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ST JOHN'S COLLEGE

Upper V

Mathematics Paper I

Date: July 2023

Time: 3 hours

Examiner: Dr Sproule

150 marks

Moderator: Mrs Garofoli

NAME: _____

TEACHER: SLS GdV DHG DC PWN MSM BB

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- The paper consists of 17 pages, which includes this cover page. An Information Sheet is handed out separately. Check that your paper is complete.
- Write your name and circle your teacher's initials.
- A non-programmable, non-graphing calculator may be used unless otherwise specified in the question.
- All necessary calculations must be clearly shown, and writing should be legible.
- Diagrams have not been drawn to scale.
- Round off your answers to **two decimal digits**, unless otherwise indicated.
- Answer all questions on the question paper.

Question	1	2	3	4	5	6	7	8	9	10	Total
Out of	22	14	12	16	11	21	16	10	16	12	150
Mark											

Question 1 [22 marks]

(a) Solve for x:

(1) $(x+2)(2x-7) = (x-1)(x+2)$ (3)

(2) $16^{2x-3} = \frac{4^{5-x}}{4}$ (4)

(3) $1 - \sqrt{4x+6} = 2x$ (4)

(b) Solve for x if $(x-3)^2 \geq 4$ (4)

(c) If -1 and 4 are the roots of the equation $x^2 + bx + c = 0$,
determine the values of b and c (3)

(d) Solve for x if $\log_x(x+3) - \log_x 2 = 2$ (4)

Question 2 [14 marks]

- (a) Mpho deposits R12 500 for exactly k years. She earns interest on the investment at a rate of 9% per annum, compounded quarterly. At the end of k years, her investment is worth R30 440.

(1) Calculate the effective annual interest rate. (2)

(2) Determine the value of k . (4)

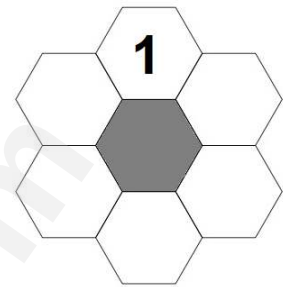
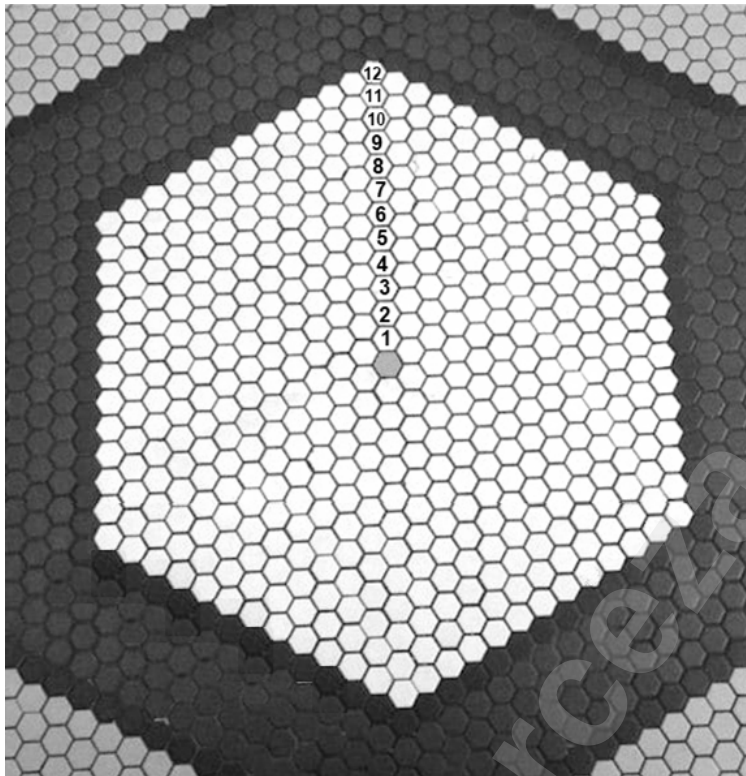
- (b) A local carpenter wishes to purchase a 5-axis CNC router for R1 621 000. He is offered a loan by the bank at an interest rate of 11,25% per annum compounded monthly over an 8 year period.

(1) Calculate the monthly instalment, if the first payment is made one month after receiving the loan. (4)

(2) Calculate the outstanding balance at the end of 4 years. (4)

Question 3 [12 marks]

- (a) A large dining hall is covered with small hexagonal tiles. Layers 1 to 12 of white tiles are numbered in the image. A graphic of layer 1, around the shaded centre tile, is shown on the right.



Layer 1

- (1) Determine the number of hexagonal tiles in the 12th layer of white tiles. (2)
- (2) Determine the number of hexagonal tiles in n -th layer of this sequence. (1)
- (3) Determine the total number of tiles that must be purchased to lay tiles from the centre and complete the first n layers. (4)

- (b) The height of a stack of polystyrene cups forms an arithmetic sequence. A stack of 7 cups has a height of 17 cm and a stack of 13 cups has a height of 24,8 cm.

The cups are sold in a set of 50 cups packaged in two stacks of 25 cups each. Calculate the height of the pack. (5)



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Question 4 [16 marks]

- (a) Given $P(A) = 0,45$; $P(B) = c$ and $P(A \text{ or } B) = 0,74$

Determine the value of c if A and B are mutually exclusive.

(3)

- (b) There are 24 chairs in a B&B, half of them have cushions. The number of chairs with arms but no cushion is 11, and 5 of the chairs have cushions but no arms.

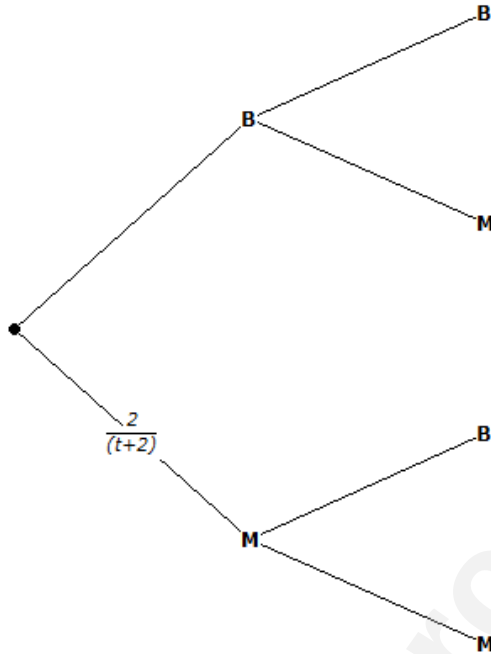
(1) Represent the information in a Venn diagram. (4)

(2) How many chairs have neither arms nor a cushion? (1)

(3) What is the probability of selecting a chair at random that has both cushions and arms? (2)

- (c) There are t blue balls and 2 maroon balls in a bag. Litha randomly selects one ball from the bag and records her choice. Then returns the ball to the bag. She randomly selects a second ball from the bag, again records her choice and returns the ball to the bag. It is known that the probability that Litha will select two balls of the same colour from the bag is 52%.

(1) Complete the probabilities on each arm of the tree diagram. (2)



(2) Calculate how many blue balls are in the bag. (4)

Question 5 [11 marks]

(a) $g(x) = (x+1)^2$

(1) Determine $g'(x)$ from first principles (5)

(2) Calculate the gradient of the tangent of g at $x = -3$ (2)

(b) Determine $\frac{dy}{dx}$ if $y = \frac{2x + \sqrt{x}}{4x^2}$ (4)

Question 6 [21 marks]

(a) $A(0; \frac{1}{3})$ and $B(-2; 3)$ are two points on the graph of $y = f(x) = a(b^x)$

(1) Show that $a = b = \frac{1}{3}$ (3)

(2) Draw a sketch of $f(x) = 3^{-x-1}$. Show the intercepts with the axes. (3)

(3) State the range of f (1)

(4) Determine f^{-1} , the inverse of f in the form $y = \dots$ (3)

(5) Sketch the graph of $y = f^{-1}(x)$, on the same set of axes as the graph of $y = f(x)$. Show the intercepts with the axes. (3)

(b) Consider the graphs of $g(x) = \frac{-4}{x+1} - 4$ and $h(x) = \frac{-4}{x-2} + 1$

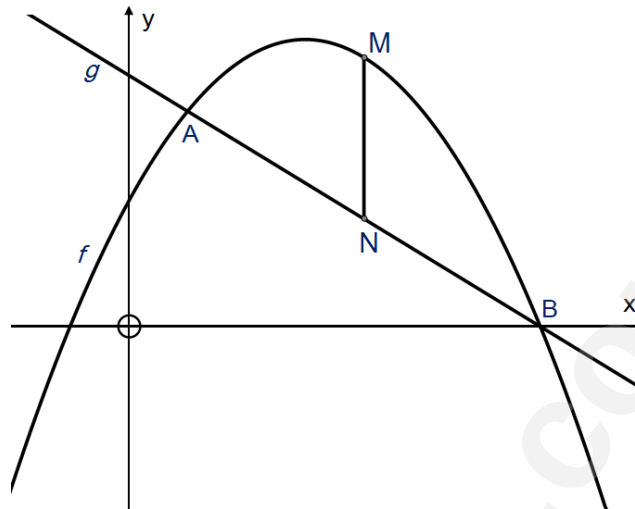
(1) State the domain of g (1)

(2) Solve for x such that $g(x) \times h(x) > 0$ (7)

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Question 7 [16 marks]

The sketch represents the graphs of the functions $f(x) = -x^2 + 6x + 7$ and $g(x) = -2x + 14$



(a) Find the turning point of the graph of f (3)

(b) (1) Determine the co-ordinates of A and B, the points of intersection of the two graphs. (4)

- (2) Hence, determine the value(s) of k for which the equation
 $-(x+k)^2 + 6(x+k) + 7 = -2(x+k) + 14$
has one positive and one negative root. (3)

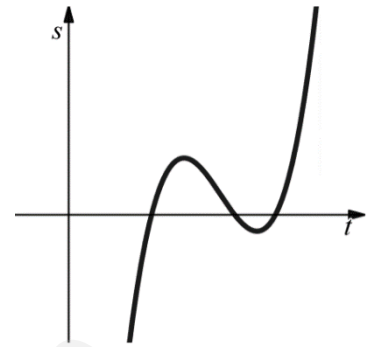
- (c) Calculate the largest possible value of MN , if MN is parallel to the y -axis
with M a point on the graph of f and N a point on the graph of g . (6)

Question 8 [10 marks]

A particle moves along a straight line so that, t seconds after observations start, its distance, s metres, from a fixed point A is given by:

$$s(t) = \frac{1}{2}t^3 - \frac{11}{2}t^2 + 19t - 20$$

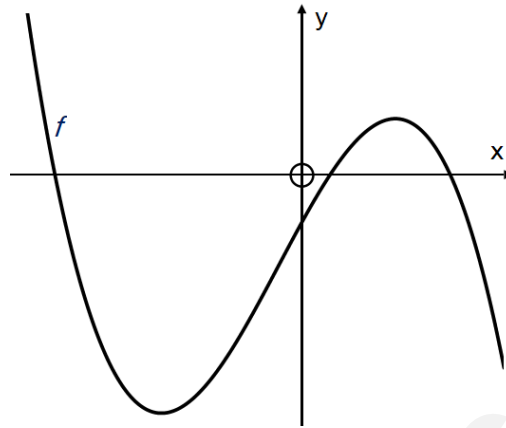
The graph of $s(t)$ is illustrated to the right.



- (a) How far will the particle be from point A, at the moment observations start? (2)
- (b) Calculate the values of t for which the particle is momentarily at rest. (4)
- (c) How many times will the particle be at the fixed point A. (1)
- (d) At what time does the motion of the particle change from decelerating to accelerating? (3)

Question 9 [16 marks]

(a) The graph of a cubic function f is sketched below.



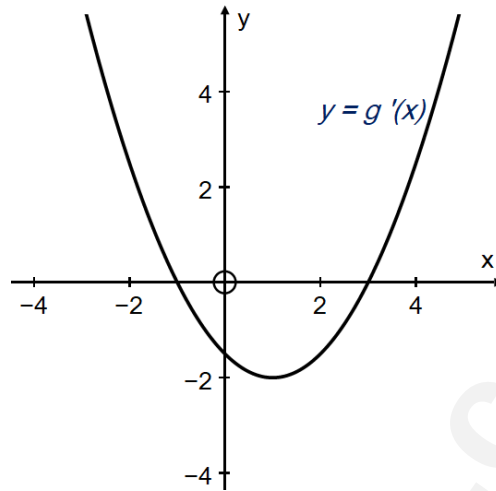
The equation of the curve is given as $f(x) = -x^3 + bx^2 + cx - 3$, where b and c are constants.

$$f(1) = 3 \quad \text{and} \quad f''(-1) = 4$$

(1) Determine the values of b and c (7)

(2) Determine the values of x for the graph is concave up (3)

- (b) The sketch represents the graph of $y = g'(x)$, with x-intercepts at $(-1; 0)$ and $(3; 0)$.
The graph has a turning point at $(1; -2)$.



- (1) State the value(s) of x for which the tangent to g is horizontal. (2)
- (2) For which values of x will the graph of g be increasing? (2)
- (3) Draw a sketch of $y = g''(x)$ on the axes above. (2)

Question 10 [12 marks]

- (a) The sum of 11 terms of the series $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}} x^3 + \log_{\frac{1}{2}} x^5 + \dots$ is 242.

Calculate the value of x

(4)

- (b) Prove that the infinite geometric series

$$1 + \frac{2x}{3+x^2} + \left(\frac{2x}{3+x^2}\right)^2 + \dots$$

is convergent for all real values of x

(8)

ADDITIONAL SPACE

(Clearly indicate at the question that you have used the additional space to ensure all questions are marked.)

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(Clearly indicate at the question that you have used the additional space to ensure all questions are marked.)

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