



eduresourceza.com



ST ANNE'S DIOCESAN COLLEGE
NOVEMBER EXAMINATION – 2023
GRADE 12

MATHEMATICS – PAPER 2

Time: 3 hours

NAME:/exam no. _____

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated. Ensure your calculator is in **DEGREE** mode.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. Three blank pages (pages 22 to 24) are included at the end of the paper.
If you run out of space for a question, use these pages.
Clearly indicate the question number of your answer should you use this extra space.

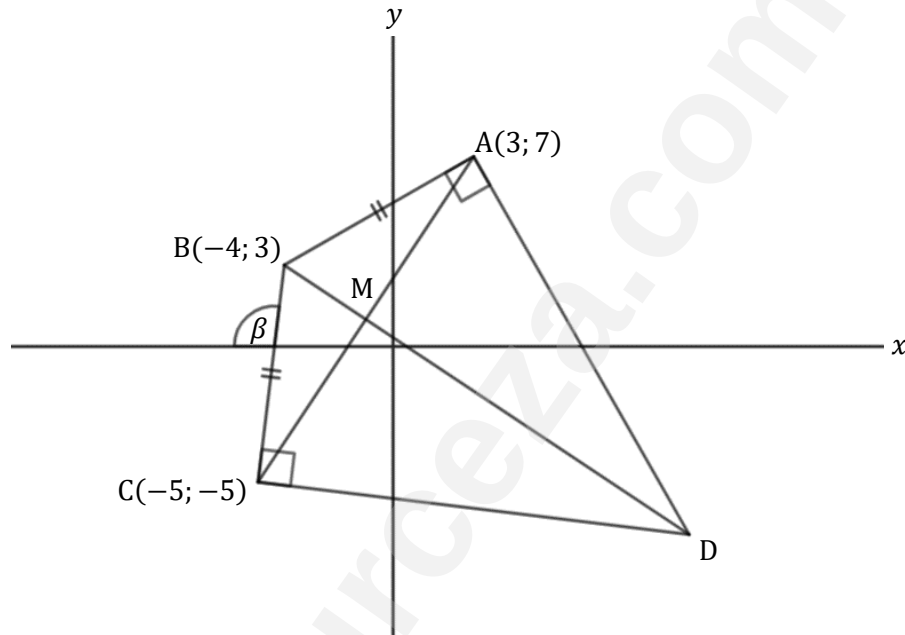
For official use only.

	Analytical Geometry	Statistics	Statistics	Euclidean Geometry	Euclidean Geometry	Euclidean Geometry	Trigonometry	Trigonometry	Trigonometry	Euclidean Geometry	Euclidean Geometry	Analytical Geometry
Question	1	2	3	4	5	6	7	8	9	10	11	12
Mark												
Total	17	12	12	11	4	9	27	19	5	11	5	18
Total												
							150					
Percentage												
							100					

SECTION A**QUESTION 1**

In the given diagram:

- Kite ABCD is given in the diagram where diagonals BD and AC intersect at M.
- $\hat{A} = \hat{C} = 90^\circ$ and $AB = CB$.
- The coordinates of the vertices are $A(3; 7)$, $B(-4; 3)$, $C(-5; -5)$ and $D(x; y)$.



(a) Calculate the gradient of BC

(2)

(b) Determine the angle of inclination, β .

(2)

(c) Calculate the gradient of CD.

(2)

(d) Determine the coordinates of M.

(2)

(e) Determine the coordinates of E for the parm ABCE.

(3)

(f) Calculate the length of AB. Leave your answer in surd form.

(2)

- (g) 1. The perpendicular drawn from the vertex of the right angle of a right-angled triangle to the hypotenuse, divides the triangle into two triangles that are similar to each other and the original triangle. Use this theorem to complete the statement for the given diagram given that $AC \perp BD$:

$$AB^2 = \dots$$

(1)

2. Calculate the length of MD given that $BM = \sqrt{13}$ units.
Give a reason for your working and leave your answer in its simplest surd form.

(3)

[17]

QUESTION 2

The Tour de France is a bicycle race where cyclists race in 20 different stages. They tend to lose significant weight by the end of the tour. The table below shows the total weight lost by 150 different cyclists after the duration of a similar race.

Interval of weight lost (in grams)	Number of cyclists			
$1\,000 \leq w < 2\,000$	5			
$2\,000 \leq w < 3\,000$	10			
$3\,000 \leq w < 4\,000$	15			
$4\,000 \leq w < 5\,000$	35			
$5\,000 \leq w < 6\,000$	50			
$6\,000 \leq w < 7\,000$	30			
$7\,000 \leq w < 8\,000$	5			
Total	150			

(a) Write down the modal class of the data.

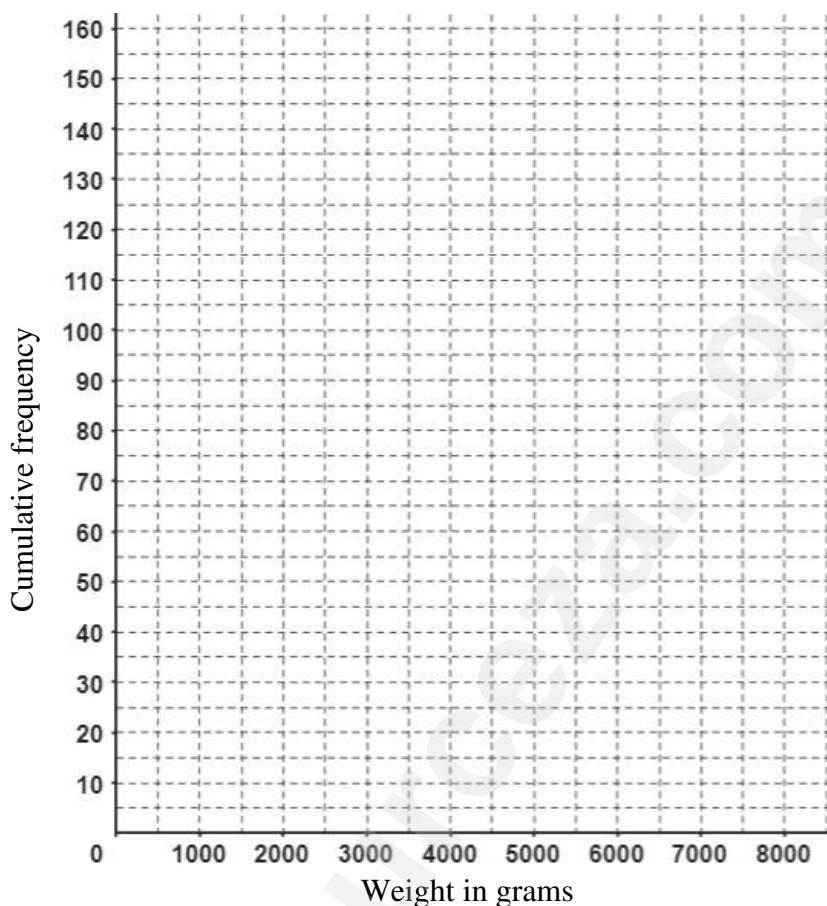
(1)

(b) Calculate the estimated mean weight loss of the cyclists.

(3)

- (c) Represent the data on a cumulative frequency curve.

Cumulative Frequency Graph to show weight loss of cyclists.

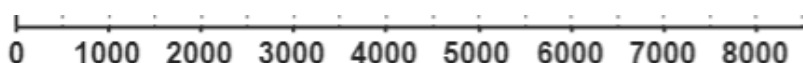


(3)

- (d) Use the graph to calculate the IQR. Show all your working.

(3)

- (e) If it is given that the lowest weight loss was 1500 grams and the highest weight loss was 7500 grams, sketch a box and whisker plot from the cumulative frequency curve.



(2)

[12]

QUESTION 3

Medical researchers collect data to gain an understanding of the relationship between variables related to health. Below is data collected on the age and resting heart rate (per minute) of ten patients.

Age (x)	18	20	24	28	29	30	36	39	45	49
Resting heart rate (y)	51	57	59	55	67	68	64	71	67	72

- (a) Use your calculator to determine the equation of the least squares regression line for the data. Leave your answer in the form $\hat{y} = A + Bx$.

3 decimal places

(2)

- (b) (1) Calculate the correlation coefficient for the data.

(1)

- (2) What does the correlation coefficient suggest about the relationship between age and heart rates?

(2)

- (c) A new patient is 33 years old. Use the model in (a) to predict his heart rate.

(2)

- (d) Write down the standard deviation for the **heart rates**.
Correct your answer to one decimal place.

(2)

- (e) What percentage of the patients fell within 1 standard deviation of the mean of the **heart rates**?

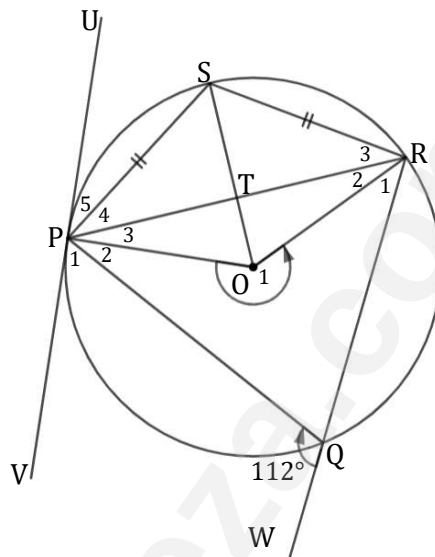
(3)

[12]

QUESTION 4

In the diagram below:

- Points: P, Q, R and S are points on a circle with centre O.
- UV is a tangent to the circle at P.
- PR and OS intersect at T and RQ is produced to W.
- $\widehat{PQW} = 112^\circ$ and $SP = SR$.



Calculate, with reasons, the size of the following angles:

(a) $\widehat{PSR}$

(b) $\widehat{R}_3$ (Show all logical steps)

(c) $\widehat{P}_5$

(d) $\widehat{O}_1$

(e) $\widehat{P}_3$

(2)

(3)

(2)

(2)

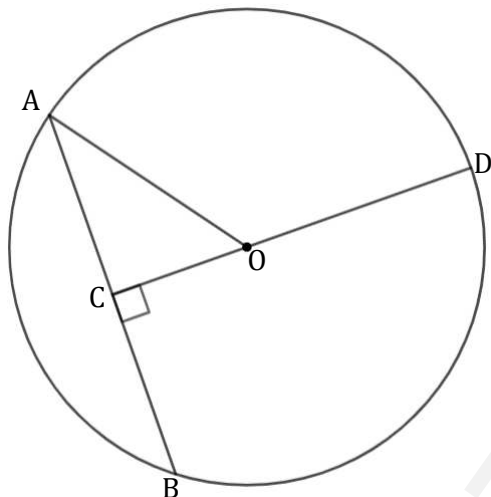
(2)

[11]

QUESTION 5

In the given diagram,

- A, B and D lie on the circle with centre O.
- $AB = CD = 10$ units and $AB \perp CD$
- Let $CO = x$



Determine the length of the radius, AO.

[4]

QUESTION 6

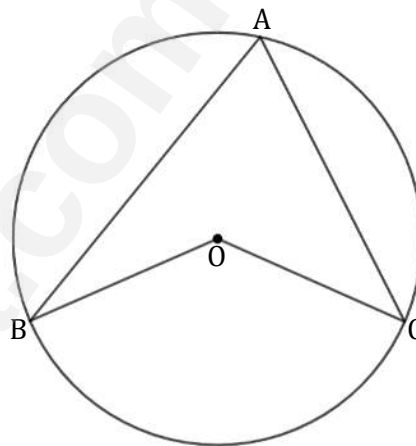
- (a) Prove the theorem which states that the angle which an arc of a circle subtends at the centre of the circle is twice the angle it subtends at the circumference of the circle. Use the given diagram to aid your proof.

Given: Circle with centre O. A, B and C are points on the circumference.

RTP:

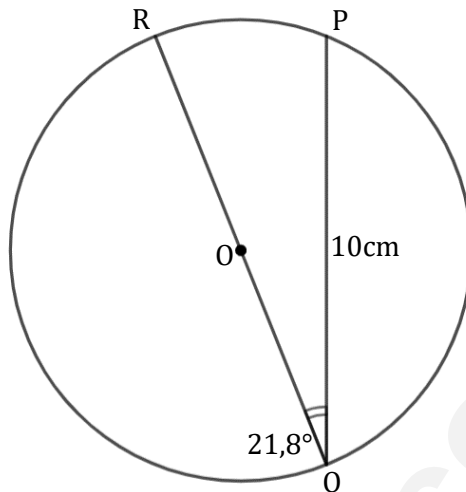
Construction:

Proof:



(5)

- (b) In the given diagram, R, P and Q are points on the circumference of circle with centre, O.
 $PQ = 10\text{cm}$ and $\widehat{Q} = 21,8^\circ$



Calculate the length of RQ.

(4)

[9]

QUESTION 7

(a) Simplify the following **without the use of a calculator**. Show all calculations.

1. $\cos 73^\circ \cos 13^\circ + \sin 73^\circ \sin 13^\circ$

(3)

2. $(1 - \sqrt{2} \sin 15^\circ)(\sqrt{2} \sin 15^\circ + 1)$

(3)

(b) Simplify: $\tan(-\theta) \cdot \sin(90^\circ + \theta) + \frac{\sin 2\theta}{2 \cos(360^\circ + \theta)}$

(6)

(c) Given: $\cos 43^\circ = \sqrt{1 - m^2}$

Express each of the following in terms of m .

1. $\sin 43^\circ$

(2)

2. $\sin 86^\circ$

(2)

(d) 1. Prove the identity: $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$

(5)

2. For what values of x is $2 \tan 2x$ undefined?

(2)

(e) Determine the general solution for: $3 \sin(x + 25^\circ) = \cos(x + 25^\circ)$

(4)

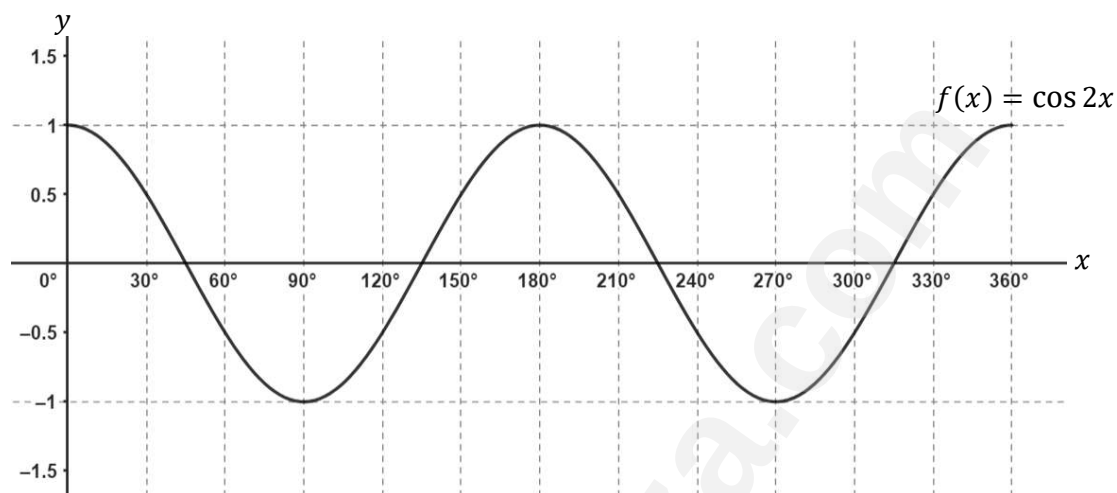
[27]

QUESTION 8

Given $f(x) = \cos 2x$ and $g(x) = \sin(x - 60^\circ)$ for $x \in [0^\circ; 360^\circ]$.

(a) Sketch the graph of g on the set of axes below.

Label intercepts with the y -axis, turning points and end points.



(b) 1. Determine the period of f . (3)

2. Give the range for the function $g(x) - 3$. (1)

3. The function $h(x)$ is obtained by translating the graph of $g(x)$ 10° to the right. Write down the equation of h . (2)

4. For what values of x is $g(x) > 0$ in the given interval? (2)

(c) 1. Determine the values of x for which $\cos 2x = \sin (x - 60^\circ)$ where $x \in [0^\circ; 360^\circ]$.

(5)

2. Hence, determine for what values of x is $g(x) \geq f(x)$ where $x \in [0^\circ; 180^\circ]$.

(2)

(d) Show that the graph of $g(x) = \sin (x - 60^\circ)$ can be written as $g(x) = \frac{1}{2}(\sin x - \sqrt{3} \cos x)$

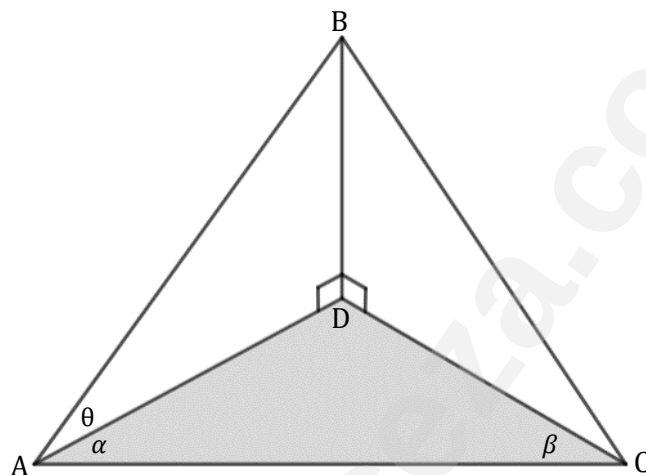
(2)

[19]

QUESTION 9

In the diagram below:

- ADC represents a horizontal triangular field.
- BD represents a vertical building standing on ADC at D.
- $AC = 120$ metres
- $\widehat{DAC} = \alpha$
- $\widehat{ACD} = \beta$
- The angle of elevation of B from A is θ .



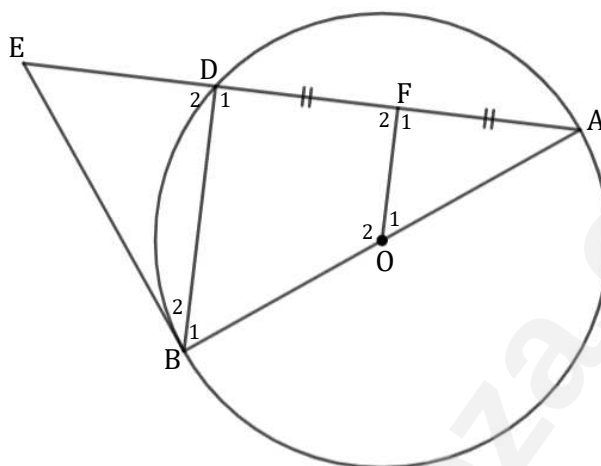
Determine the height of the building BD in terms of α , β and θ .

[5]

QUESTION 10

In the diagram below,

- BOA is the diameter of the circle centred at O.
- BE is a tangent to the circle at B and EA cuts the circle at D.
- F is the midpoint of AD.



Prove, giving reasons, that:

(a) OBEF is a cyclic quadrilateral

(b) $\triangle ADB \parallel \triangle BDE$

(3)

(4)

(c) $OF^2 = \frac{AD \times DE}{4}$

(3)

(d) OB is a tangent to the circle passing through B, D and E.

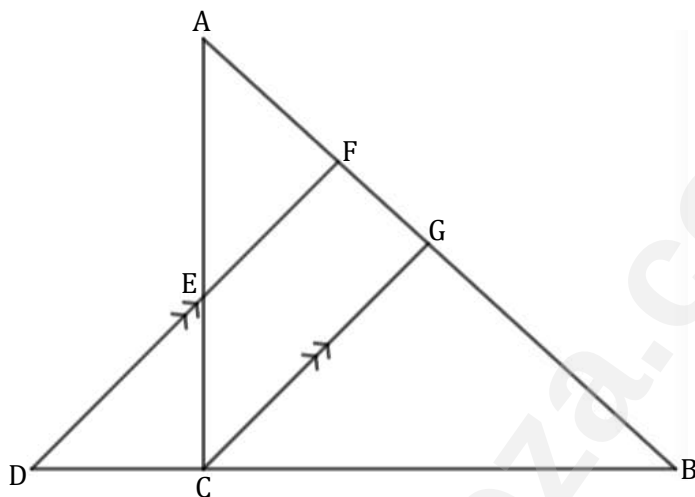
(1)

[11]

QUESTION 11

In the diagram, $\triangle ABC$ is drawn.

- E and F are points on AC and AB respectively such that $\frac{AE}{EC} = \frac{3}{2}$ and $\frac{AF}{FB} = \frac{2}{5}$
- BC produced meets FE produced at D.
- G is a point on FB so that $FD \parallel GC$.



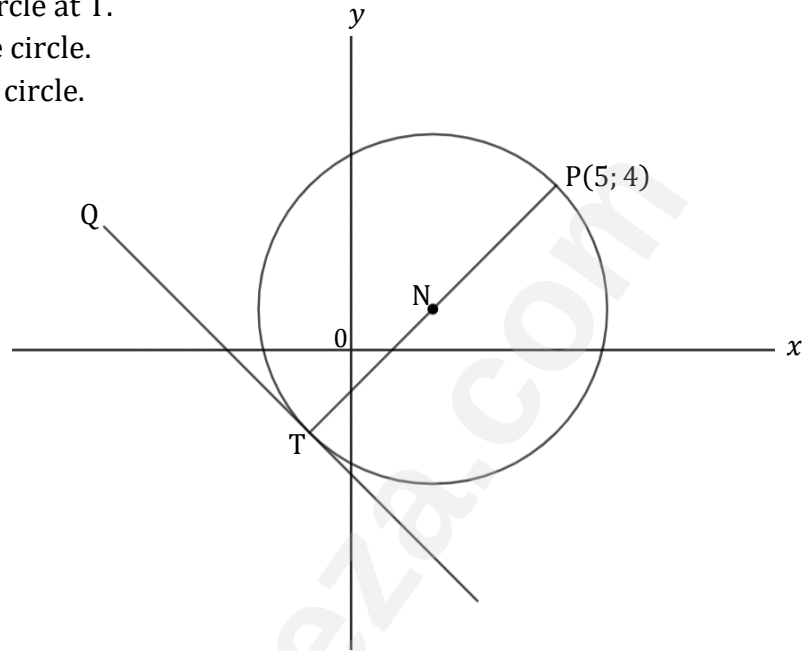
Calculate, giving reasons: $\frac{BC}{CD}$

[5]

QUESTION 12

In the diagram below:

- The equation of the circle with centre N is $x^2 + y^2 - 4x - 2y - 13 = 0$.
- QT is a tangent to the circle at T.
- TNP is a diameter of the circle.
- Point P(5; 4) lies on the circle.



(a) Write the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$

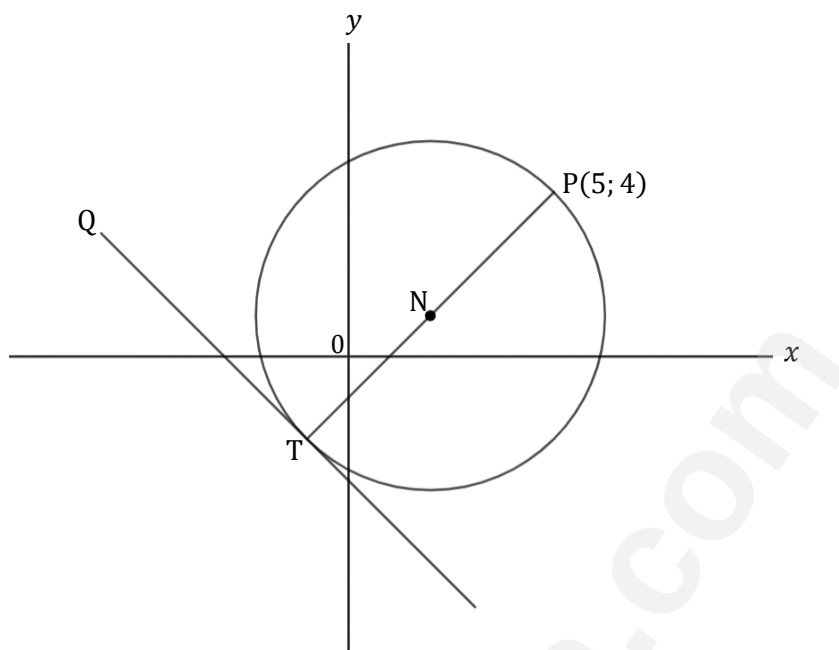
(3)

(b) Write down the coordinates of N and the length of NT. Leave your answer in simplified surd form.

(2)

(c) Show that the point (6; -1) lies outside the circle with centre N.

(3)



(d) Show the coordinates of T are $(-1; -2)$.

(2)

(e) Determine the equation of the tangent QT in the form $y = mx + c$.

(4)

- (f) The circle with centre $S(a; b)$ touches the circle with centre N externally at T .
QT is a tangent to both these circles. If $NS = 3NT$, determine the coordinates of S .

(4)

[18]

eduresourceza.com

Downloaded from eduresourceza.com

eduresourceza.com

Downloaded from eduresourceza.com

eduresourceza.com