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St Alban's College



FORM 5

MATHEMATICS

PRELIM EXAMINATION – MATHEMATICS PAPER 1

9 September 2023

Name & Surname:

Memo

Teacher:

Examiner: T Knoetze

Moderator: K Charalambous

Time: 3 Hours

Total: 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer **ALL** the questions on the question paper and hand it in at the end of the examination. Remember to write your name in the space provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. TWO blank pages (pages 27 and 28) are included at the end of the paper. Should you run out of space for a question, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	A	Q6	Q7	Q8	Q9	Q10	Q11	B	TOTAL
22	11	14	16	16	79	17	7	11	8	16	12	71	150
TK	TK	KS	KS	NM		NM	TK	Mdk	KS	Mdk	Mdk		

SECTION A

QUESTION 1

- (a) (1) Solve for
- x
- :

$$(2x - 3)^2 = 4$$

(4)

$$4x^2 - 12x + 9 - 4 = 0$$

$$4x^2 - 12x + 5 = 0$$

$$(2x - 5)(2x - 1) = 0$$

$$x = \frac{5}{2} \quad \text{or} \quad x = \frac{1}{2}$$

- (2) Hence or otherwise solve for
- x
- if:

$$(2x - 3)^2 < 4$$

(3)

$$\frac{1}{2} < x < \frac{5}{2}$$

$$\text{or } x \in \left(\frac{1}{2}; \frac{5}{2}\right)$$



notation

(b) Solve for x :

(1) $x + 3 = \frac{5}{x}$

(4)

$$x^2 + 3x - 5 = 0 \quad \checkmark a$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)} \quad \checkmark \text{mv subst into formula}$$

$$x = 1, 2 \quad \checkmark \text{ca} \quad \text{or} \quad x = -4, 2 \quad \checkmark \text{ca}$$

(2) $2x = 1 + \sqrt{2x+5}$

(4)

$$(2x-1)^2 = (\sqrt{2x+5})^2 \quad \checkmark a$$

$$4x^2 - 4x + 1 = 2x + 5 \quad \checkmark m$$

$$4x^2 - 6x - 4 = 0$$

$$2x^2 - 3x - 2 = 0 \quad \checkmark ca$$

$$(2x+1)(x-2) = 0$$

$$x \neq -\frac{1}{2} \quad \text{or} \quad x = 2 \quad \checkmark \text{ca}$$

NA -1 if
not

(3) $\log(x+10) = \log x + 1$

(4)

$$\log(x+10) - \log x = 1 \quad \checkmark a$$

$$\log \frac{x+10}{x} = 1 \quad \checkmark m$$

$$10 = \frac{x+10}{x} \quad \checkmark ca$$

$$10x = x+10$$

$$9x = 10$$

$$x = \frac{10}{9} \quad \checkmark ca$$

(c) Determine the value(s) of x for which $2x^2 + x + m = 0$ has non-real roots.

(3)

$$2x^2 + x + m = 0$$

$$\Delta < 0 \quad \checkmark a \quad \checkmark a$$

$$(1)^2 - 4(2)(m) < 0$$

$$1 - 8m < 0$$

$$-8m < -1$$

$$m > \frac{1}{8} \quad \checkmark ca$$

[22]

QUESTION 2

- (a) Matches are used to make the pattern below.

Diagram 1



Diagram 2



Diagram 3



If this pattern continues, calculate the maximum number of houses that can be formed in a row if a total of 196 matches are used.

(4)

$$6 + (n-1)(5) = 196 \quad \checkmark \text{ ca}$$

$$6 + 5n - 5 = 196$$

$$5n + 1 = 196 \quad \checkmark \text{ ca}$$

$$5n = 195 \quad \checkmark \text{ ca}$$

$$n = 39 \quad \checkmark \text{ a}$$

- (b) Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \dots = k$

- (1) Determine the value of k if the series has 21 terms.

(3)

$$S_{21} = \frac{\frac{5}{8} \left(1 - \left(\frac{1}{2} \right)^{21} \right)}{1 - \frac{1}{2}}$$

$$= 1,3 \quad \checkmark \text{ ca}$$

- (2) Determine the largest value of n for which $T_n > \frac{5}{8192}$

(4)

$$\frac{5}{8} \left(\frac{1}{2}\right)^{n-1} > \frac{5}{8192} \quad \checkmark \text{ ca}$$

$$\left(\frac{1}{2}\right)^{n-1} > \frac{1}{1024}$$

$$\left(\frac{1}{2}\right)^{n-1} > \left(\frac{1}{2}\right)^{10} \quad \checkmark \text{ ca}$$

$$n-1 < 10 \quad \checkmark \text{ ca} \quad \left(\frac{1}{2}^x \text{ is a decreasing function}\right)$$

$$n < 11$$

$$\therefore n = 10$$

or

$$2^{-n+1} > 2^{-10}$$

$$-n+1 > -10$$

$$-n > -11$$

$$n < 11$$

$$\therefore n = 10$$

[11]

QUESTION 3

- (a) An amount of R5 600 is invested at $r\%$ per annum compounded quarterly. The investment grows to R7 024, 64 after 2 years. Calculate the value of r . (4)

$$5600 \left(1 + \frac{i}{4}\right)^8 = 7024,64$$

$$\frac{i}{4} = \sqrt[8]{\frac{784}{625}} - 1$$

$$= 0,0287$$

$$i = 0,1149$$

$$\therefore i = 11,5\% \text{ ca}$$

- (b) Mr Jacobs takes out a loan of R850 000 to buy a house. The loan will be repaid over a period of 20 years. He repays the loan by means of equal monthly installments starting one month after the granting of the loan. The interest rate is 8,5 % per annum compounded monthly.

- (1) Determine Mr Jacobs monthly installments. (4)

$$P = \frac{x [1 - (1 + i)^{-n}]}{i}$$

$$850\,000 = \frac{x \left[1 - \left(1 + \frac{0,085}{12}\right)^{-240}\right]}{\frac{0,085}{12}}$$

correct for umulq

$$= R\,7\,376,50 \text{ ca}$$

- (2) Determine the balance outstanding on the loan, immediately after the 144th payment. (3)

$$\begin{aligned}
 BO &= \frac{7376,50 \left[1 - \left(1 + \frac{0,085}{12} \right)^{-96} \right]}{\frac{0,085}{12}} \quad \checkmark \text{ ca} \\
 &= R 512\,537,11 \quad \checkmark \text{ ca} \\
 \text{or } BO &= 850\,000 \left(1 + \frac{0,085}{12} \right)^{144} - \frac{7376,50 \left[\left(1 + \frac{0,085}{12} \right)^{144} - 1 \right]}{\frac{0,085}{12}} \quad \checkmark \text{ a} \\
 &= R 512\,536,31 \quad \checkmark \text{ ca}
 \end{aligned}$$

- (3) What percentage of the 144 payments made has gone towards reducing the amount outstanding? (3)

$$\begin{aligned}
 \text{Amount paid in 12 yrs: } &7376,50 \times 144 \\
 &= R 1062216 \quad \checkmark \text{ ca}
 \end{aligned}$$

$$\begin{aligned}
 \text{BO outstanding after 12 yrs: } &R 512\,537,11 \\
 \therefore &R 850\,000 - 512\,536,31 \\
 &= R 337\,463,69 \quad \checkmark \text{ ca}
 \end{aligned}$$

$$\therefore \frac{337463,69}{1062216} \times 100 = 31,8\% \quad \checkmark \text{ ca} \quad [14]$$

QUESTION 4

- (a) Determine from first principles the derivative of $f(x) = -7x^2 + 7$ (5)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-7(x+h)^2 + 7 - (-7x^2 + 7)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-7(x^2 + 2xh + h^2) + 7 + 7x^2 - 7}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-7x^2 - 14xh - 7h^2 + 7x^2 - 7 + 7}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-14xh - 7h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(-14x - 7h)}{h} \\
 \therefore f'(x) &= -14x
 \end{aligned}$$

(b) Determine $\frac{dy}{dx}$ in each of the following:

(1) $y = (2x - 3)^2$

(3)

$$y = 4x^2 - 12x + 9 \quad \checkmark \quad a$$

$$\frac{dy}{dx} = 8x - 12$$

$\checkmark \quad ca$
 $\checkmark \quad ca$

(2) $y = \frac{2x^2 - 1}{\sqrt{x}}$

(4)

$$y = \frac{2x^2 - 1}{x^{1/2}}$$

$$= \frac{2x^2}{x^{1/2}} - \frac{1}{x^{1/2}}$$

$$= 2x^{3/2} - x^{-1/2} \quad \checkmark \quad a$$

$$\frac{dy}{dx} = 3x^{1/2} + \frac{1}{2}x^{-3/2}$$

$\checkmark \quad ca$
 $\checkmark \quad ca$

- (c) For which value(s) of m will the curve of $y = mx - 2x^3$ have a local minimum at $x = \frac{1}{2}$?

(4)

$$\frac{dy}{dx} = m - 6x^2 \quad \checkmark a$$

For local min $\frac{dy}{dx} = 0$ at $x = \frac{1}{2}$

$$m - 6\left(\frac{1}{2}\right)^2 = 0 \quad \checkmark a$$

$$m - 6\left(\frac{1}{4}\right) = 0$$

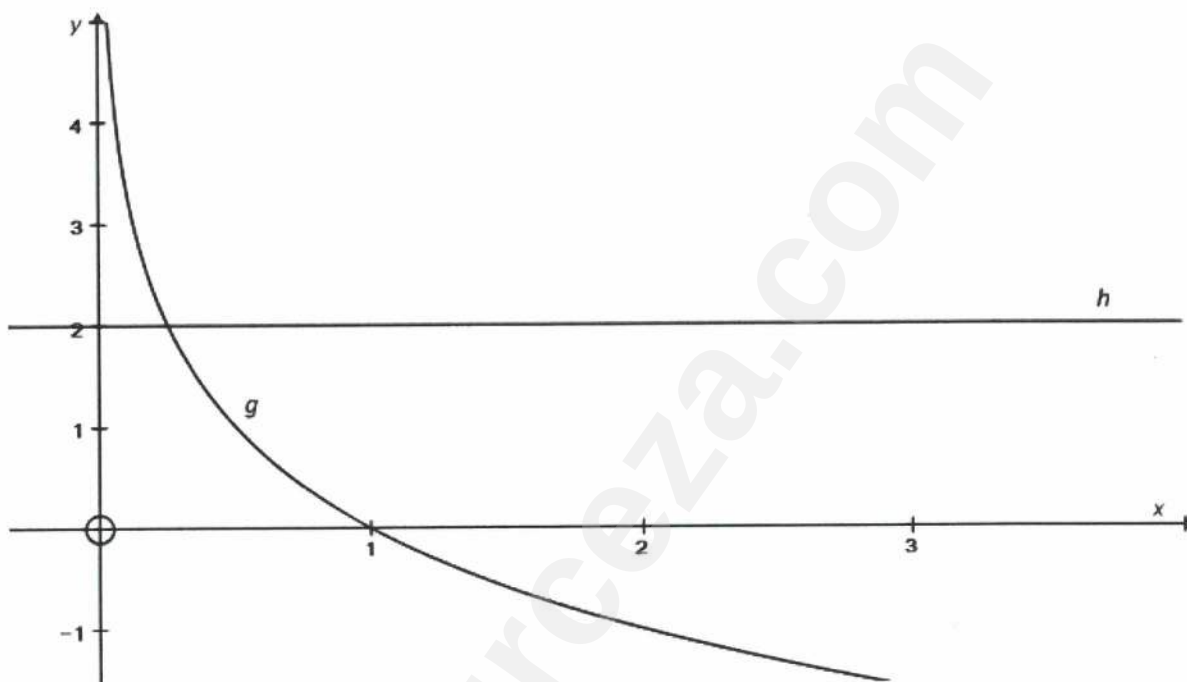
$$m = \frac{6}{4}$$

$$= \frac{3}{2} \quad \checkmark ca$$

[16]

QUESTION 5

- (a) The graphs of functions g and h with equations $g(x) = \log_{\frac{1}{2}} x$ and $h(x) = 2$ are sketched on the Cartesian plane as shown below.



- (1) Write down the domain of g . (1)

$$x \in (0; \infty) \quad \checkmark a$$

- (2) Calculate the values of x for which $h(x) = g(x)$ (2)

$$\log_{\frac{1}{2}} x = 2 \quad \checkmark a$$

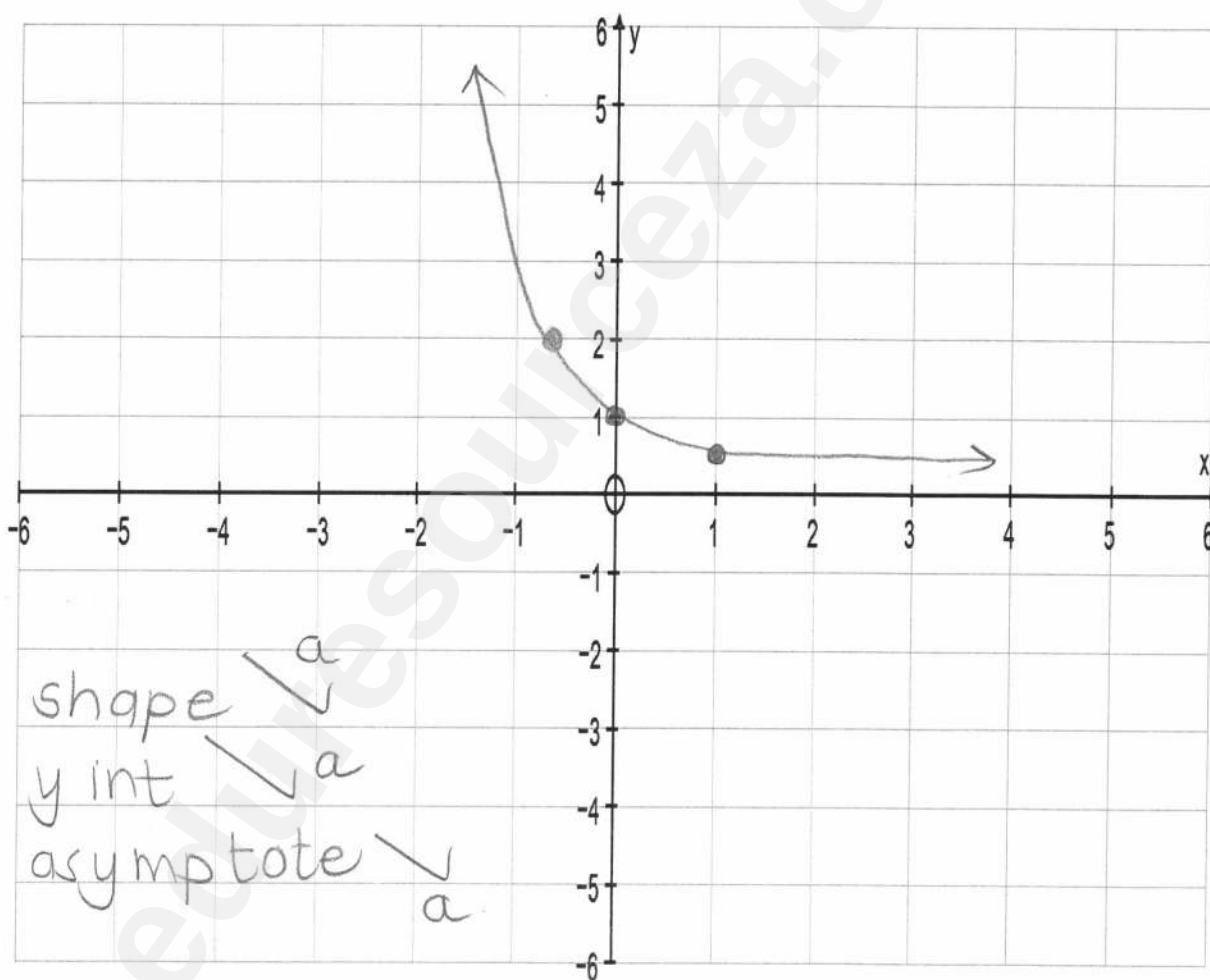
$$\left(\frac{1}{2}\right)^2 = x$$

$$\therefore x = \frac{1}{4} \quad \checkmark ca$$

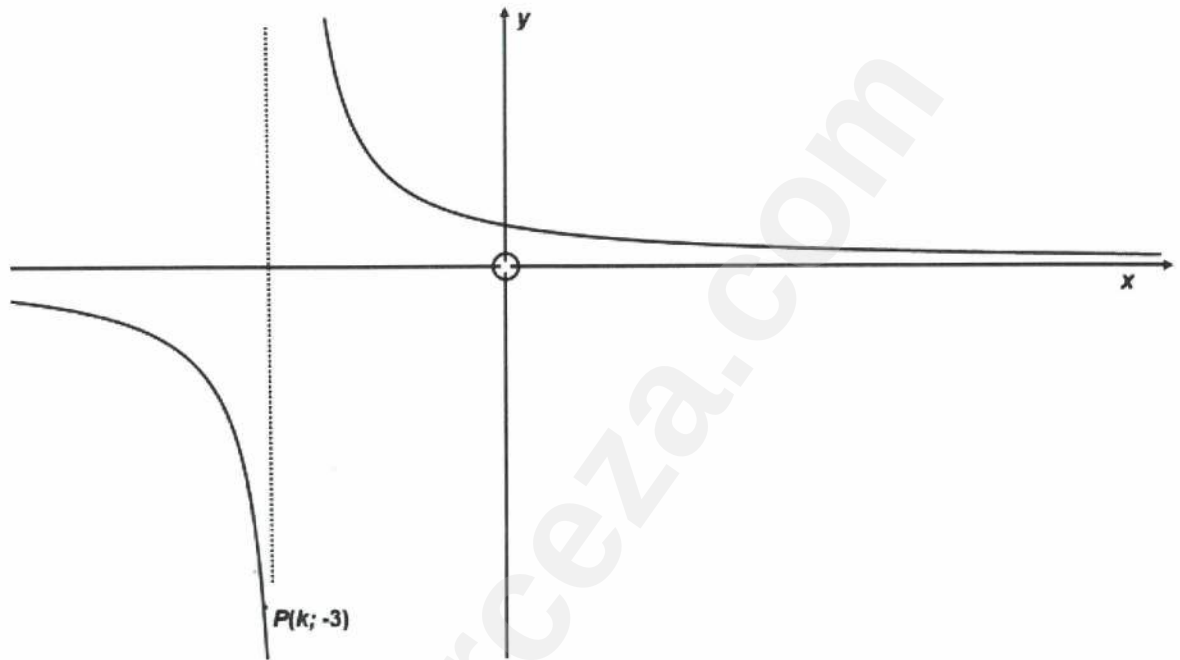
- (3) Describe the transformation of g to f if $f(x) = \log_2 x$. (1)

f is a reflection of g about the x -axis $\searrow a$

- (4) On the cartesian plane below, sketch the graph of p if $p(x) = g^{-1}(x)$. Clearly indicate the intercepts with axes and one additional point. (3)



- (b) The sketch represents the function of g with equation $g(x) = \frac{1}{x+1}$.
The point $P(k; -3)$ lies on the graph of g .



- (1) Determine the value of k , the x coordinate of P .

(3)

$$\begin{aligned}
 g(x) &= \frac{1}{x+1} \\
 -3 &= \frac{1}{k+1} \quad \checkmark \text{ sub} \\
 -3k - 3 &= 1 \quad \checkmark \text{ ca} \\
 -3k &= 4 \\
 k &= -\frac{4}{3} \quad \checkmark \text{ ca}
 \end{aligned}$$

- (2) Determine the equation of the axis of symmetry of g which has a negative gradient. (2)

$$y = -x + c$$

$$0 = -(-1) + c$$

$$1 = c$$

$$\therefore y = -x - 1$$

- (3) For which values of x will $g(x) < 1$. (2)

$$x < -1 \quad \text{or} \quad x > 0$$

- (4) Determine the vertical and horizontal asymptotes of h , if $h(x) = g(-x) + 1$ (2)

$$x = 1 \quad \text{or} \quad y = 1$$

[16]

SECTION A [79]

SECTION B

QUESTION 6

- (a) The 3rd and 4th terms of a geometric sequence are $\frac{1}{3}$ and $\frac{2}{9}$ respectively.

Find the sum to infinity of this sequence.

(5)

$$\begin{aligned}
 T_3 &= ar^2 & ar^2 &= \frac{1}{3} & \dots\dots\dots (1) & \checkmark a \\
 T_4 &= ar^3 & ar^3 &= \frac{2}{9} & \dots\dots\dots (2) & \checkmark a \\
 (2) \div (1) & & ar^3 &= \frac{2}{9} & & \\
 & & ar^2 &= \frac{1}{3} & & \\
 & & \therefore r &= \frac{2}{3} & \checkmark ca & \\
 a\left(\frac{2}{3}\right)^2 &= \frac{1}{3} & \therefore a &= \frac{3}{4} & \checkmark ca & \\
 S_{\infty} &= \frac{\frac{3}{4}}{1 - \frac{2}{3}} & & & & \\
 &= \frac{3/4}{1/3} & & & & \\
 &= \frac{9}{4} & \checkmark ca & & &
 \end{aligned}$$

- (b) Determine n if $\sum_{k=1}^n (12 - 5k) = -2256$

(5)

$$\begin{aligned}
 7 + 2 - 3 &\dots\dots\dots \checkmark a \\
 a &= 7 & -2256 &= \frac{n}{2} [2(7) + (n-1)(-5)] & \checkmark a \\
 d &= -5 & & & \text{subst} \\
 -4512 &= n(14 - 5n + 5) & & & \\
 -4512 &= 14n - 5n^2 + 5n & & & \\
 -4512 &= 19n - 5n^2 & & & \\
 5n^2 - 19n - 4512 &= 0 & \checkmark ca & & \\
 n &= 32 & \text{or } n &= \frac{-141}{5} & \checkmark ca
 \end{aligned}$$

- (c) It is given that the common difference of an arithmetic series is 4.

Prove that $\sqrt{S_{2n} - 2S_n} = 2n$.

(7)

LHS

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$2S_n = n(2a + (n-1)(4)) \quad \checkmark^m \text{ subst.}$$

$$2S_n = n(2a + 4n - 4)$$

$$2S_n = 2an + 4n^2 - 4n \quad \checkmark^a$$

$$S_{2n} = \frac{2n}{2} [2a + (2n-1)(4)] \quad \checkmark^m \text{ subst.}$$

$$= n(2a + 8n - 4)$$

$$= 2an + 8n^2 - 4n \quad \checkmark^a$$

$$\begin{aligned} S_{2n} - 2S_n &= 2an + 8n^2 - 4n - 2an - 8n^2 + 4n \quad \checkmark^a \\ &= 4n^2 \quad \checkmark^a \end{aligned}$$

$$\therefore \sqrt{4n^2} = 2n \quad \checkmark^a$$

$$\therefore \text{LHS} = \text{RHS}$$

[17]

QUESTION 7Given: $f(x) = x^3$

- (a) Show that the graph of f has a point of inflection which is also a stationary point at $(0; 0)$. (2)

$$f'(x) = 3x^2$$

$$\text{at } x = 0$$

$$f'(0) = 3(0)^2$$

$$= 0$$

$$\therefore \text{SP } (0; 0) \checkmark a$$

$$f''(x) = 6x$$

$$6x = 0$$

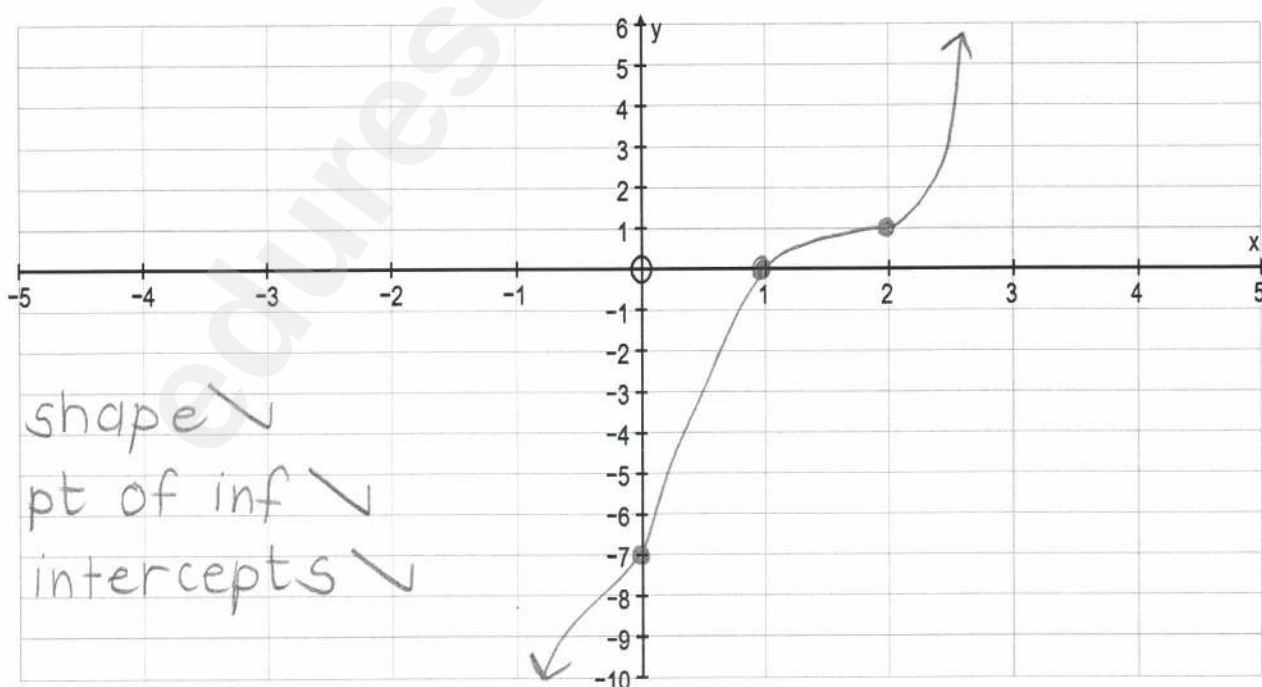
$$x = 0$$

$$\therefore (0; 0) \text{ pt of inf. } \checkmark a$$

- (b) If $g(x) = f(x - 2) + 1$, write down the coordinates of the point of inflection of g . (2)

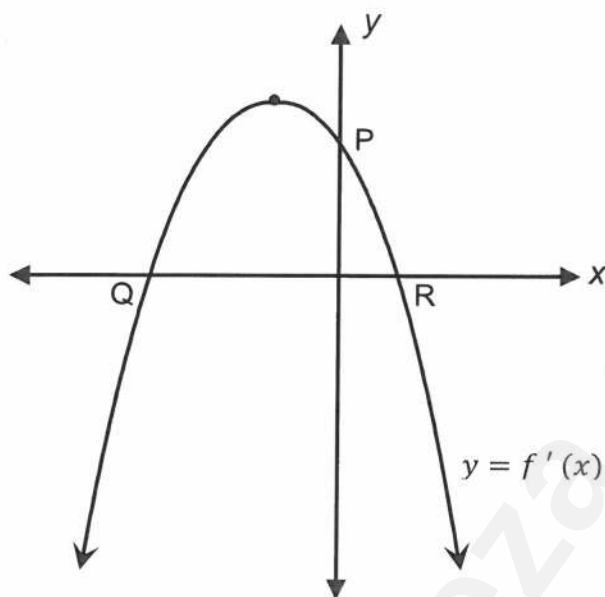
$$(2; 1) \checkmark a$$

- (c) Hence, or otherwise, sketch the graph of g . Show the point of inflection and intercepts with the axes. (3)

**[7]**

QUESTION 8

Sketched below is the graph of $f'(x)$, the derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$.



- (a) Determine the coordinates of P.

(2)

$$f(x) = -2x^3 - 3x^2 + 12x + 20$$

$$f'(x) = -6x^2 - 6x + 12 \quad \checkmark a$$

$$\therefore P(0, 12) \quad \checkmark ca$$

- (b) Determine the coordinates of Q and R. (4)

$$-6x^2 - 6x + 12 = 0 \quad \checkmark a$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \quad \checkmark ca$$

$$x = -2 \quad \text{or} \quad x = 1$$

$$Q(-2; 0)$$

$$R(1; 0)$$

$\checkmark ca$

$\checkmark ca$

- (c) For which values of x will $f(x)$ be increasing? (2)

$$\checkmark ca \quad \checkmark ca$$

$$-2 < x < 1$$

or $x \in (-2; 1)$

- (d) Does Q represent a local minimum or maximum turning point of the graph of f ? Verify your answer. (3)

$$a = -2 \quad \checkmark$$

$$\text{local min} \quad f''(x) > 0$$

$$f''(x) = -12x - 6 \quad \checkmark a$$

$$f''(-2) = -12(-2) - 6 \quad (\text{Q } x = -2)$$

$$= 18$$

$$\therefore 18 > 0 \quad \checkmark a$$

$$\therefore \text{local min at } x = -2 \quad \checkmark a$$

[11]

QUESTION 9

A magazine company has 6 480 subscribers, each of whom pays an annual price of R400. Advisers have told them that for every R1 decrease in the annual subscription, there will be 24 new people prepared to become subscribers. Suppose that the company decides to reduce the annual subscription by x Rands.

- (a) Show that the Revenue (R) that the magazine company will now generate can be represented by the equation:

$$R(x) = -24x^2 + 3120x + 2592000 \quad (4)$$

Reduced subsc $400 - x$
 No of subsc $6480 + 24(x)$
 Revenue: $(400 - x)(6480 + 24x)$ product
 $= 2592000 + 9600x - 6480x - 24x^2$
 $= -24x^2 + 3120x + 2592000$ a.

- (b) The company wishes to maximise its revenue. Determine the number of subscribers and the reduced price that will generate the maximum revenue. (4)

$$R(x) = -24x^2 + 3120x + 2592000$$

$$R'(x) = -48x + 3120 \quad \text{a.}$$

$$-48x + 3120 = 0$$

$$-48x = -3120$$

$$x = 65 \quad \text{ca.}$$

No of subsc $6480 + 24(65) = 8040$ a.

Reduced price $400 - 65 = R335$ a.

[8]

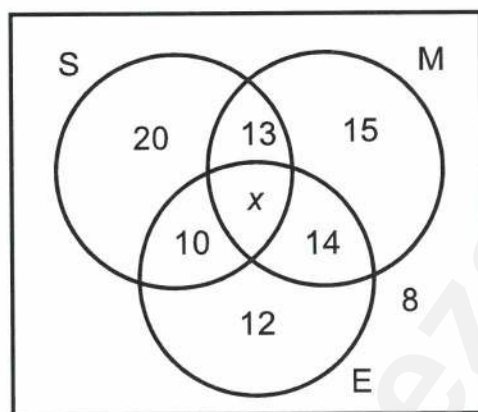
QUESTION 10

- (a) A group of students were asked about the subjects they study. The numbers who study Science, Mathematics and Economics are shown in the Venn diagram below. The number of students who study all three subjects is x .

S is the event that a student studies Science.

M is the event that a student studies Mathematics.

E is the event that a student studies Economics.



Given that the events M and E are **independent**, calculate the value of x . (7)

$$P(M \text{ and } E) = P(M) \times P(E)$$

$$P(M \text{ and } E) = \frac{x + 14}{x + 20 + 13 + 10 + 12 + 14 + 15 + 8}$$

$$= \frac{x + 14}{x + 92} \quad \checkmark a$$

$$P(M) \times P(E) = \frac{13 + 14 + 15 + x}{x + 92} \times \frac{10 + 14 + 12 + x}{x + 92}$$

$$= \frac{42 + x}{x + 92} \times \frac{36 + x}{x + 92}$$

$$\therefore \frac{x + 14}{x + 92} = \frac{(42 + x)(36 + x)}{(x + 92)^2} \quad \checkmark a$$

$$(x + 14)(x + 92) = 1512 + 78x + x^2 \quad \checkmark a$$

$$x^2 + 106x + 1288 = 1512 + 78x + x^2$$

- (b) Find the number of different arrangements that can be made with all nine letters in the word CAMERAMAN in each of the following cases.
Assume that the repeated letters are identical.

(1) There are no restrictions.

(3)

$$\begin{array}{r} 9! \quad \checkmark a \\ \hline 3! \quad 2! \quad \checkmark a \\ = 30240 \quad \checkmark a \end{array}$$

(2) The A's occupy the 2nd, 5th and 9th positions.

(3)

$$\begin{array}{r} 6! \quad \checkmark a \\ \hline 2! \quad \checkmark a \\ = 360 \quad \checkmark a \end{array}$$

- (c) A laptop security code consists of a sequence of four digits. What is the probability that a person will choose a code that has no duplicate digits in the sequence?

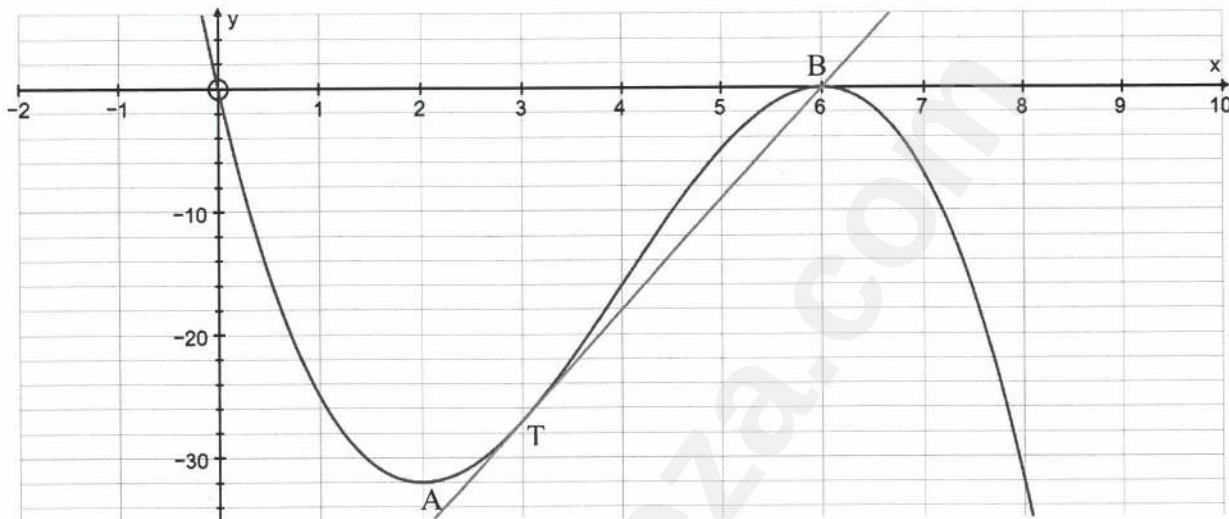
(3)

$$\begin{array}{r} 10 \times 9 \times 8 \times 7 \quad \checkmark a \\ \hline 10 \times 10 \times 10 \times 10 \quad \checkmark a \\ = 5040 \\ \hline 10000 \\ = 0,5 \quad \checkmark ca \end{array}$$

[16]

QUESTION 11

Given below is the graph of $f(x) = -x^3 + bx^2 - cx$. A and B(6; 0) are turning points of $f(x)$.



(a) Show that $b = 12$ and $c = 36$.

(4)

$$f(x) = -x^3 + bx^2 - cx$$

$$f'(x) = -3x^2 + 2bx - c \quad \checkmark a$$

$$f'(b) = 0$$

$$-3(6)^2 + 2b(6) - c = 0 \quad \checkmark a$$

$$-108 + 12b - c = 0$$

$$12b - c = 108 \quad \text{-----} \quad (1)$$

(6; 0) into $f(x)$

$$0 = -(6)^3 + b(6)^2 - c(6) \quad \checkmark a$$

$$0 = -216 + 36b - 6c$$

$$36 = 6b - c \quad \text{-----} \quad (2)$$

$$(2) - (1) \quad 36 = 6b - c$$

$$108 = 12b - c \quad \checkmark a$$

$$\underline{-72 = -6b}$$

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- (b) For which value(s) of x is the graph of f concave down?

(3)

$$f(x) = -x^3 + 12x^2 - 36x$$

$$f'(x) = -3x^2 + 24x - 36$$

$$f''(x) = -6x + 24 \quad \checkmark \text{ ca.}$$

$$-6x + 24 = 0$$

$$-6x = -24$$

$$x = 4 \quad \checkmark \text{ ca.}$$

$$\therefore x > 4 \quad \checkmark \text{ ca.}$$

$\therefore$ concave down

- (c) Determine the value(s) of k for which $f(x) = k$ has only one positive root.

(1)

$$k > 0 \quad \checkmark \text{ a.}$$

- (d) If the x -coordinate of T is 3, determine the equation of the tangent to the curve at T. (4)

$$f'(x) = -3x^2 + 24x - 36$$

at T $x = 3$

$$\begin{aligned} f'(3) &= -3(3)^2 + 24(3) - 36 \quad \checkmark \text{ ca} \\ &= 9 \quad \checkmark \text{ ca} \end{aligned}$$

$$\therefore y = 9x + c$$

$$(3; -27)$$

$$-27 = 9(3) + c \quad \checkmark \text{ ca}$$

$$-54 = c$$

$$\therefore y = 9x - 54 \quad \checkmark \text{ ca}$$

[12]

SECTION B [71]

(Extra Space)

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