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斐京華僑公學

Pretoria Chinese School

MATHEMATICS GRADE 12

Prelim Paper 1 - 2023

DATE: 18 September 2023

TOTAL: 150 Marks

DURATION: 3 hours

EXAMINER: IEB Cluster

MODERATOR: A van Niekerk (snr)

INSTRUCTIONS:

1. This question paper consists of 29 pages (including the cover page). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions in blue pen.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs et cetera that you have used in determining your answering.

Answers only will NOT necessarily be awarded full marks.

6. Diagrams are not necessarily drawn to scale.
7. Round off answers to **TWO** decimal places, unless stated otherwise.
8. It is in your own interest to write legibly and to present your work neatly.

Q 1 (22)	Q 2 (19)	Q 3 (14)	Q 4 (14)	Q 5 (8)	Q 6 (11)		
Q 7 (9)	Q 8 (15)	Q 9 (7)	Q 10 (7)	Q 11 (11)	Q 12 (6)	Q 13 (7)	TOTAL
							150
							%

Name: Memo

SECTION A**[75 MARKS]****QUESTION 1****[22]**a) Given: $(x - k)(x + 3) = k$. Find x if:

(1) $k = 0$

(2)

$$(x - 0)(x + 3) = 0$$

$$\therefore x = 0 \text{ or } x = -3$$

(2) $k = -1$

(3)

$$(x + 1)(x + 3) = -1$$

$$x^2 + 4x + 3 + 1 = 0$$

$$x^2 + 4x + 4 = 0$$

$$(x + 2)^2 = 0$$

$$x = -2$$

b) Solve for x :

(1) $3^{2x-3} = \frac{27}{3^x}$

(3)

$$3^{2x-3} = 3^{-x} \cdot 3^3$$

$$\therefore 2x - 3 = -x + 3$$

$$3x = 6$$

$$x = 2$$

$$(2) \quad \sqrt{x^2 + 3} - \frac{4}{\sqrt{x^2 + 3}} = 0 \quad (4)$$

$$\text{Let } k = \sqrt{x^2 + 3}$$

$$\therefore k - \frac{4}{k} = 0$$

$$k^2 - 4 = 0 \quad \checkmark$$

$$(k - 2)(k + 2) = 0$$

$$\sqrt{x^2 + 3} = 2 \quad \checkmark \text{ or } \sqrt{x^2 + 3} = -2$$

$$x^2 + 3 = 2^2 = 4$$

$$\text{na} \quad \checkmark$$

$$x^2 = 1$$

$$x = \pm 1 \quad \checkmark$$

$$(3) \quad 2 \log_2 5 - \log_2(x - 1) = 1 \quad (4)$$

$$\log_2 5^2 - \log_2(x - 1) = 1$$

$$\log_2 \left(\frac{25}{x-1} \right) = 1$$

$$\frac{25}{x-1} = 2^1 \quad \checkmark$$

$$\frac{25}{2} = x - 1$$

$$\frac{27}{2} = x \quad \checkmark$$

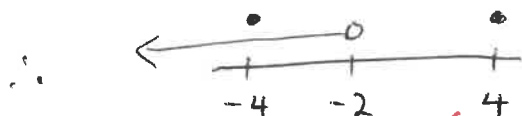
$$13,5 = x$$

c) If $g(a) = \frac{\sqrt{a+2}}{a^2-16}$, determine the values of a for which:

(1) $g(a)$ is undefined (2)

$$a+2 < 0 \quad \text{or} \quad a^2 - 16 = 0$$

$$a < -2 \quad a = \pm 4$$



$$\therefore a < -2 \quad \text{or} \quad a = 4$$

(2) $g(a)$ non-real (1)

$$a < -2$$

d) Determine all values of k for which the equation $kx^2 + kx + 3 - k = 0$ has real roots. (3)

$$b^2 - 4ac \geq 0$$

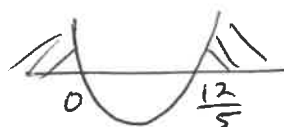
$$k^2 - 4(k)(3-k) \geq 0$$

$$k^2 - 12k + 4k^2 \geq 0$$

$$5k^2 - 12k \geq 0$$

$$k(5k - 12) \geq 0$$

$$CV: k = 0 / \frac{12}{5}$$



$$\therefore k \leq 0 \quad \text{or} \quad k \geq \frac{12}{5}$$

QUESTION 2**[19]**

- a) Determine the n^{th} term of the sequence $3 ; -2 ; -11 ; -24 ; \dots$ (4)

$$\begin{array}{ccc} & \swarrow & \searrow \\ 3 & & -2 \\ & \swarrow & \searrow \\ & -5 & -9 \\ & \swarrow & \searrow \\ & & -13 \\ & \swarrow & \searrow \\ & -4 & -4 \end{array}$$

$$2a = -4 \quad | \quad 3(-2) + b = -5 \quad | \quad -2 + 1 + c = 3$$

$$a = -2 \quad | \quad b = 1 \quad | \quad c = 4$$

$$\therefore T_n = -2n^2 + n + 4$$

- b) Given the sequence: $\frac{3}{4} ; \frac{3}{2} ; 3 ; \dots$

- (1) Determine (in simplest exponential form), T_{21} . (2)

$$4 ; 2 ; 1$$

$$T_1 = 2^{3-1}$$

$$T_2 = 2^{3-2}$$

$$T_3 = 2^{3-3}$$

$$\therefore T_n = 2^{3-n}$$

OR

$$a = \frac{3}{4}, \quad r = 2$$

$$T_{21} = \frac{3}{4} (2)^{20}$$

$$= 3 \cdot 2^2 \cdot 2^{20}$$

$$= 3 \times 2^{18}$$

$$\text{In sequence: } T_n = \frac{3}{2^{3-n}}$$

$$\therefore T_{21} = \frac{3}{2^{3-21}} = 3 \times 2^{18}$$

(2) Which term equals 24 576 ?

(3)

$$T_n = \frac{3}{2^{3-n}} = 24576 \checkmark$$

$$2^{3-n} = \frac{3}{24576}$$

$$3-n = \log_2 \left(\frac{3}{24576} \right) \checkmark$$

$$= -13$$

$$n = 16 \checkmark$$

c) For which values of x will this geometric series $\sum_{k=1}^{\infty} 3(4-x)^k$ converge? (4)

$$T_1 = 3(4-x)^1$$

$$T_2 = 3(4-x)^2$$

$$T_3 = 3(4-x)^3$$

$$r = \frac{3(4-x)^2}{3(4-x)} = 4-x$$

$$-1 < r < 1 \checkmark$$

$$-1 < 4-x < 1 \checkmark$$

$$-5 < -x < -3 \checkmark$$

$$\therefore 3 < x < 5 \checkmark$$

d) Determine the value of x if:

$$\sum_{t=1}^{14} (24 - 3t) = \sum_{k=1}^{\infty} 9x^k$$

$$\text{LHS: } a = 24 - 3(1) = 21 \quad (6)$$

$$d = -3$$

$$\begin{aligned} \therefore S_{14} &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{14}{2} [2(21) + 13(-3)] \\ &= 21 \end{aligned}$$

$$\begin{aligned} \text{RHS: } S_n &= \frac{a(r^n - 1)}{r - 1} \quad / \quad S_{\infty} = \frac{a}{1 - x} \\ &= \frac{9x}{1 - x} \end{aligned}$$

$$\begin{aligned} a &= 9x \\ ar &= 9x^2 \end{aligned} \quad \left[\begin{aligned} r &= \frac{9x^2}{9x} = x \end{aligned} \right]$$

$$\therefore 21 = \frac{9x}{1 - x}$$

$$21(1 - x) = 9x$$

$$21 - 21x = 9x$$

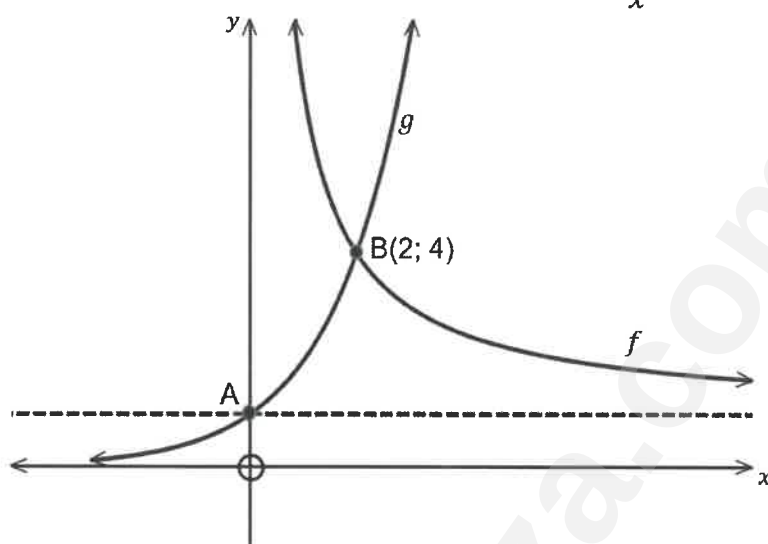
$$21 = 30x$$

$$\frac{21}{30} = x$$

$$\frac{7}{10} =$$

QUESTION 3**[14]**

Sketched below are the graphs of $g(x) = 2^x$ and $f(x) = \frac{a}{x} + q$; $x > 0$.



- a) Determine the values of a and q . (3)

$$q = 1$$

$$y = \frac{a}{x} + 1$$

$$\text{Sub } (2; 4) : 4 = \frac{a}{2} + 1$$

$$3 = \frac{a}{2}$$

$$6 = a$$

- b) Determine the equation of the axis of symmetry of f that has a positive gradient. (2)

$$y = x + c$$

$$\text{Sub } (0; 1) : 1 = 0 + c$$

$$\therefore y = x + 1$$

c) Write down the equations for the following transformations:

(1) k , the inverse of g in the form $k(x) = \dots$ (2)

$$x \leftrightarrow y: x = 2^y \checkmark$$

$$y = \log_2 x$$

$$k(x) = \log_2 x \checkmark$$

(2) p , if it is obtained by moving f , 2 units to the right and 4 units down. (2)

$$\begin{aligned} p(x) &= f(x-2) - 4 \\ &= \frac{6}{x-2} + 1 - 4 \\ &= \frac{6}{x-2} - 3 \checkmark \end{aligned}$$

(3) t , if it is obtained by reflecting g in the y -axis. (1)

$$\begin{aligned} t(x) &= g(-x) \\ &= 2^{-x} \checkmark \end{aligned}$$

d) For which values of x is:

(1) $g(x) \leq f(x)$? (2)

$$x \in (0; 2] \checkmark$$

-1 : incorrect brackets.

(2) $k(x) < 0$? (2)

$$x \in (0; 1) \checkmark$$

QUESTION 4**[14]**

- a) Using first principles, find $f'(x)$ if $f(x) = -3x^2$. (4)

$$\begin{aligned} \textcircled{1} \quad f(x+h) &= -3(x+h)^2 \\ &= -3(x^2 + 2xh + h^2) \\ &= -3x^2 - 6xh - 3h^2 \checkmark \end{aligned}$$

$$\textcircled{2} \quad f(x) = \underline{-3x^2}$$

$$\textcircled{1} - \textcircled{2} \quad \cdot \quad -6xh - 3h^2$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2 \checkmark}{h} = \frac{h(-6x - 3h) \checkmark}{h} \\ &= \lim_{h \rightarrow 0} -6x - 3h = -6x \checkmark \end{aligned}$$

- b) Determine $\frac{dy}{dx}$ in each of the following:

$$(1) \quad y = \frac{-8}{x} - 6\sqrt[3]{x} \quad (4)$$

$$= -8x^{-1} \checkmark - 6x^{\frac{1}{3}} \checkmark$$

$$\frac{dy}{dx} = 8x^{-2} \checkmark - 2x^{-\frac{2}{3}} \checkmark$$

$$(2) \quad y = \frac{25x^2 - 9}{5x + 3} = \frac{(5x - 3)(5x + 3)}{5x + 3} \quad (2)$$

$$= 5x - 3$$

$$\frac{dy}{dx} = 5$$

- c) Find the co-ordinates of a point on the curve $f(x) = x^2 - 3x + 7$ where a tangent to the curve is parallel to the line $y = 3x + 4$. (4)

$$f'(x) = 2x - 3$$

$$m_{\text{tan}} = 3$$

$$\therefore 2x - 3 = 3$$

$$2x = 6$$

$$x = 3$$

$$f(3) = 3^2 - 3(3) + 7$$

$$= 7$$

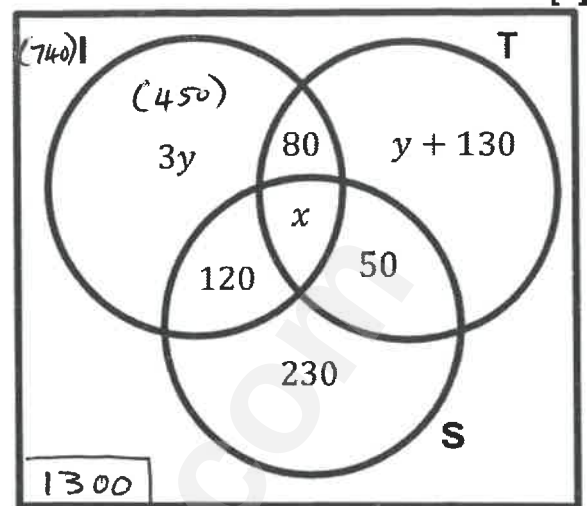
$$\therefore (3; 7)$$

QUESTION 5

The Venn diagram alongside represents the following information:

Vuyi has 1 300 followers on three social media platforms: Twitter, Instagram and Snapchat.

740 follow her on Instagram and 450 follow her on Instagram only.



- a) Determine the values of x and y . (2)

$$\begin{array}{l|l} 3y = 450 & 3(150) + 80 + x + 120 = 740 \\ y = 150 & x = 90 \end{array}$$

- b) Determine the probability that if a follower is randomly selected, they do not follow Vuyi on Twitter but do follow her on Instagram and Snapchat. (2)

$$\begin{aligned} & 120 \div (740 + 150 + 130 + 50 + 230) \\ &= 120 \div (1300) = \frac{6}{65} = 0,09 \end{aligned}$$

- c) Determine, showing all your calculations, whether the event of being Vuyi's follower on Snapchat is independent of being her follower on Instagram. (4)

$$\begin{array}{ll} P(S) \times P(I) & P(S \cap I) \\ = \frac{490}{1300} \times \frac{740}{1300} & = \frac{210}{1300} \\ = 0,2145... & = 0,1615... \end{array}$$

$$\therefore P(S) \times P(I) \neq P(S \cap I)$$

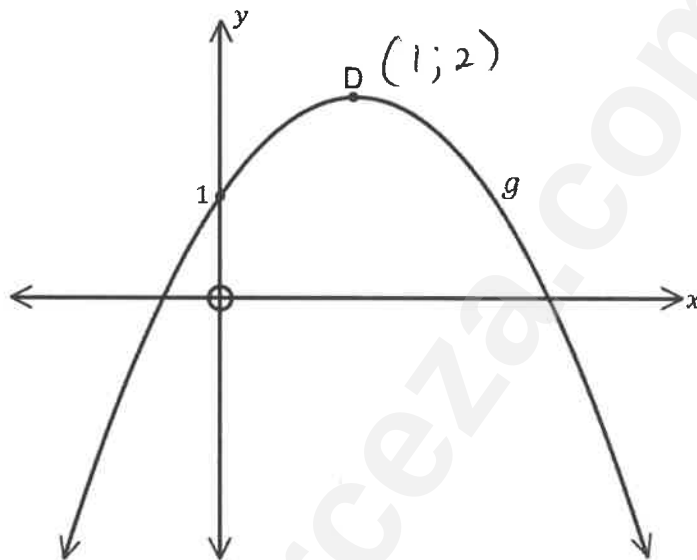
$\therefore$ Not independent.

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SECTION B**[75 MARKS]****QUESTION 6****[11]**

Sketched below is the graph $g(x) = -(x - 1)^2 + q$, where q is a constant. The y -intercept is 1, and D is the turning point.



- a) Determine the coordinates of the D. (2)

$$\text{Sub } (0; 1): 1 = -(0 - 1)^2 + q$$

$$1 = -(1) + q$$

$$2 = q$$

$$D(1; 2)$$

- b) Determine the value of x for which $g(x) > 0$. (3)

$$-(x - 1)^2 + 2 = 0$$

$$(x - 1)^2 = 2$$

$$x = 1 \pm \sqrt{2}$$

$$\therefore x \in (1 - \sqrt{2}; 1 + \sqrt{2})$$

$$(-0,4; 2,4)$$

- c) Explain why the inverse of g is not a function. (1)

The relationship will be 1 to many ✓

- d) Write down the equation of the inverse of $g(x)$ in the form $y = \dots$ (3)

$$x \leftrightarrow y: x = -(y-1)^2 + 2 \quad \checkmark$$

$$(y-1)^2 = 2-x$$

$$y-1 = \pm \sqrt{2-x} \quad \checkmark$$

$$y = 1 \pm \sqrt{2-x} \quad \checkmark$$

- e) How can the domain of g be restricted so that g^{-1} is called a function. (1)

$$x \geq 1 \quad \checkmark \quad \text{or} \quad x \leq 1 \quad \checkmark$$

- f) Give the domain of $g^{-1}(x)$. (1)

$$x \in (-\infty; 2] \quad \checkmark$$

QUESTION 7**[9]**

In the construction of a 5 km national road, a truck delivers materials from a depot containing materials. The truck returns to the depot after depositing each load of materials. The first load is deposited 200 metres from the depot, the second 350 metres from the depot, and the third 500 metres from the depot. Each subsequent load is deposited 150 metres from the previous one.

- a) How far is the fourteenth load deposited from the depot? (3)

$$a = 200, d = 150 \quad \checkmark$$

$$\begin{aligned} T_n &= 200 + (n-1)(150) \\ &= 200 + 150n - 150 \\ &= 150n + 50 \quad \checkmark \end{aligned}$$

$$\begin{aligned} T_{14} &= 150(14) + 50 \\ &= 2150 \text{ m} \quad \checkmark \end{aligned}$$

- b) How many loads are deposited in total? (3)

$$T_n = 150n + 50 = 5000 \quad \checkmark$$

$$150n = 4950$$

$$n = 33 \quad \checkmark$$

c) How many kilometres has the truck travelled in total?

(3)

$$\begin{aligned} S_{33} &= \frac{33}{2} [2a + (n-1)d] \\ &= \frac{33}{2} [2(400) + 32(300)] \\ &= 171\,600 \text{ m} \\ &= 171,6 \text{ km} \end{aligned}$$

✓ double up
✓ correct
F, correct
subs

QUESTION 8**[15]**

- a) Danica buys a Suzuki Swift for R200 000.

The value of car depreciate at the rate of 20 % per annum on the reducing balance depreciation, while the price of a new similar car is expected to appreciate by 10 % per annum based on the previous year's value. The car is expected to last for 8 years.



- (1) Find the price of a new Suzuki Swift in 8 years' time.

(2)

$$A = 200\,000(1 + 0,1)^8 \quad \checkmark \text{ subs}$$

$$= R\,428\,717,76 \quad \checkmark$$

- (2) Find the trade-in value of the Suzuki Swift in 8 years' time.

(2)

$$A = 200(1 - 0,2)^8 \quad \checkmark \text{ subs}$$

$$= 33\,554,43 \quad \checkmark$$

- (3) Find the value of the sinking fund that will be required to purchase a new Suzuki Swift in 8 years' time.

(1)

$$428\,717,76 - 33\,554,43$$

$$= 395\,163,33 \quad \checkmark$$

- (4) Monthly payments are to be made into the sinking fund. The account generates interest at the rate of 18 % p.a. compounded monthly. Payments commence one month after the car is bought and continues until the end of 8 years. Determine the value of the monthly payments into the sinking fund. (3)

$$395\,163,33 = \frac{x \left[\left(1 + \frac{0,18}{12} \right)^{96} - 1 \right]}{\frac{0,18}{12}} \quad \checkmark F \quad \checkmark \text{Subs}$$

$$x = 1\,866,44 \quad \checkmark$$

- b) Tino also bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 15,4 % p.a. compounded monthly.

- (1) Calculate Tino's monthly instalments.

$$500\,000 = \frac{x \left[1 - \left(1 + \frac{0,154}{12} \right)^{-60} \right]}{\frac{0,154}{12}} \quad (3) \quad \checkmark F \quad \checkmark \text{Subs}$$

$$x = 12\,000,21 \quad \checkmark$$

- (2) Tino decided to pay R12 000 each month as his repayment. Calculate the outstanding balance on the car after 2 years.

$$OB = 500\,000 \left(1 + \frac{0,154}{12} \right)^{24} - \frac{12\,000 \left[\left(1 + \frac{0,154}{12} \right)^{24} - 1 \right]}{\frac{0,154}{12}} \quad (4)$$

$$= 344\,230,57 \quad \checkmark$$

QUESTION 9**[7]**

- a) The digits 1 to 7 are used to create a four-digit code to enter a locked room. (2)
How many different codes are possible if the digits may not be repeated?

$$\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} = 840 \checkmark \checkmark$$

- b) What is the probability that any random number chosen, will be a number that is even and bigger than 5000? Show the method you use in detail. (5)
No repeats.

1 2 3 4 5 6 7

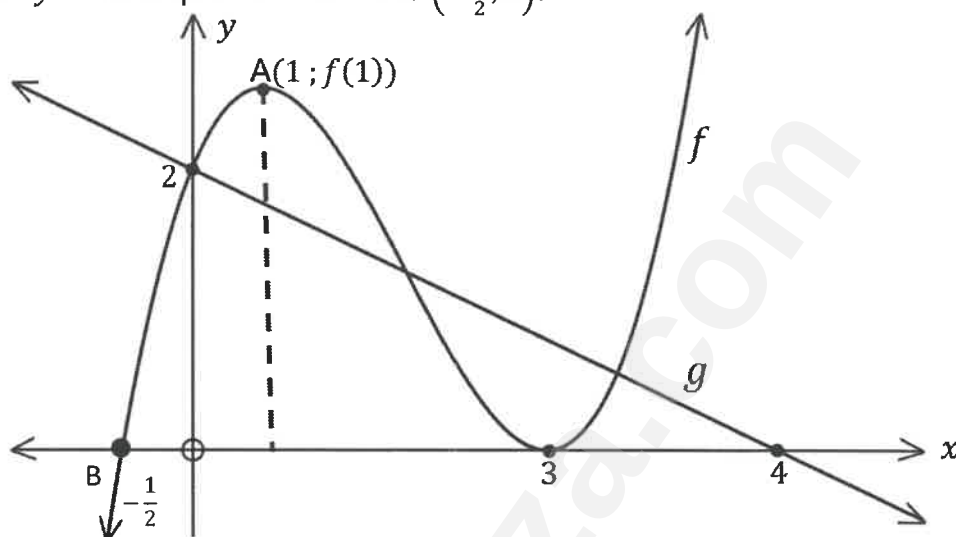
Start:

$$\begin{array}{l} \boxed{5} \underline{5} \underline{4} \underline{3} \rightarrow 2, 4, 6 = 60 \checkmark \\ \boxed{6} \underline{5} \underline{4} \underline{2} \rightarrow 2, 4 = 40 \checkmark \\ \boxed{7} \underline{6} \underline{5} \underline{3} \rightarrow 2, 4, 6 = 90 \checkmark \\ \hline 190 \end{array}$$

$$\begin{aligned} P &= \frac{C}{T} = \frac{190 \checkmark}{840} \\ &= \frac{19}{84} \checkmark \\ &= 0,23 \end{aligned}$$

QUESTION 10**[7]**

In the diagram below $f(x)$ has turning points at $x = 1$ and $x = 3$. The x -intercept of g is 4 and the y -intercept is 2. Point B is $(-\frac{1}{2}; 0)$.



a) Evaluate the following:

- (1) For which values of k will $f(x+k) = 0$ have 1 positive and 1 negative root? (2)

$$-\frac{1}{2} < k < 3$$

- (2) $f'(3)$ (1)

$$= 0$$

b) Determine the value(s) of x for which $f(x)$ is concave down. (2)

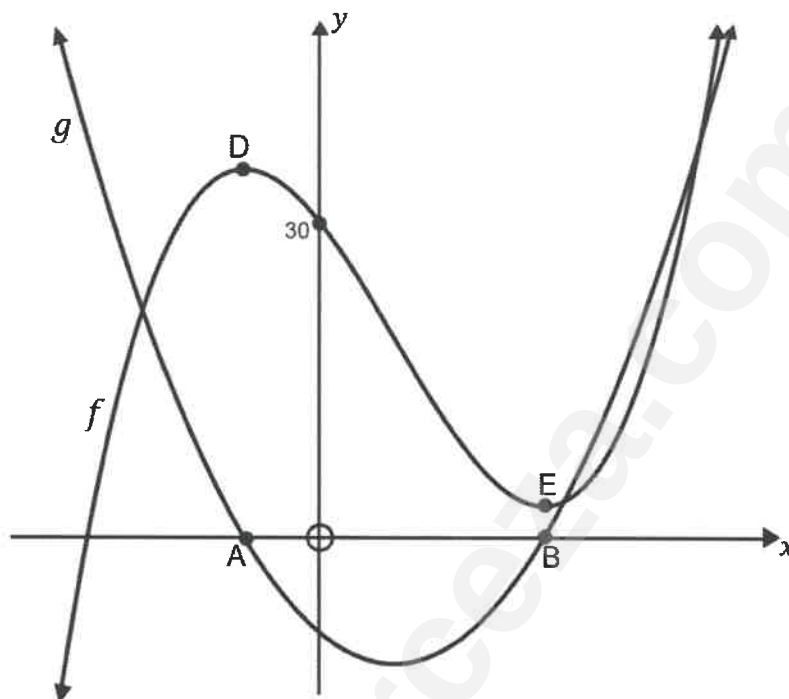
$$POI_x = \frac{1+3}{2} = 2 \Rightarrow x < 2$$

c) Find x if $\frac{f'(x)}{g(x)} \geq 0$ when $x \geq 0$. (2)

$$\begin{aligned} + & : x < 1 \text{ or } 3 < x < 4 \\ + & : \therefore 0 \leq x < 1 \\ - & : \text{na} \end{aligned}$$

QUESTION 11**[11]**

In the diagram below, the graph of $y = g(x) = 3x^2 - 6x - 9$, has x intercepts at A and B. D and E are stationary points of the cubic graph $y = f(x)$.



Determine:

- a) the coordinates of A and B.

(3)

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$\therefore x = 3 / -1$$

$$\therefore A(-1; 0)$$

$$B(3; 0)$$

- b) the equation of the cubic graph, if $g(x) = f'(x)$. (4)

$$f(x) = ax^3 + bx^2 + cx + 30 \quad \checkmark$$

$$f'(x) = 3x^2 - 6x - 9$$

also $f'(x) = 3ax^2 + 2bx + c \quad \checkmark$

$$\therefore \begin{array}{c|c|c} 3a = 3 & 2b = -6 & c = -9 \\ a = 1 & b = -3 & \end{array} \quad m \quad \checkmark$$

$$\therefore f(x) = x^3 - 3x^2 - 9x + 30 \quad \checkmark$$

- c) the coordinates of D and E. (2)

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 30 = 35$$

$$f(3) = 3^3 - 3(3)^2 - 9(3) + 30 = 3$$

$$\therefore D(-1; 35) \quad \checkmark, \quad E(3; 3) \quad \checkmark$$

- d) for what values of k will $f(x) = k$ have three solutions? (2)

$$1 < k < 35 \quad \checkmark \quad \checkmark$$

QUESTION 12**[6]**

Vigo, a grade 12 boy, put up his hand during a calculus lesson on sketching cubic graphs and said that he had found a general formula for the x – coordinates of the turning points of the graph: $y = ax^3 + bx^2 + cx + d$.

His formula was as follows: $x = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$. It is interesting that this is so similar to the quadratic formula which we know so well.

- a) Find $\frac{dy}{dx}$ and thus derive Vigo's formula by showing all working. (4)

$$\frac{dy}{dx} = 3ax^2 + 2bx + c = 0$$

$$x = \frac{-2b \pm \sqrt{(2b)^2 - 4(3a)(c)}}{2(3a)} \quad \checkmark \text{ subs}$$

$$= \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$$

$$= \frac{-2b \pm \sqrt{4(b^2 - 3ac)}}{6a}$$

$$= \frac{-2b \pm 2\sqrt{b^2 - 3ac}}{6a} \quad \checkmark$$

$$= \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

- b) Hence, or otherwise, prove that the graph of $f(x) = x^3 + 2x^2 + 10x - 18$ has no turning points. (2)

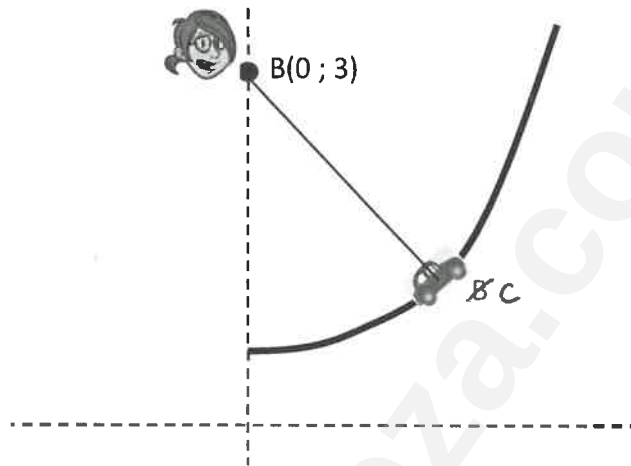
$$\begin{aligned}\sqrt{b^2 - 3ac} &= \sqrt{2^2 - 3(1)(10)} \checkmark \text{subs} \\ &= \sqrt{-26} \checkmark\end{aligned}$$

$\therefore$ No R solution

QUESTION 13**[7]**

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2, x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Helen sits at a vantage point $B(0;3)$ and observes a car, ~~B~~^C, travelling along the road.



Calculate the distance between Helen and the car, when the car is closest to Helen.

$$C = (x; x^2 + 2)$$

$$\begin{aligned} \therefore BC^2 &= (0 - x)^2 + (3 - (x^2 + 2))^2 \quad \checkmark \\ &= x^2 + (1 - x^2)^2 \\ &= x^2 + 1 - 2x^2 + x^4 = x^4 - x^2 + 1 \quad \checkmark \end{aligned}$$

BC will be minimum if BC^2 is min.

$$\therefore (BC^2)' = 4x^3 - 2x = 0 \quad \checkmark$$

$$2x(2x^2 - 1) = 0 \quad \checkmark$$

$$x = 0 \quad \text{or} \quad 2x^2 - 1 = 0$$

$$x^2 = \frac{1}{2}$$

$$x = \sqrt{\frac{1}{2}}, x \geq 0 \quad \checkmark$$

$$\begin{aligned} BC^2 &= \left(\sqrt{\frac{1}{2}}\right)^4 - \left(\sqrt{\frac{1}{2}}\right)^2 + 1 \quad \checkmark \\ &= \frac{1}{4} - \frac{1}{2} + 1 \\ &= \frac{3}{4} \end{aligned}$$

$$BC = \frac{\sqrt{3}}{2}$$

$$= 0,87 \quad \checkmark$$

Extra space:

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**MATHEMATICS
INFORMATION SHEET**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^n 1 = n$$

$$T_n = a + (n - 1)d$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 + i)^n$$

$$F = x \left[\frac{(1 + i)^n - 1}{i} \right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 - i)^n$$

$$P = x \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

$$M \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$$

$$y - y_1 = m(x - x_1)$$

$$m = \tan \theta$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$
