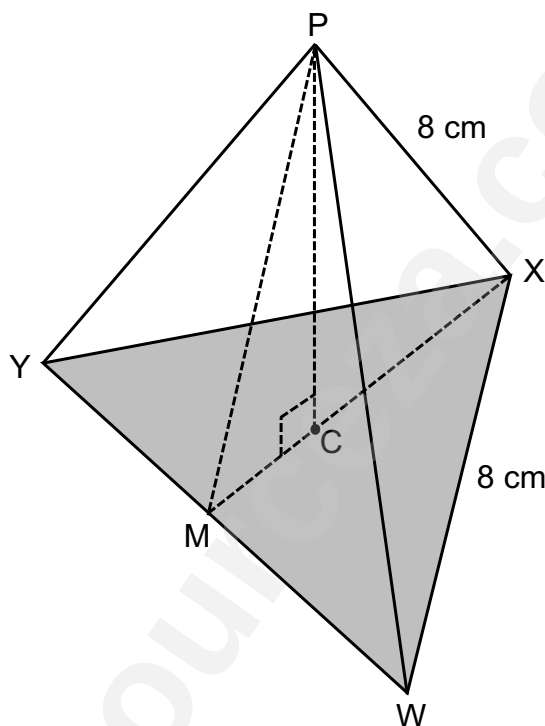
(6)

[18]

QUESTION 14

A regular tetrahedron is a special type of pyramid where all 4 faces are equilateral triangles. The regular tetrahedron below has sides of 8 cm. The apex of the tetrahedron is P. In $\triangle WXY$, XM is the median of side YW (i.e. YM = MW) which has been drawn from vertex X. C is the centroid of $\triangle WXY$.

Hint: The centroid divides the median in the ratio 2:1.



Determine the volume of the tetrahedron if $V = \frac{1}{3}(\text{Area of base})(\text{Perpendicular height})$

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[7]

75 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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