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ST JOHN'S COLLEGE

Upper V

Mathematics Paper I - MEMO

Date: July 2023

Time: 3 hours

Examiner: Dr Sproule

150 marks

Moderator: Mrs Garofoli

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- The paper consists of 17 pages, which includes this cover page. An Information Sheet is handed out separately. Check that your paper is complete.
- Write your name and circle your teacher's initials.
- A non-programmable, non-graphing calculator may be used unless otherwise specified in the question.
- All necessary calculations must be clearly shown, and writing should be legible.
- Diagrams have not been drawn to scale.
- Round off your answers to **two decimal digits**, unless otherwise indicated.
- Answer all questions on the question paper.

Question	1	2	3	4	5	6	7	8	9	10	Total
Out of	22	14	12	16	11	21	16	10	16	12	150
Mark											

MEMO – 2023 GR12 Paper 1

Question 1 [22 marks]

(a) (1) $(x+2)(2x-7) = (x-1)(x+2)$ (3)

$(x+2)[(2x-7)-(x-1)] = 0$ ✓ a

$(x+2)(x-6) = 0$ ✓ a

$x = -2$ or $x = 6$ ✓ ca

(2) $16^{2x-3} = \frac{4^{5-x}}{4}$ (4)

$4^{4x-6} = 4^{4-x}$ ✓ a

$4x-6 = 4-x$ ✓ ca

$5x = 10$

$x = 2$ ✓ ca

(3) $1 - \sqrt{4x+6} = 2x$ (4)

$1 - 2x = \sqrt{4x+6}$

$(1-2x)^2 = 4x+6$ ✓ m

$4x^2 - 4x + 1 = 4x + 6$ ✓ a

$4x^2 - 8x - 5 = 0$

$(2x-5)(2x+1) = 0$ ✓ a

$x \neq 2\frac{1}{2}$ or $x = -\frac{1}{2}$ ✓ ca

(b) $(x-3)^2 \geq 4$ (4)

$x^2 - 6x + 9 - 4 \geq 0$ ✓ m

$(x-5)(x-1) \geq 0$ ✓ a

$x \leq 1$ or $x \geq 5$ ✓ a

(c) $(x+1)(x-4) = 0$ ✓ subst (m) (3)

$x^2 - 3x - 4 = 0$ ✓ a

$b = -3$ and $c = -4$ ✓ ca

(d) $\log_x(x+3) - \log_x 2 = 2$ (4)

$\log_x \frac{(x+3)}{2} = 2$ ✓ log law (m)

$x+3 = 2x^2$ ✓ log defn (m)

$0 = 2x^2 - x - 3$

$0 = (2x-3)(x+1)$

$x = 1\frac{1}{2}$ ✓ ca or $x \neq -1$ ✓ ca

Question 2 [14 marks]

(a) (1) $\left(1 + \frac{0,09}{4}\right)^4 = (1+i_e)$ ✓ m (2)

$i_e = 0,0930833$ ✓ a

(2) $30\,440 = 12\,500(1+i_e)^k$ ✓ m (4)

$2,4352 = (1+i_e)^k$ ✓ a

$k = \frac{\log 2,4352}{\log 1,09308}$ ✓ m

$k = 10$ ✓ ca

(b) (1) $i = \frac{0,1125}{12}$ ✓ a (4)

$1\,621\,000 = \frac{x[1 - (1+i)^{-96}]}{i}$ ✓ a ✓ m

$x = R\,25\,682,45$ ✓ ca

(2) $OB = \frac{25\,682,45[1 - (1+i)^{-48}]}{i}$ ✓ ca ✓ a

✓ m

$OB = R\,989\,038,13$ ✓ a (4)

Question 3

[12 marks]

- (a) (1) ✓ method to see pattern (2)

$$T = 6(12) = 72 \text{ tiles } \checkmark a$$

$$(2) T = 6n \checkmark a \quad (1)$$

$$(3) 1 \checkmark + (6 + 12 + \dots) \checkmark m \text{ (AS)}$$

$$1 + S_n = 1 + \frac{n}{2}[12 + (n-1)6] \checkmark \text{subst}$$

$$= 3n^2 + 3n + 1 \checkmark ca \quad (4)$$

$$(b) T_7 = 17 = a + 6d \checkmark a \quad (5)$$

$$T_{13} = 24,8 = a + 12d$$

$$\therefore 6d = 7,8 \checkmark m \text{ (sim solve)}$$

$$d = 1,3 \checkmark a$$

$$a = 17 - 6(1,3) = 9,2 \checkmark ca$$

$$T_{25} = 9,2 + 24(1,3) = 40,4 \text{ cm } \checkmark ca$$

Question 4

[16 marks]

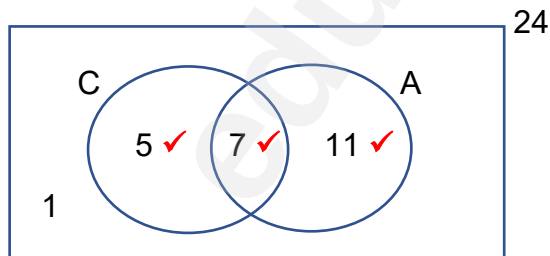
$$(a) P(A \cap B) = 0 \checkmark \quad (3)$$

$$P(A \text{ or } B) = P(A) + P(B) \checkmark m$$

$$0,74 = 0,45 + c$$

$$c = 0,3 \checkmark a$$

$$(b) (1) \checkmark \text{ Venn layout correct} \quad (4)$$



$$(2) 1 \checkmark \quad (1)$$

$$(3) P(C \text{ and } A) = \frac{7}{24} \checkmark \checkmark \quad (2)$$

$$(c) (1) \checkmark \frac{t}{t+2} \text{ for blue} \quad (2)$$

✓ second arms the same as first

$$(2) \frac{2}{t+2} \cdot \frac{2}{t+2} \checkmark + \frac{t}{t+2} \cdot \frac{t}{t+2} \checkmark = 0,52 \quad (4)$$

$$4 + t^2 = 0,52(t+2)^2 \checkmark ca$$

$$0,48t^2 - 2,08t + 1,92 = 0$$

$$t = 3 \text{ blue balls } \checkmark ca$$

Question 5

[11 marks]

$$(a) (1) g'(x) = \lim_{h \rightarrow 0} \frac{(x+h+1)^2 - (x+1)^2}{h} \checkmark a \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + 2x + 2h + h^2 + 1 \checkmark a - x^2 - 2x - 1 \checkmark a}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + 2h + h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x + 2 + h)}{h} \checkmark ca$$

$$= \lim_{h \rightarrow 0} (2x + 2 + h)$$

$$= 2x + 2 \checkmark a$$

$$(2) m_T = 2(-3) + 2 = -4 \checkmark m \checkmark a \quad (2)$$

$$(b) y = \frac{2x + \sqrt{x}}{4x^2} = \frac{1}{2x} \checkmark a + \frac{1}{4} x^{-\frac{3}{2}} \checkmark a \quad (4)$$

$$\frac{dy}{dx} = -\frac{1}{2} x^{-2} \checkmark ca - \frac{3}{8} x^{-\frac{5}{2}} \checkmark ca$$

Question 6

[21 marks]

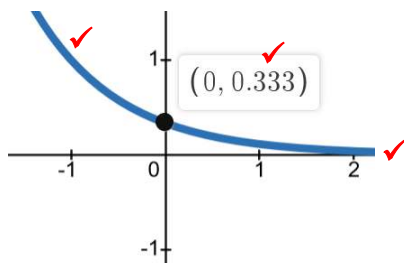
(a) (1) $\frac{1}{3} = a(b^0)$ ✓ a $\therefore a = \frac{1}{3}$ (3)

$3 = \frac{1}{3}(b^{-2})$ ✓ m

$9 = b^{-2}$ ✓ ca

$b = \frac{1}{3}$

(2) (3)



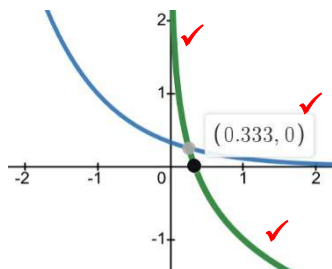
(3) $y > 0$ ✓ (1)

(4) $x = 3^{-y-1}$ ✓ m (3)

$\log_3 x = -y - 1$

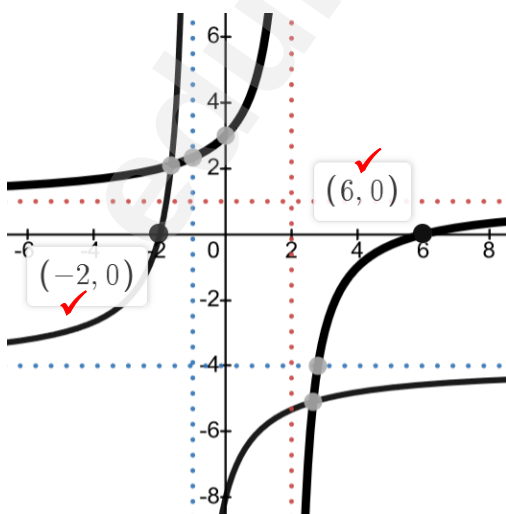
$y = -\log_3 x - 1$ ✓ ca

(5) (3)



(b) (1) $x \in \mathbb{R} \quad x \neq -1$ ✓ (1)

(2) ✓✓ asymptotes (7)
✓ hyperbole

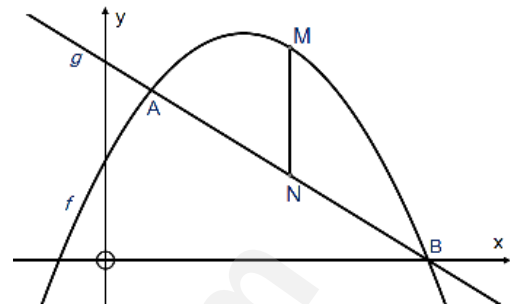


$-2 < x < -1$ ✓ a ; $2 < x < 6$ ✓ a

Question 7

[16 marks]

$f(x) = -x^2 + 6x + 7$ and $g(x) = -2x + 14$



(a) $x = \frac{6}{-2(-1)} = 3$ ✓ a (3)

$y = -(3)^2 + 6(3) + 7$ ✓ subs = 16 ✓ ca

(b) (1) $-x^2 + 6x + 7 = -2x + 14$ ✓ m (4)

$0 = x^2 - 8x + 7$ ✓ a

$0 = (x-7)(x-1)$

$x = 7$ or $x = 1$ ✓ a

$y = 0$ $y = 12$ ✓ ca

(2) recognising a horizontal shift ✓ (3)

a ✓ $1 < k < 7$ ✓ a

(c) $MN = -x^2 + 6x + 7 - (-2x + 14)$ ✓ a (6)

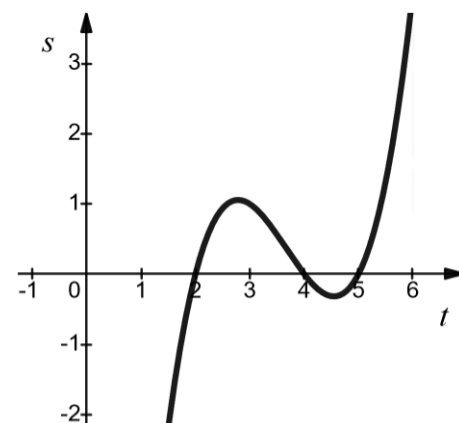
$MN = -x^2 + 8x - 7$ ✓ ca

$MN' = -2x + 8$ ✓ m

$0 = -2x + 8$

$x = 4$ ✓ a

$MN = -(4)^2 + 8(4) - 7 = 9$ ✓ ca



Question 8 [10 marks] Downloaded from eduresourceza.com

$$s(t) = \frac{1}{2}t^3 - \frac{11}{2}t^2 + 19t - 20$$

(a) $t = 0$ ✓ then $s = 20$ metres ✓ (2)

(b) $v(t) = s'(t) = \frac{3}{2}t^2 - 11t + 19$ ✓ m ✓ a (4)

$$v(t) = s'(t) = \frac{3}{2}t^2 - 11t + 19$$

$$\therefore s'(t) = 0$$
 ✓

$$0 = \frac{3}{2}t^2 - 11t + 19$$

$$t = 2,78 \text{ or } t = 4,55$$
 ✓ a

(c) 3 times ✓ (1)

(d) $a(t) = 0$ ✓ (3)

$$a(t) = s''(t) = 3t - 11$$
 ✓ a

$$0 = 3t - 11$$

$$t = \frac{11}{3} \text{ sec}$$
 ✓ ca

Question 9 [16 marks]

(a) $f(x) = -x^3 + bx^2 + cx - 3$

(1) $f(1) = 3 = -(1)^3 + b(1)^2 + c(1) - 3$ ✓ m (7)

$$b + c = 7$$
 ✓ a

$$f'(x) = -3x^2 + 2bx + c$$
 ✓ m

$$f''(x) = -6x + 2b$$
 ✓ a

$$f''(-1) = 4 = 6 + 2b$$
 ✓ ca

$$\therefore b = -1$$
 ✓ a

$$\therefore c = 8$$
 ✓ ca

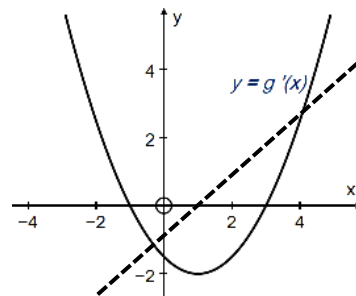
(2) $f''(x) = -6x - 2$ ✓ ca (3)

$$-6x - 2 = 0$$

$$x = -\frac{1}{3}$$
 ✓ ca

$$\therefore x \leq -\frac{1}{3}$$
 ✓ ca

(b)



(1) $x = -1$ ✓ or $x = 3$ ✓ (2)

(2) $x \leq -1$ ✓ or $x \geq 3$ ✓ (2)

(3) x-int = 1 ✓ + slope ✓ (2)

Question 10 [12 marks]

(a) $d = \log_{\frac{1}{2}} x^3 - \log_{\frac{1}{2}} x = 2\log_{\frac{1}{2}} x$ ✓ (4)

$$S_n \text{ ✓ m} = 242 = \frac{11}{2} \left[2\log_{\frac{1}{2}} x + 10 \left(2\log_{\frac{1}{2}} x \right) \right] \text{ ✓ a}$$

$$242 = 11 \left[11\log_{\frac{1}{2}} x \right]$$

$$2 = \log_{\frac{1}{2}} x$$

$$x = \frac{1}{4}$$
 ✓ ca

(b) 2 marks for the series

3 marks for the inequalities

3 marks for the proof of no real roots

$$-1 < r < 1 \quad \text{knowing } \checkmark \quad (8)$$

$$r = \frac{2x}{3+x^2} \quad \checkmark \text{ m}$$

$$-1 < \frac{2x}{3+x^2} < 1$$

Evidence understanding $3+x^2 > 0$ $\checkmark$

[They can't just cross multiple by $3+x^2$ - if they do they lose this mark.]

$$\begin{array}{ll} -3 - x^2 < 2x & \checkmark \text{ a} \text{ or } 2x < 3 + x^2 \quad \checkmark \text{ a} \\ 0 < x^2 + 2x + 3 & 0 < x^2 - 2x + 3 \end{array}$$

[They need to show that these functions have no real roots. They can do that by:

- Nature of roots
- Completing the square and arguing that it is always pos/neg.
- Drawing the graph WITH the co-ordinates of the TP shown and arguing that it is always pos/neg.

If they just claim that the inequality always holds they get 1 out of 3]

$$\Delta = (2)^2 - 4(1)(3) = -8 \quad \checkmark \text{ m} \quad \checkmark \text{ a}$$

$\therefore$ non-real roots $\checkmark$ conclusion

[If they use limits to argue that S_∞ tends to a value then the max is $5/8$. This is not a complete proof]