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FORM V PAPER II

PRELIMINARY EXAMINATION 2023 – MEMO

SECTION A

QUESTION 1

(a)

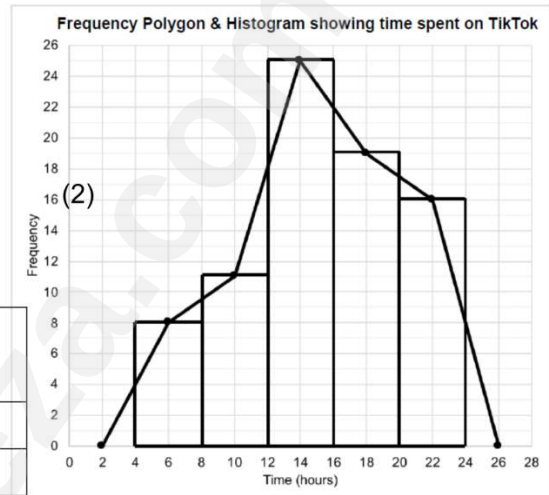
$$\text{Est. } \bar{x} = \frac{1\ 202}{79} \quad \checkmark_{79}$$

$$\text{Est. } \bar{x} = 15,22 \text{ hrs} \quad \checkmark$$

Award 2/2 for answer only. Don't penalise units.

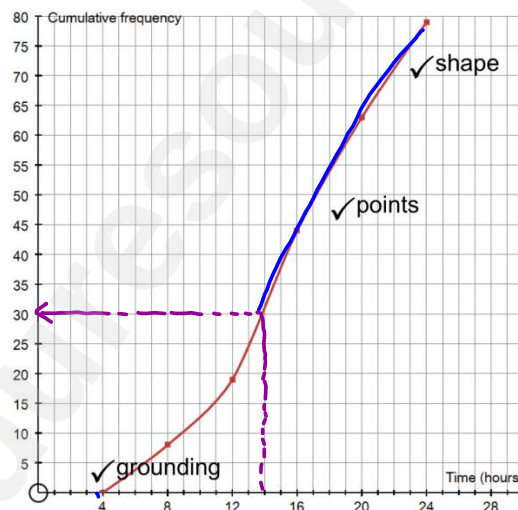
(b)

Time (hours)	Frequency	Cumulative Frequency
$4 < t \leq 8$	8	8
$8 < t \leq 12$	11	19
$12 < t \leq 16$	25	44
$16 < t \leq 20$	19 ✓	63 ✓
$20 < t \leq 24$	16	79



(2)

(c)



(4)

(d) $79 - 31 = 48$ ✓

✓ both axes labels

Accept a range of 28 - 34

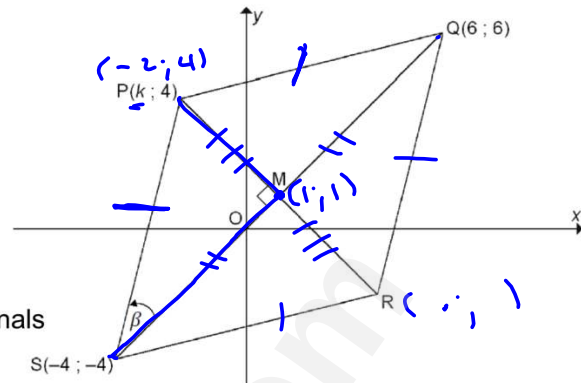
OR ca mark if student read off 14 hours correctly only if ogive shape is correct

(2)

[10]

QUESTION 2

(a) $M\left(\frac{-4+6}{2}, \frac{-4+6}{2}\right)$
 $M(1, 1)$
 ✓ ✓
 (2)



(b) Method 1: Gradients of perpendicular diagonals

$m_{PM} \times m_{SM} = -1$
 ✓ ✓ ✓

$$\frac{4-1}{k-1} \times \frac{1+4}{1+4} = -1$$

$$k = -2$$

Method 2: Adjacent sides equal

$$(k+4)^2 + (4+4)^2 = (6-k)^2 + (6-4)^2$$

$$k^2 + 8k + 16 + 64 = k^2 - 12k + 36 + 4$$

$$k = -2$$

(3)

(c) $1 = \frac{-2+x}{2}$ $1 = \frac{4+y}{2}$ ✓_m
 $\therefore R(4, -2)$
 ✓ ✓

(3)

(d) $m_{PQ} = m_{SR} = \frac{1}{4}$ ✓

$$-4 = \left(\frac{1}{4}\right)(-4) + c$$

$$c = -3$$

$$\therefore y = \frac{1}{4}x - 3$$

(e) Method 1: Exterior angle triangle

$$m_{QS} = 1 \quad \checkmark$$

$$\tan \theta = 1$$

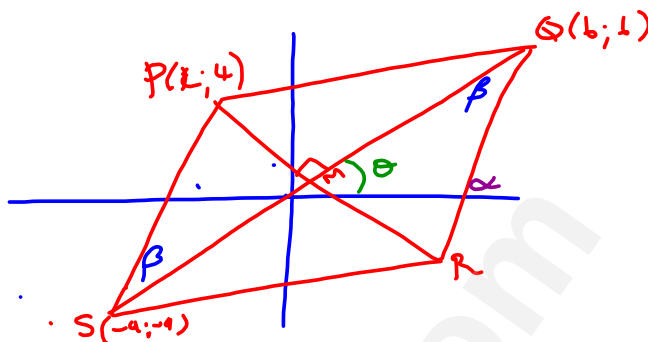
$$\theta = 45^\circ \quad \checkmark$$

$$m_{QR} = 4 \quad \checkmark$$

$$\tan \alpha = 4$$

$$\alpha = 75,96^\circ \quad \checkmark$$

$$\therefore \beta = 31^\circ \quad \text{ext} < \Delta \quad \checkmark \text{statement}$$



Method 2: Trigonometry

$$\checkmark \checkmark \cos \beta = \frac{5\sqrt{2}}{2\sqrt{17}} \quad \checkmark$$

$$\beta = 31^\circ \quad \checkmark$$

in ΔPSM

(5)

[16]

QUESTION 3

(a)

$$\frac{\cos 190^\circ - \sin(90^\circ - \alpha)}{\sin 100^\circ + \cos(-\alpha)}$$

$$= \frac{-\cos 10^\circ - \cos \alpha}{\sin 80^\circ + \cos \alpha}$$

$$= \frac{-(\cos 10^\circ + \cos \alpha)}{\cos 10^\circ + \cos \alpha} \quad \checkmark$$

$$= -1 \quad \checkmark$$

$$\begin{aligned} &\checkmark -\cos 10^\circ \\ &\checkmark -\cos \alpha \\ &\checkmark \cos \alpha \end{aligned}$$

(5)

$$(b) \quad LHS = \frac{\cos(2\alpha + \alpha)}{\cos \alpha} \quad \checkmark \quad (6)$$

$$LHS = \frac{\cos(2\alpha) \cdot \cos(\alpha) - \sin(2\alpha) \cdot \sin(\alpha)}{\cos \alpha} \quad \checkmark$$

$$LHS = \frac{(1 - 2\sin^2 \alpha) \cdot \cos \alpha - 2\sin \alpha \cdot \cos \alpha \cdot \sin \alpha}{\cos \alpha} \quad \checkmark$$

$$LHS = \frac{\cos \alpha - 2\sin^2 \alpha \cdot \cos \alpha - 2\sin^2 \alpha \cdot \cos \alpha}{\cos \alpha} \quad \checkmark$$

$$LHS = \frac{\cos \alpha (1 - 2\sin^2 \alpha - 2\sin^2 \alpha)}{\cos \alpha} \quad \checkmark$$

$$LHS = 1 - 4\sin^2 \alpha$$

$$LHS = RHS$$

(c) $\tan(2x + 10^\circ) = -\frac{5}{3}$ *Std form* (4)

$2x + 10^\circ = -59,036...^\circ + k \cdot 180^\circ, \quad k \in \mathbb{Z}$

$x = -34,5^\circ + k \cdot 90^\circ$ ✓

$\therefore x = 55,5^\circ, \quad x = 145,5^\circ$ ✓

[15]

QUESTION 4

(a) In $\triangle AKV$ and $\triangle CAB$

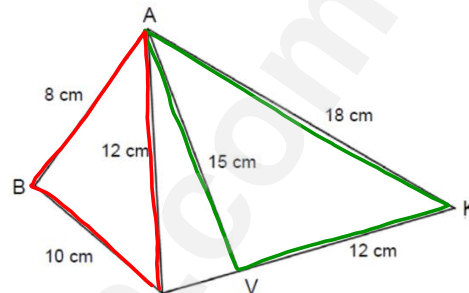
$\frac{AK}{CA} = \frac{18}{12} = \frac{3}{2}$ ✓

$\frac{KV}{AB} = \frac{12}{8} = \frac{3}{2}$ ✓

$\frac{AV}{CB} = \frac{15}{10} = \frac{3}{2}$ ✓

$\therefore \triangle AKV \parallel \triangle CAB$ sides of $\triangle$ in proportion ✓_r

(4)



(b) ✓_s ✓_r

$\hat{AK} = \hat{AC} \parallel \triangle s$

$\therefore ABCV$ is a cyclic quad conv. ext $\angle$ cyclic quad ✓_r

(3)

[7]

QUESTION 5

(a) (1) $\hat{C}_2 = 58^\circ$ tan chord theorem ✓_{s+r}

$\hat{D}_2 = 90^\circ$ $\angle$ semi circle ✓_{s+r}

$\hat{A}_2 = 32^\circ$ sum of $\angle$ s in $\triangle$ ✓_s

$\hat{A}_1 = 32^\circ =$ chords subtend $= \angle$ ✓_{s+r}

(4)

(2) $\hat{O}_2 = 64^\circ$ $\angle$ at centre $= 2x$ $\angle$ at circ. ✓_{s+r} (1)

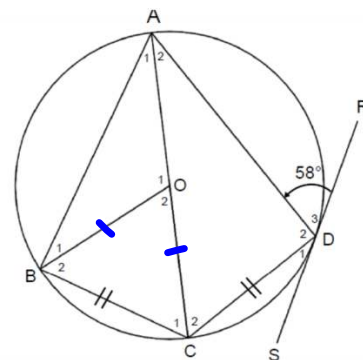
(b) $\hat{B}_2 = 58^\circ$ ✓_s sum of $\angle$ s in $\triangle$, $\angle$ s opp $=$ radii

$\therefore \hat{B}_2 = \hat{C}_2$ ✓_r

$\therefore CD$ is a tangent conv. tan chord theorem ✓_r

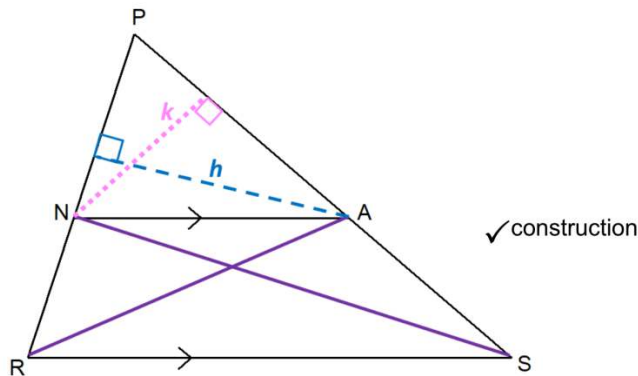
(3)

[8]



QUESTION 6

(a)



$$\frac{\text{Area } \triangle PNA}{\text{Area } \triangle RAN} = \frac{\frac{1}{2} \times PN \times h}{\frac{1}{2} \times RN \times h} = \frac{PN}{RN} \quad \checkmark$$

$$\frac{\text{Area } \triangle PNA}{\text{Area } \triangle NAS} = \frac{\frac{1}{2} \times PA \times k}{\frac{1}{2} \times AS \times k} = \frac{PA}{AS} \quad \checkmark$$

but $\text{Area } \triangle RAN = \text{Area } \triangle NAS$ Δ s with same base, same height $\checkmark$

$$\therefore \frac{\text{Area } \triangle PNA}{\text{Area } \triangle RAN} = \frac{\text{Area } \triangle PNA}{\text{Area } \triangle NAS} \quad \checkmark$$

$$\therefore \frac{PN}{NR} = \frac{PA}{AS}$$

(b) (1) $KC : CV = 2 : 3$ $\checkmark$ line $\parallel$ one side Δ $\checkmark$ (2)

(2)

$$\frac{CN}{CK} = \frac{5k + 3k}{2k} \quad \checkmark$$

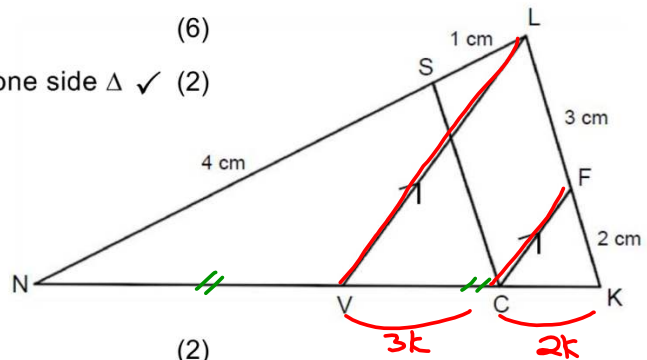
$$\frac{CN}{CK} = 4 \quad \checkmark$$

Award 2/2 for answer only.

(3) $\frac{NS}{SL} = 4 \quad \checkmark$ and $\frac{CN}{CK} = 4$

$\therefore SC \parallel LK$ line divides two sides of Δ in prop $\checkmark$

(6)



(2)

[12]

QUESTION 7

(a) $\hat{y} = -3,93 + 0,67x$ ✓✓

Penalise one mark for rounding. (2)

(b) $r = 0,98$ ✓ (1)

(c) Line A – includes outliers ✓

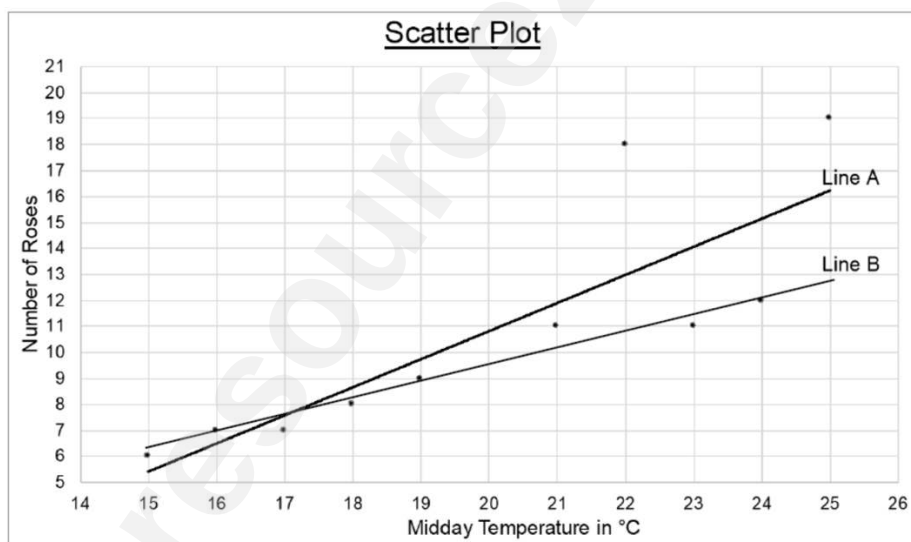
Line B – excludes outliers ✓

The correlation is closer to 1 / stronger when outliers are excluded. ✓

The gradient of the line will be less steep if the two outliers are excluded. ✓

(4)

[7]

TOTAL SECTION A: 75 marks

SECTION B

QUESTION 8

interval for 1 sd.

$$(p^2 + 4p ; 3p^2 - 2p)$$

$$\text{mean} = \frac{p^2 + 4p + 3p^2 - 2p}{2} \quad \checkmark$$

$$\text{mean} = \frac{4p^2 + 2p}{2}$$

$$\text{mean} = 2p^2 + p \quad \checkmark$$

$$\sigma = 3p^2 - 2p - (2p^2 + p) \quad \checkmark$$

$$\sigma = p^2 - 3p \quad \checkmark$$

[4]

QUESTION 9

$$(a) \quad x^2 + 2kx + y^2 - 6y + 4 = 0$$

$$x^2 + 2kx + k^2 + y^2 - 6y + 9 = -4 + k^2 + 9 \quad \checkmark \quad \checkmark$$

$$(x+k)^2 + (y-3)^2 = k^2 + 5 \quad \checkmark$$

$$r = \sqrt{k^2 + 5} \quad \checkmark$$

(4)

$$(b) \quad \sqrt{k^2 + 5} = 3 \quad \checkmark$$

$$k^2 + 5 = 9$$

$$k = \pm 2$$

$$\therefore k = -2 \text{ only} \quad \checkmark$$

$$\therefore (2, 3)$$

$$\checkmark \quad \checkmark$$

(4)

[8]

QUESTION 10

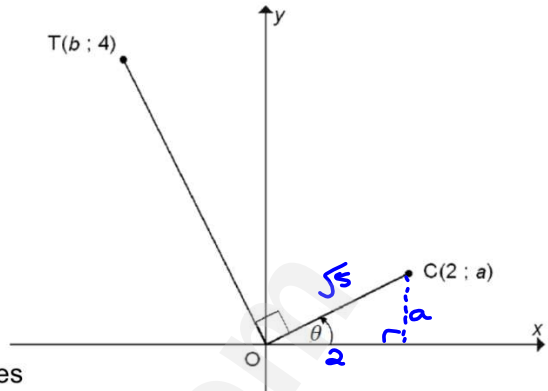
(a) (1)

$$\cos \theta = \frac{2}{\sqrt{5}} \checkmark$$

$$2^2 + a^2 = (\sqrt{5})^2 \checkmark \quad \text{pythag}$$

$$a = 1 \checkmark$$

(3)



(2) Method 1: Gradients of Perpendicular Lines

$$\frac{1-0}{2-0} \times \frac{4-0}{b-0} = -1$$

$$b = -2 \checkmark$$

Method 2: Co-functions

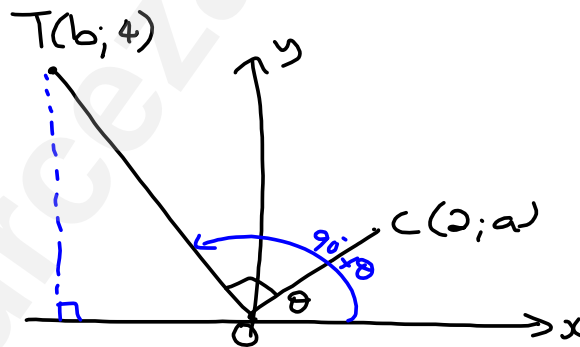
$$\tan(90^\circ + \theta) = \frac{4}{b} \checkmark$$

$$\frac{\sin(90^\circ + \theta)}{\cos(90^\circ + \theta)} = \frac{4}{b}$$

$$\frac{\cos \theta}{-\sin \theta} = \frac{4}{b} \checkmark$$

$$\left(\frac{6}{3\sqrt{5}} \right) \div \left(-\frac{3}{3\sqrt{5}} \right) = \frac{4}{b} \checkmark$$

$$b = -2 \checkmark$$



Method 3:

$$OC = \sqrt{5}$$

$$OT = \sqrt{b^2 + 4^2} \checkmark$$

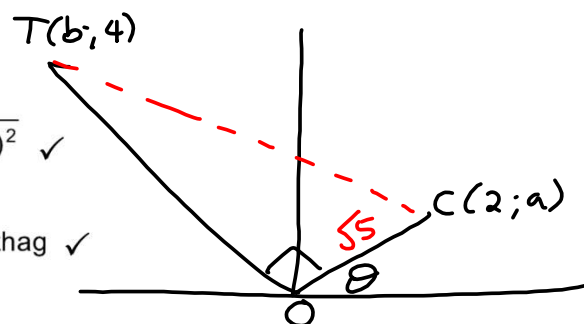
$$CT = \sqrt{(b-2)^2 + (4-1)^2} \checkmark$$

$$CT = \sqrt{b^2 - 4b + 4 + 9}$$

$$CT^2 = OT^2 + OC^2 \quad \text{pythag} \checkmark$$

$$b^2 - 4b + 13 = b^2 + 4^2 + 5 \checkmark$$

$$b = -2$$



Method 4:

$$M_{OC} = \frac{1}{2} \checkmark$$

$$\therefore M_{TO} = -2 \checkmark$$

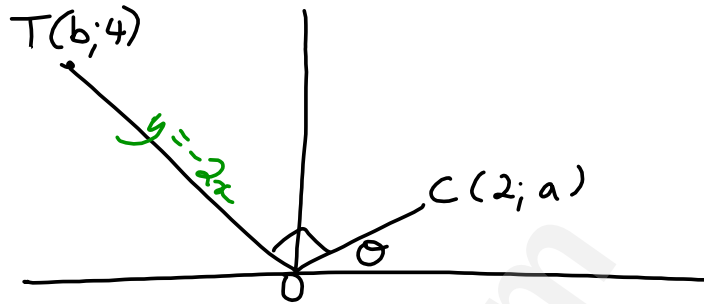
$$y = -2x \checkmark$$

$$\text{subs } y = 4$$

$$4 = -2x \checkmark$$

$$x = -2$$

$$\therefore b = -2$$



(4)

$$(b) \quad 4 \sin x \cdot \cos x = 3 \sin^2 x$$

$$0 = 3 \sin^2(x) - 4 \sin x \cdot \cos x \checkmark$$

$$0 = \sin x (3 \sin x - 4 \cos x) \checkmark$$

$$\therefore \sin x = 0$$

$$\text{or } 3 \sin x - 4 \cos x = 0$$

$$x = 0^\circ + k \cdot 180^\circ \checkmark$$

$$\tan x = \frac{4}{3} \checkmark$$

$$k \in \mathbb{Z}$$

$$x = 53,1^\circ + k \cdot 180^\circ \checkmark$$

(5)

(c)

$$\cos 10^\circ + \cos 70^\circ$$

$$\cos(40^\circ - 30^\circ) + \cos(40^\circ + 30^\circ) \checkmark$$

$$= \cos 40^\circ \cdot \cos 30^\circ + \sin 40^\circ \cdot \sin 30^\circ + \cos 40^\circ \cdot \cos 30^\circ - \sin 40^\circ \cdot \sin 30^\circ$$

$$= 2 \cos 40^\circ \cdot \cos 30^\circ \checkmark$$

$$= 2(m) \left(\frac{\sqrt{3}}{2} \right) \checkmark$$

$$= \sqrt{3} m \checkmark$$

$$\text{given: } \cos 40^\circ = m$$

Other methods:

$$\cos 10^\circ + \cos 70^\circ$$

$$= \sin 80^\circ + \cos(40^\circ + 30^\circ) \checkmark$$

$$= 2 \sin 40^\circ \cos 40^\circ + \cos 40^\circ \cos 30^\circ - \sin 40^\circ \sin 30^\circ$$

$$= 2 \sqrt{1-m^2} \cdot m + m \cdot \frac{\sqrt{3}}{2} - \sqrt{1-m^2} \cdot \frac{1}{2} \checkmark$$

$$= \frac{4m\sqrt{1-m^2} + \sqrt{3}m - \sqrt{1-m^2}}{2} \checkmark$$

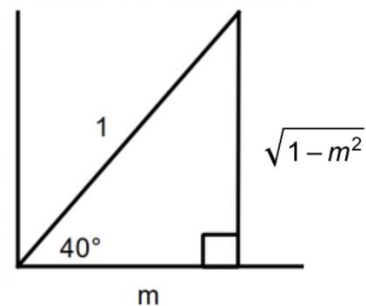
$$\text{Or } \cos 10^\circ + \cos 70^\circ$$

$$= \sin(50^\circ - 40^\circ) + \cos(40^\circ + 30^\circ)$$

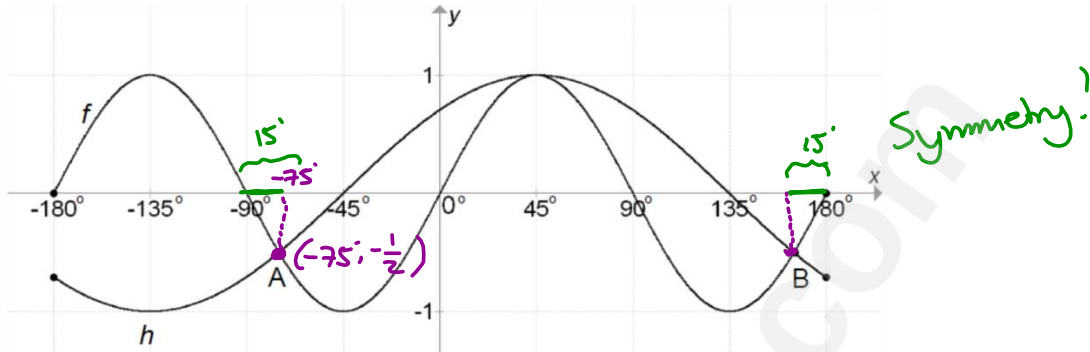
.....

(6)

[18]



QUESTION 11



(a) $x = 165^\circ$ ✓

(1)

(b) $x \in (-90^\circ; 0^\circ) \cup (90^\circ; 180^\circ)$ ✓ values ✓ notation

(2)

(c) $4 \sin x \cdot \cos x \leq \sqrt{2} (\cos x + \sin x)$
 $2 \sin 2x \leq \sqrt{2} \cos x + \sqrt{2} \sin x$

$\sin 2x \leq \frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x$ ✓

$\sin 2x \leq \cos 45^\circ \cdot \cos x + \sin 45^\circ \cdot \sin x$ ✓

$\sin 2x \leq \cos(x - 45^\circ)$

$\therefore x \in [-75^\circ; 165^\circ]$ ✓

(4)

[7]

$$\begin{aligned} f(x) &= \sin 2x \\ h(x) &= \cos(x - 45^\circ) \end{aligned}$$

$$\begin{aligned} h(x) &= \cos x \cos 45^\circ + \sin x \sin 45^\circ \\ &= \frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x \end{aligned}$$

QUESTION 12

(a)

In $\triangle AOC$ and $\triangle BCK$

$$\hat{O}_1 = \hat{B}_1 \quad \angle s \text{ semi circle } \checkmark^{s+r}$$

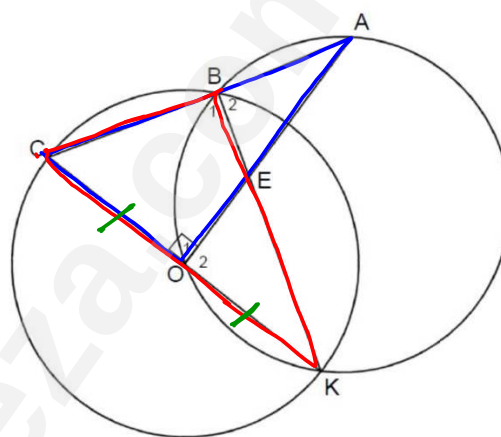
$$\hat{A} = \hat{K} \quad \angle s \text{ same seg } \checkmark^{s+r}$$

$\hat{C}$ is common $\checkmark$

$$\therefore \triangle OAC \equiv \triangle BKC \quad AAA \quad \checkmark^{s+r}$$

$$\therefore \frac{AC}{KC} = \frac{OC}{BC} \quad \equiv \triangle s \quad \checkmark^{s+r}$$

(5)



(2)

(b)

$$KC \cdot OC = AC \cdot BC \quad \checkmark$$

$$\text{But } KC = 2OC \quad \checkmark \text{ radii}$$

$$2OC \cdot OC = AC \cdot BC$$

(c)

Join AK $\checkmark$

In $\triangle ACO$ and $\triangle AKO$

AO is common $\checkmark$

$$\hat{O}_1 = \hat{O}_2 = 90^\circ \quad \checkmark$$

$$OC = OK \quad \checkmark \text{ radii}$$

$$\therefore \triangle ACO \equiv \triangle AKO \quad SAS \quad \checkmark$$

$$\therefore \hat{CAO} = \hat{KAO} \quad \checkmark \quad \equiv \triangle s$$

(6)

[13]

QUESTION 13

(a) x -int of str. line

$$4(0) + 3x - 36 = 0 \quad \checkmark$$

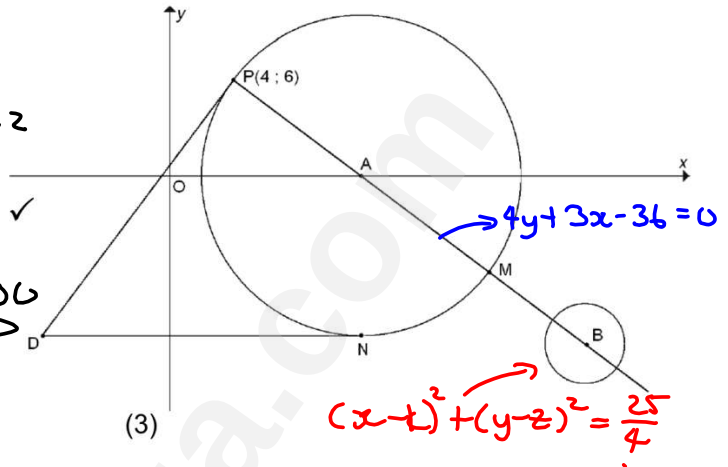
$$x = 12 \quad \checkmark$$

$$\therefore (x-12)^2 + (y-0)^2 = r^2$$

Sub (4): $r = \sqrt{(6-0)^2 + (4-12)^2} \quad \checkmark$

$$r = 10 \quad \checkmark$$

$$(4) \quad \therefore (x-12)^2 + y^2 = 100$$



(b) $m_{AP} = -\frac{3}{4} \quad \therefore m_{PD} = \frac{4}{3} \quad \checkmark$

$$y-6 = \frac{4}{3}(x-4) \quad \checkmark$$

$$y = \frac{4}{3}x + \frac{2}{3} \quad \checkmark$$

(c) $AN = 10 \quad (3)$

$$\sqrt{10} = \frac{4}{3}x + \frac{2}{3}$$

$$x = -8 \quad \checkmark$$

$$D(-8, -10) \quad \checkmark$$

(d) Centre B(k; z) $\checkmark \quad (2)$

$$4z + 3k - 36 = 0$$

$$3k = -4z + 36 \quad \checkmark$$

$$4y + 3x - 36 = 0$$

(e) $d_{AB} = R_A - r_B \quad \checkmark$

$$\sqrt{\left(-\frac{4}{3}z + 12 - 12\right)^2 + (z-0)^2} = 10 - \frac{5}{2} \quad \checkmark$$

$$\frac{25}{9}z^2 - \frac{225}{4} = 0 \quad \checkmark$$

$$z = \frac{9}{2} \quad \text{or} \quad z = -\frac{9}{2}$$

Or

$$\left(x + \frac{4}{3}z - 12\right)^2 + (y - z)^2 = \frac{25}{4} \quad \checkmark$$

Sub $P(4;6)$

$$\left(4 + \frac{4}{3}z - 12\right)^2 + (6 - z)^2 = \frac{25}{4} \quad \checkmark$$

✓

$$z = 4,5$$

$$\left(x + \frac{4}{3}z - 12\right)^2 + (y - z)^2 = \frac{25}{4}$$

Sub $M(20;-6)$ ✓

$$\left(20 + \frac{4}{3}z - 12\right)^2 + (-6 - z)^2 = \frac{25}{4} \quad \checkmark$$

✓

$$z = -4,5$$

(6)

[18]

QUESTION 14

$$XM = 4\sqrt{3} \quad \text{pythag} \quad \checkmark$$

$$\text{Area Base} = \frac{1}{2} (8)(4\sqrt{3}) \quad \checkmark \quad \text{or} \quad \frac{1}{2} (8)(8) \sin 60^\circ$$

$$\text{Area Base} = 16\sqrt{3} \quad \checkmark$$

$$CM = \frac{4\sqrt{3}}{3} \quad \checkmark$$

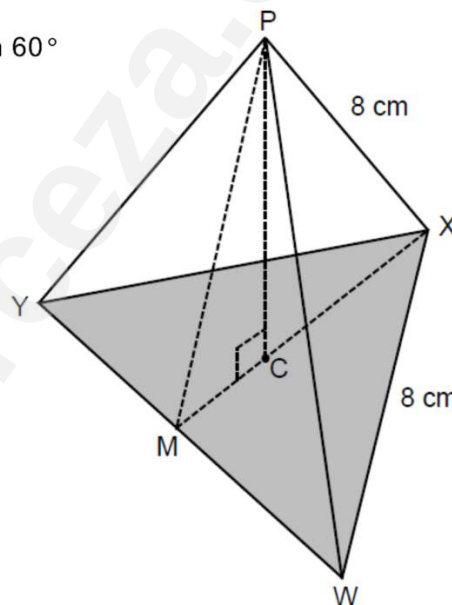
$$PC^2 = (4\sqrt{3})^2 - \left(\frac{4\sqrt{3}}{3}\right)^2 \quad \checkmark$$

$$PC = \frac{8\sqrt{6}}{3} \quad \checkmark$$

$$V = \frac{1}{3} (16\sqrt{3}) \left(\frac{8\sqrt{6}}{3}\right)$$

$$V = 60,3 \text{ cm}^3 \quad \checkmark$$

[7]



OR In $\triangle PCX$ where $XC = \frac{8\sqrt{3}}{3}$

$$PC^2 = 8^2 - \left(\frac{8\sqrt{3}}{3}\right)^2$$

$$PC = \frac{8\sqrt{6}}{3}$$

TOTAL SECTION B: 75 marks

TOTAL: 150 marks