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HILTON COLLEGE
TRIAL EXAMINATIONS
SEPTEMBER 2023

MATHEMATICS PAPER 2
MARKING GUIDELINES

EXAMINATION NUMBER:

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TIME: 3 Hours

MARKS: 150

Examiner: Mrs van Heeswijk

Moderator: Mathematics Department

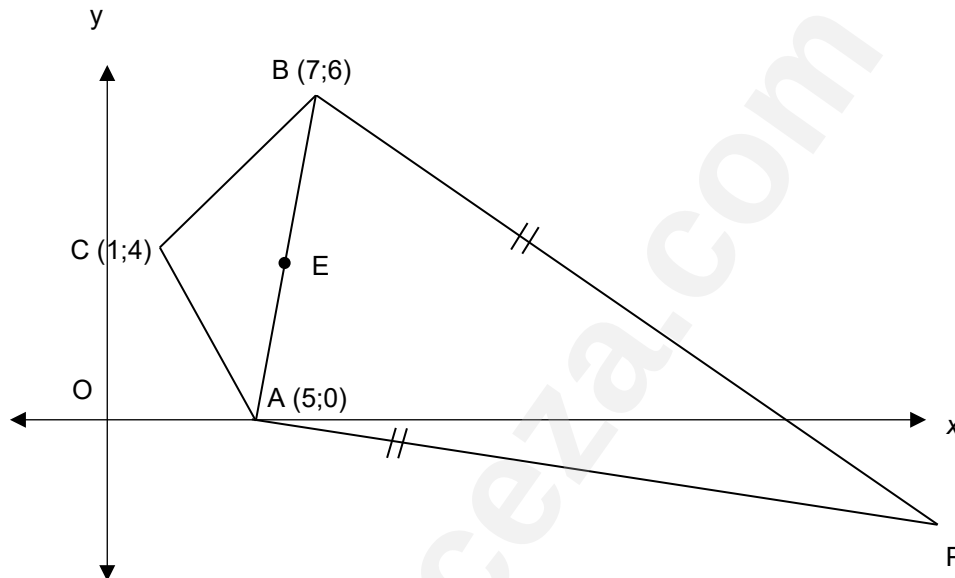
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This Question paper consists of 26 pages as well as a yellow Information sheet of 2 pages.
2. This Question paper consists of 12 questions. Answer all of them.
3. You may use a non-programmable and non-graphical calculator unless a specific question prohibits the use of a calculator.
4. Round off your answers to **one decimal place**, where necessary, unless otherwise stated.
5. All necessary working details must be shown.
6. It is in your own interest to write legibly and present your work neatly.
7. Please note that diagrams are not drawn to scale.

	SECTION A						SECTION B						TOTAL
	1	2	3	4	5	6	7	8	9	10	11	12	
	12	13	12	20	15	6	14	9	17	10	8	14	
Mark													
Total													

SECTION A**[78]****QUESTION 1****[12]**

In the diagram below, which is not drawn to scale, points A (5;0), C (1;4), B (7;6) and P form a quadrilateral. BP = AP and E is the midpoint of AB.



- (a) Determine the co-ordinates of E. (3)

$$E \left(\frac{7+5}{2}, \frac{6+0}{2} \right) \quad \checkmark \text{M to find midpoint}$$

$$= (6;3) \quad \checkmark \text{A}$$

- (b) Determine the gradient of AC. (2)

$$m_{AC} = \frac{4-0}{1-5} \quad \checkmark \text{M find gradient}$$

$$= -1 \quad \checkmark \text{A}$$

- (c) Hence, determine the equation of the line parallel to AC and passing through E. (3)

$$y = -x + c \quad \checkmark \text{CA gradient from (b)}$$

$$\text{Subst (6; 3)} \quad \checkmark \text{M subst point}$$

$$3 = -(6) + c$$

$$c = 9 \quad \checkmark \text{CA with gradient}$$

$$y = -x + 9$$

- (d) Given the length of AC is $4\sqrt{2}$, determine the length of BC and, hence state with reasons whether CBPA is a kite. (4)

$$BC^2 = (7 - 1)^2 + (6 - 4)^2 \quad \checkmark \text{M distance}$$

$$= 40 \quad \checkmark \text{A}$$

$$BC = 2\sqrt{10} \quad \checkmark \text{CA with previous step}$$

$\therefore$ not a kite as not two pairs of adjacent sides equal in length $\checkmark \text{A with reason}$

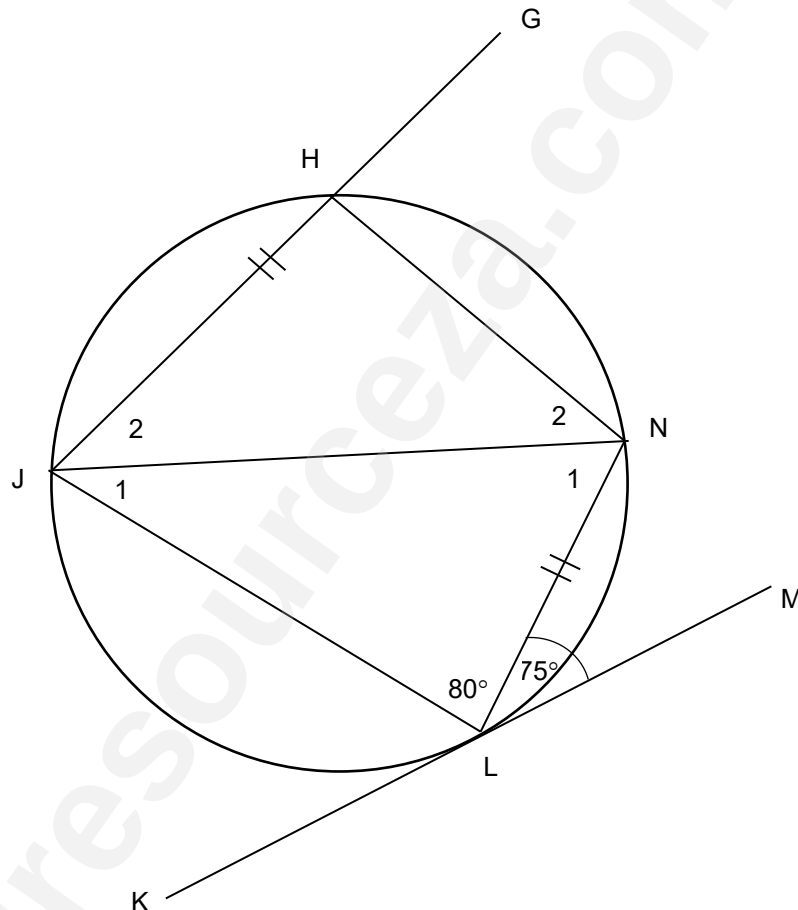
QUESTION 2

[13]

- (a) Complete the statement: The angle formed between a tangent to the circle and the chord drawn at the point of contact, is ...
equal to the angle in the alternate segment ✓A

(1)

- (b) In the diagram, KLM is a tangent to the circle at L. $\hat{NLM} = 75^\circ$ and $\hat{JLN} = 80^\circ$. $JH = LN$.



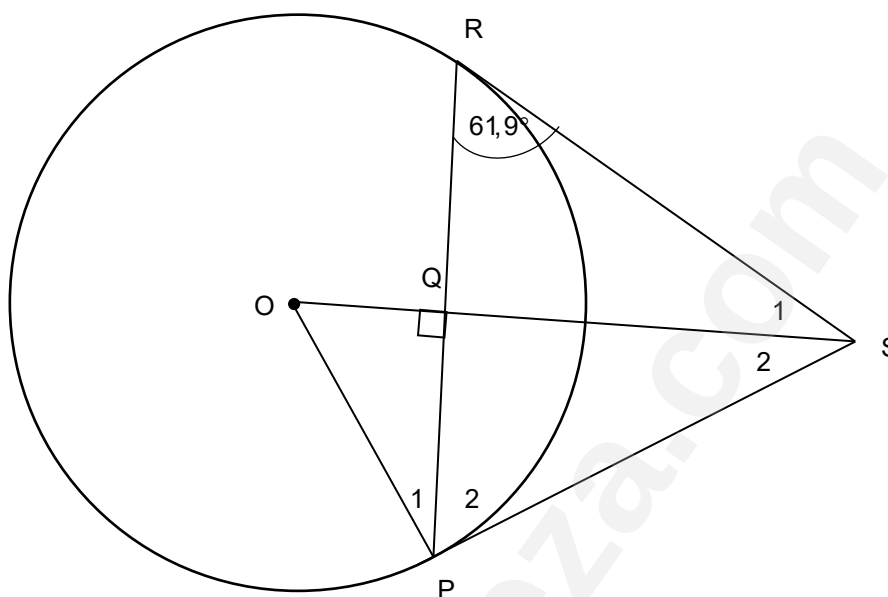
Determine, with reasons, the size of:

- (1) $\hat{J}_1$ (2)
 $\hat{J}_1 = 75^\circ$ tan chord theorem ✓s ✓r

- (2) $\hat{GHN}$ (2)
 $\hat{GHN} = 80^\circ$ ext < cyc quad ✓s ✓r

- (3) $\hat{N}_2$ (2)
 $\hat{N}_2 = \hat{J}_1 = 75^\circ$ <s opp equal chords ✓s ✓r

- (c) In the diagram below, SP and SR are tangents to the circle, centre O.
 $\hat{PRS} = 61,9^\circ$.



- (1) Give the reason why $SP = SR$. (1)

Tangents from common point ✓A

- (2) Prove that ORSP is a cyclic quad. (5)

$$\hat{P}_2 = 61,9^\circ \quad \angle s \text{ opp} = \text{sides} \quad \checkmark s/r$$

$$\hat{OPS} = 90^\circ \quad \text{rad} \perp \text{tangent} \quad \checkmark s/r$$

$$\therefore \hat{P}_1 = 22,1^\circ \quad \checkmark A$$

In $\triangle OQP$

$$\begin{aligned} \hat{O} &= 90^\circ - 22,1^\circ \quad \angle \text{sum } \triangle \quad \checkmark s/r \\ &= 61,9^\circ \end{aligned}$$

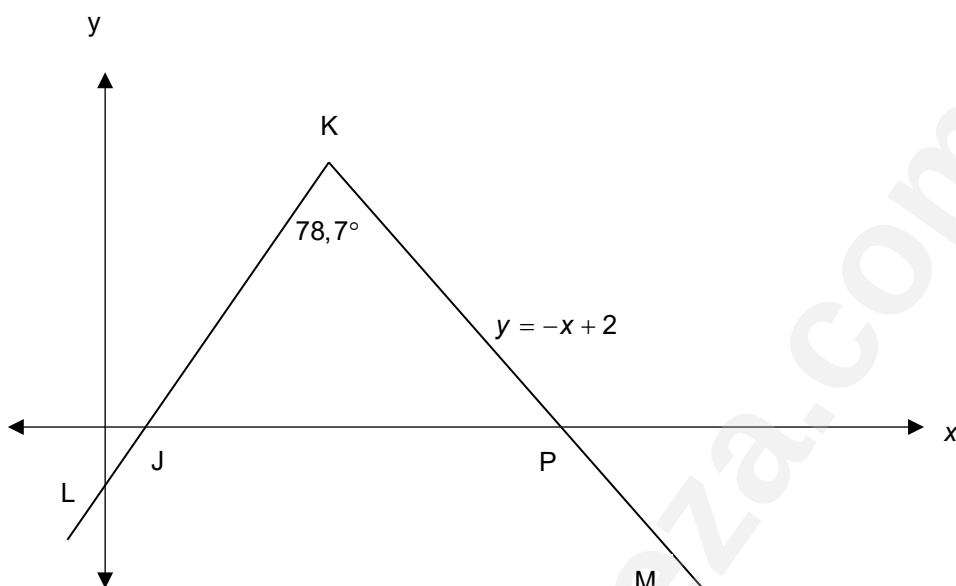
$$\therefore \hat{O} = \hat{R}$$

$\therefore$ ORSP is a cyclic quad $\quad \text{conv } \angle s \text{ in same seg } \quad \checkmark A \text{ concl /reason}$

QUESTION 3

[12]

In the diagram, the equation of KM is $y = -x + 2$. $\hat{K} = 78,7^\circ$.



- (a) Determine the gradient of KL.

(4)

$$\begin{aligned}
 \hat{P}_1 &= \tan^{-1}(1) \\
 &= 45^\circ \quad \checkmark A \\
 \therefore \hat{J}_1 &= 56,3^\circ \quad < \text{sum } \Delta \quad \checkmark s/r \\
 \therefore m_{KL} &= \tan 56,3^\circ \quad \checkmark M \text{ use } m = \tan \theta \quad \checkmark CA \text{ with } \hat{J}_1 \\
 &= 1,5
 \end{aligned}$$

- (b) If L has a y – co-ordinate of -1 and $KL = LP$, determine the length of KL. Leave your answer in simplified surd form.

(4)

$$\begin{aligned}
 -x + 2 &= 0 \quad \checkmark M \text{ find intercept} \\
 \therefore x &= 2 \\
 \therefore P &(2; 0) \quad \checkmark A \\
 \therefore LP^2 &= (0 - 2)^2 - (-1 - 0)^2 \quad \checkmark M \text{ distance formula} \\
 &= 5 \\
 \therefore LP &= \sqrt{5} = KL \quad \checkmark CA \text{ with } P
 \end{aligned}$$

- (c) Determine the value of x for which R $(x; -2)$, L and P are collinear. (4)

$$m_{LP} = \frac{-1-0}{0-2} \quad \checkmark \text{ A gradient}$$
$$= \frac{1}{2}$$

$$\therefore m_{RL} = \frac{1}{2} \quad \checkmark \text{ A}$$

$$\therefore \frac{-2-(-1)}{x-0} = \frac{1}{2} \quad \checkmark \text{ A equation}$$

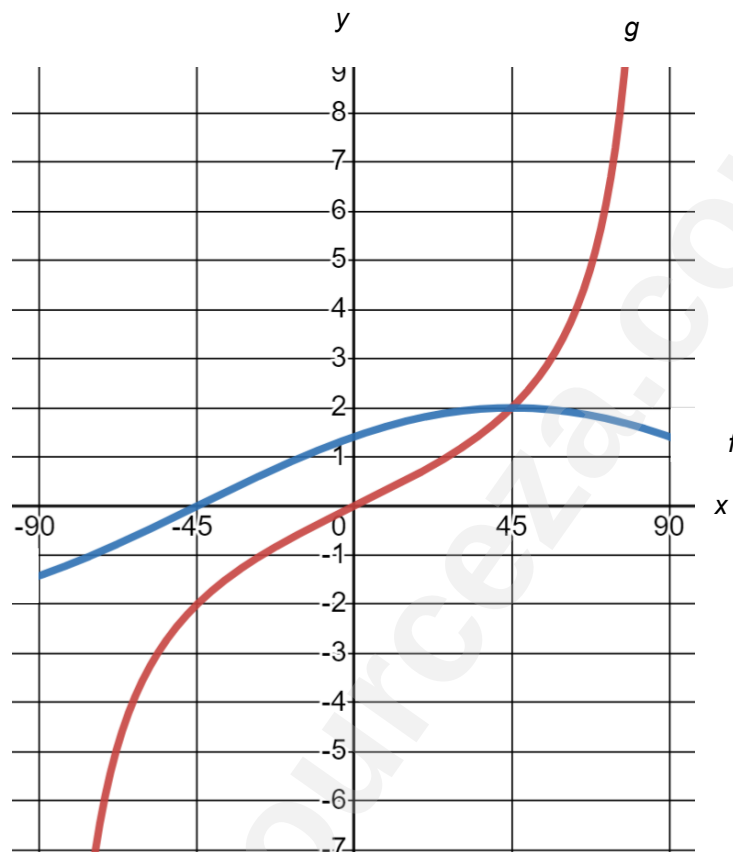
$$-2 = x - 0$$

$$-2 = x$$

$$\checkmark \text{ CA with } m_{RL}$$

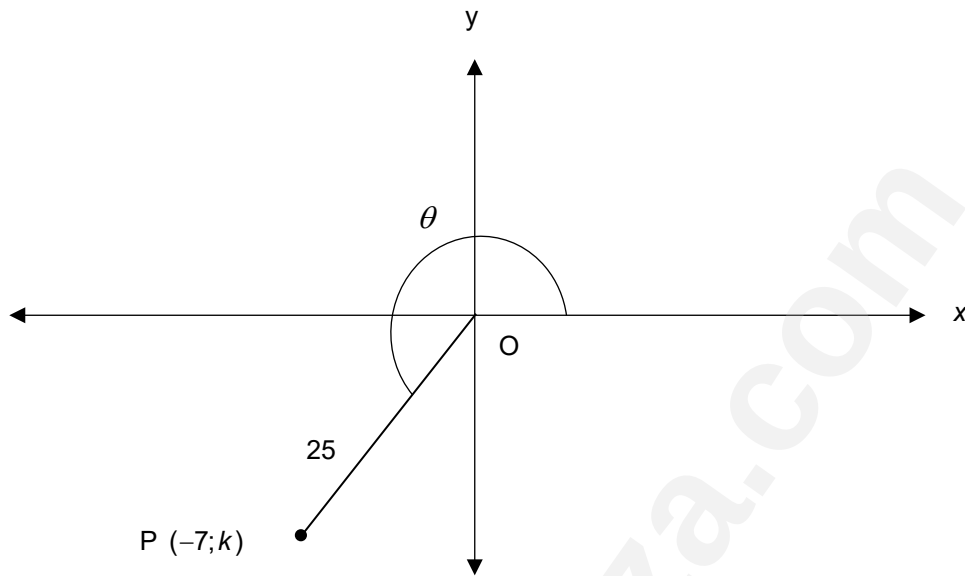
QUESTION 4**[20]**

- (a) The graphs of $g(x) = 2\tan x$ and $f(x) = 2\sin(x + 45^\circ)$ over the interval $x \in [-90^\circ; 90^\circ]$ are shown. Use the graph to answer the questions that follow.



- (1) What is the amplitude of f ? (1)
 2 ✓A
- (2) For which values of x is $f(x) \leq g(x)$ over the given interval? (2)
 ✓A ✓A must not include 90
 $x \in [45^\circ; 90^\circ)$ or $45^\circ \leq x < 90^\circ$
- (3) If $g(x)$ is translated 30° to the right and 2 units up to form $h(x)$, give the equation of h . (2)
 ✓A ✓A
 $h(x) = 2\tan(x - 30^\circ) + 2$

- (b) In the diagram, $P(-7;k)$ is a point in the third quadrant such that $OP = 25$ units.



Determine the value of the following:

(1) k (1)
 $k = -24$ ✓A must be negative

(2) $\sin(360^\circ - \theta)$ (2)

$\sin(360^\circ - \theta)$ ✓A reduction

$= -\sin \theta$

$= -\frac{-24}{25}$

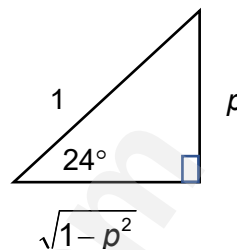
$= \frac{24}{25}$

✓CA with k

- (c) If $\sin 24^\circ = p$, express, **without using a calculator**, the following in terms of p :

(1) $\cos 24^\circ$ (2)

$$\begin{aligned}
 &= \frac{\sqrt{1-p^2}}{1} \quad \checkmark \text{M find adjacent side} \\
 &= \sqrt{1-p^2} \\
 &\checkmark \text{A}
 \end{aligned}$$



(2) $(\sin 168^\circ) \cdot (-\sin 78^\circ)$ (4)

$$\begin{aligned}
 &= (\sin 12^\circ) \cdot (-\cos 12^\circ) \quad \checkmark \text{M reduction} \\
 &= -\sin 12^\circ \cdot \cos 12^\circ \\
 &= -\frac{1}{2} (2 \sin 12^\circ \cdot \cos 12^\circ) \quad \checkmark \text{M double angle} \\
 &= -\frac{1}{2} \cdot \sin(2 \times 12^\circ) \quad \checkmark \text{A} \\
 &= -\frac{1}{2} \cdot \sin 24^\circ \\
 &= -\frac{1}{2} p \\
 &\checkmark \text{A}
 \end{aligned}$$

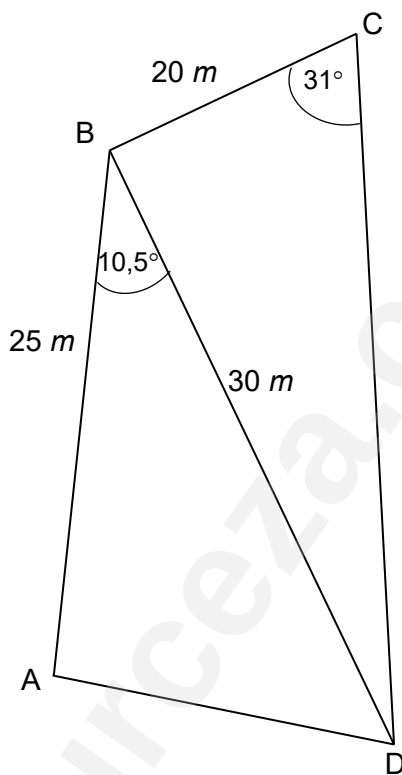
- (d) Prove, **without the use of a calculator**, and showing all working: (6)

$$\frac{\sqrt{3} \sin x \cdot \sin^2 58^\circ + \sqrt{3} \sin^2 212^\circ \cdot \cos(x - 90^\circ)}{\tan 120^\circ \cdot \sin x} = -1$$

$$\begin{aligned}
 \text{LHS} &= \frac{\sqrt{3} \cdot \sin x \cdot \sin^2 58^\circ + \sqrt{3} \cdot \sin^2 32^\circ \cdot \sin x}{-\tan 60^\circ \cdot \sin x} \quad \checkmark \text{A} \quad \checkmark \text{A} \\
 &\quad \checkmark \text{A} \\
 &\quad \checkmark \text{A co-ratio} \\
 &= \frac{\sqrt{3} \cdot \sin x (\sin^2 58^\circ + \cos^2 58^\circ)}{-\sqrt{3} \cdot \sin x} \quad \checkmark \text{M factorise} \\
 &= \frac{\sqrt{3} \cdot \sin x (1)}{-\sqrt{3} \cdot \sin x} = -1 \quad \checkmark \text{M identity} = 1 \\
 \therefore \text{LHS} &= \text{RHS}
 \end{aligned}$$

QUESTION 5**[15]**

- (a) In quadrilateral ABCD, AB = 25 metres, BC = 20 metres, BD = 30 metres, $\hat{ABD} = 10,5^\circ$ and $\hat{BCD} = 31^\circ$.



Determine:

- (1) the size of $\hat{BDC}$. (3)

Using sin rule:

$$\frac{\sin D}{20} = \frac{\sin 31^\circ}{30} \quad \checkmark \text{M sin rule}$$

 $\checkmark$ A value for sinD

$$\therefore \sin D = 0,344...$$

 $\checkmark$ CA with previous step

$$\therefore \hat{D} = 20,1^\circ$$

- (2) the length of AD. (4)

Using cos rule in $\triangle ABD$:

$$\begin{aligned} AD^2 &= 25^2 + 30^2 - 2(25)(30)\cos 10,5^\circ \quad \checkmark \text{M use cos rule} \quad \checkmark \text{A values} \\ &= 50,117... \quad \checkmark \text{CA with values} \end{aligned}$$

$$\therefore AD = 7,1 \text{ m} \quad \checkmark \text{CA with step before}$$

(b) Given the identity: $\frac{\cos 2x + \cos x}{\sin 2x - \sin x} = \frac{\cos x + 1}{\sin x}$

(1) Prove the identity.

(5)

$$\begin{aligned}
 \text{LHS} &= \frac{2\cos^2 x - 1 + \cos x}{2\sin x \cos x - \sin x} && \checkmark \text{A double } \angle s \\
 &= \frac{2\cos^2 x + \cos x - 1}{2\sin x \cos x - \sin x} \\
 &= \frac{\overset{\checkmark \text{A}}{(2\cos x - 1)} \overset{\checkmark \text{A}}{(\cos x + 1)}}{\underset{\checkmark \text{A}}{\sin x} (2\cos x - 1)} && \checkmark \text{M factorise} \\
 &= \frac{\cos x + 1}{\sin x} \\
 &= \text{RHS}
 \end{aligned}$$

(2) For which value(s) of x is the identity undefined for $-90^\circ \leq x \leq 90^\circ$.

(3)

Undefined if $\sin x = 0$

$$\therefore x = 0^\circ \quad \checkmark \text{A}$$

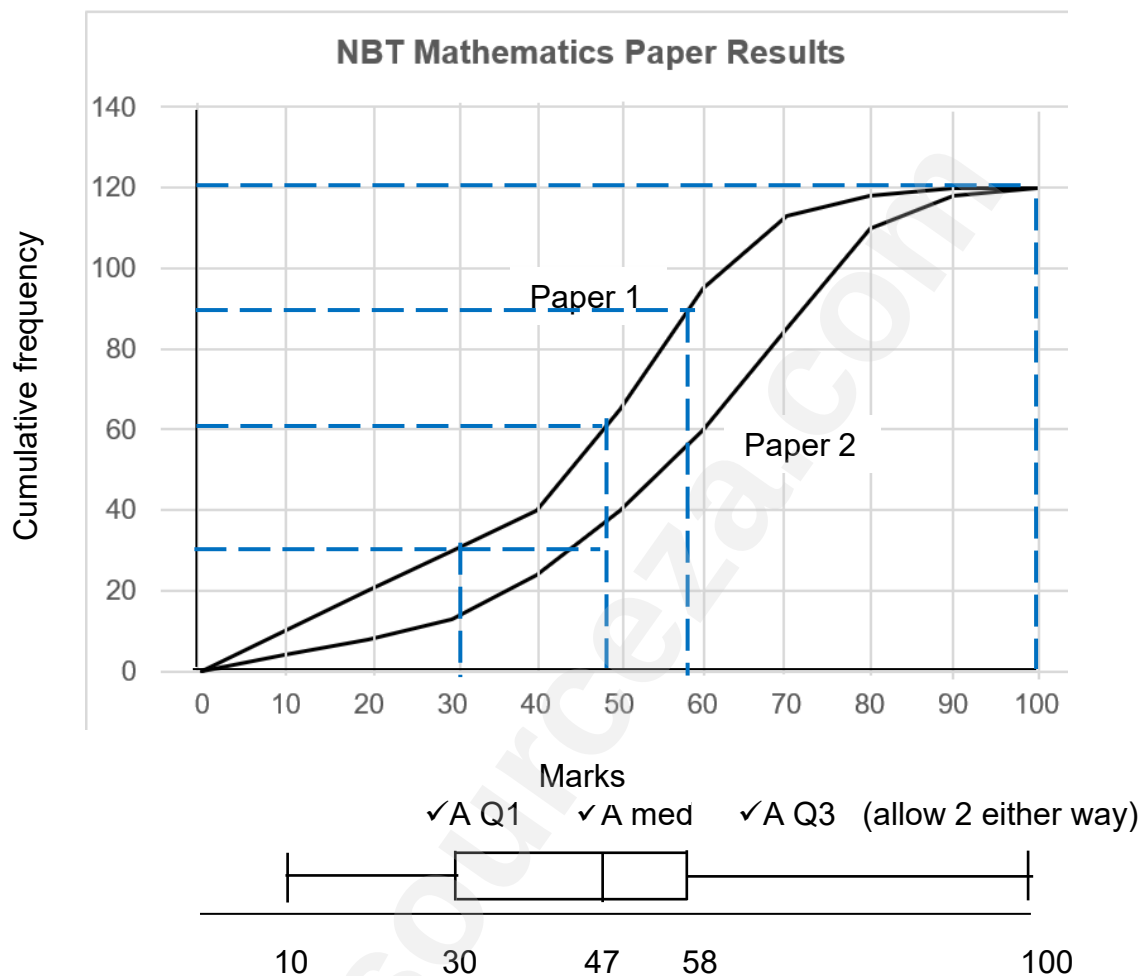
Undefined if $2\cos x - 1 = 0$

$$\therefore \cos x = \frac{1}{2}$$

$$\therefore x = 60^\circ \quad \checkmark \text{A} \quad \text{or} \quad x = -60^\circ \quad \checkmark \text{A}$$

QUESTION 6**[6]**

A sample of marks from two benchmark tests is shown.



- (a) In which paper were the results more favourable? Explain your reasoning. (2)

Paper 2: higher quartiles and median ✓A answer ✓A reason

- (b) Draw a box and whisker plot under the graph using the cumulative frequency graph of Paper 1. The lowest mark was 10%. Using your plot, comment on the distribution of the results for this paper. (4)

Data is skewed left/ negatively skewed as $Q3 - M < M - Q1$ ✓CA with box and whisker

SECTION B

[72]

QUESTION 7

[14]

- (a) **Without the use of a calculator**, and showing all working, determine the values of α and β if: (7)

- $\sin(\alpha - \beta) = \frac{1}{2}$
- $\sin(\alpha + \beta) = -1$
- α and β are both positive
- $\sin \alpha < 0$
- $\beta < 90^\circ$

$$\alpha - \beta = 30^\circ$$

$$\alpha + \beta = 270^\circ$$

✓M set up equations ✓A equations

$$\therefore 2\alpha = 300^\circ$$

✓M solve sim eqns

$$\therefore \alpha = 150^\circ$$

✓A with reject

but $\sin \alpha > 0$

$$\therefore \alpha \neq 150^\circ$$

or

$$\alpha - \beta = 150^\circ$$

$$\alpha + \beta = 270^\circ$$

✓A equations

$$\therefore 2\alpha = 420^\circ$$

$$\therefore \alpha = 210^\circ$$

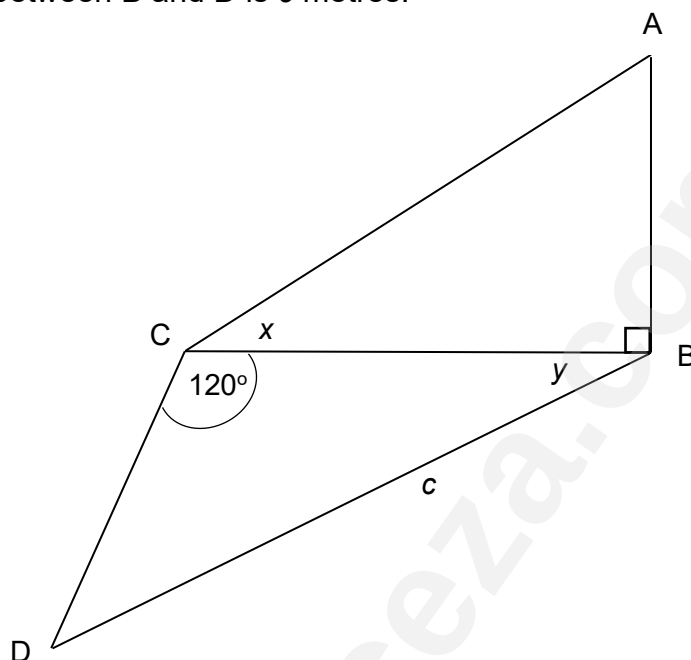
✓A

$$\therefore \beta = 60^\circ$$

✓A

and $\sin \alpha < 0$
 $\beta < 90^\circ$

- (b) In the diagram below, AB is a building. B, the foot of the building, and the points C and D are in the same horizontal plane. From C, the angle of elevation to the top of the building is x . $\hat{BCD} = 120^\circ$ and $\hat{CBD} = y$, and the distance between B and D is c metres.



Prove, **without the use of a calculator** and showing all working, that (7)

$$AB = \frac{2c \tan x \cdot \sin(60^\circ - y)}{\sqrt{3}}$$

$$\hat{D} = 60^\circ - y \quad \checkmark \text{ sum } \triangle \quad \checkmark A$$

In $\triangle DCB$

$$\frac{CB}{\sin(60^\circ - y)} = \frac{c}{\sin 120^\circ} \quad \checkmark M \text{ sin rule}$$

$$\therefore CB = \frac{c \cdot \sin(60^\circ - y)}{\sin 60^\circ} \quad \checkmark A \text{ reduction}$$

$$CB = \frac{c \cdot \sin(60^\circ - y)}{\frac{\sqrt{3}}{2}} \quad \checkmark A \text{ value for sin } 60$$

$$= \frac{2c \cdot \sin(60^\circ - y)}{\sqrt{3}}$$

$\checkmark A$ simplifying

In $\triangle ABC$

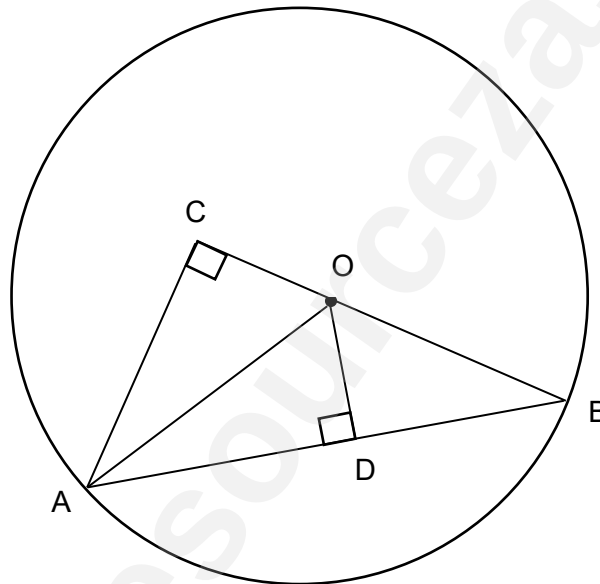
$$\tan x = \frac{AB}{\frac{2c \cdot \sin(60^\circ - y)}{\sqrt{3}}} \quad \checkmark \text{M def of tan in } \triangle ABC \quad \checkmark \text{A sub}$$

$$\therefore AB = \frac{2c \cdot \tan x \cdot \sin(60^\circ - y)}{\sqrt{3}}$$

QUESTION 8

[9]

In the diagram, O is the centre of the circle. $AC \perp CB$ and $OD \perp DA$.



(a) Prove $\triangle ODB \parallel \triangle ACB$.

(3)

In $\triangle$ s ODB, ACB

$$\hat{D}_1 = \hat{C} = 90^\circ \quad \text{given} \quad \checkmark \text{s/r}$$

$$\hat{B} \text{ common} \quad \checkmark \text{s/r}$$

$$\hat{O}_1 = \hat{CAB} < \text{sum } \triangle$$

$$\therefore \triangle ADB \parallel \triangle ACB \quad \text{AAA} \quad \checkmark \text{A must have condition}$$

(b) Hence, prove that $AD = \sqrt{\frac{OA \cdot BC}{2}}$ (6)

$$\frac{OB}{AB} = \frac{DB}{CB} \quad ||| \Delta \text{ s proved} \quad \checkmark \text{ M use } ||| \Delta \text{ s}$$

but $OA = OB$ radii $\checkmark \text{ s/r}$

and $AD = BD$ line from centre $\perp$ to chord $\checkmark \text{ s } \checkmark \text{ r}$

$$\therefore \frac{OA}{2AD} = \frac{AD}{BC} \quad \checkmark \text{ A subst = sides}$$

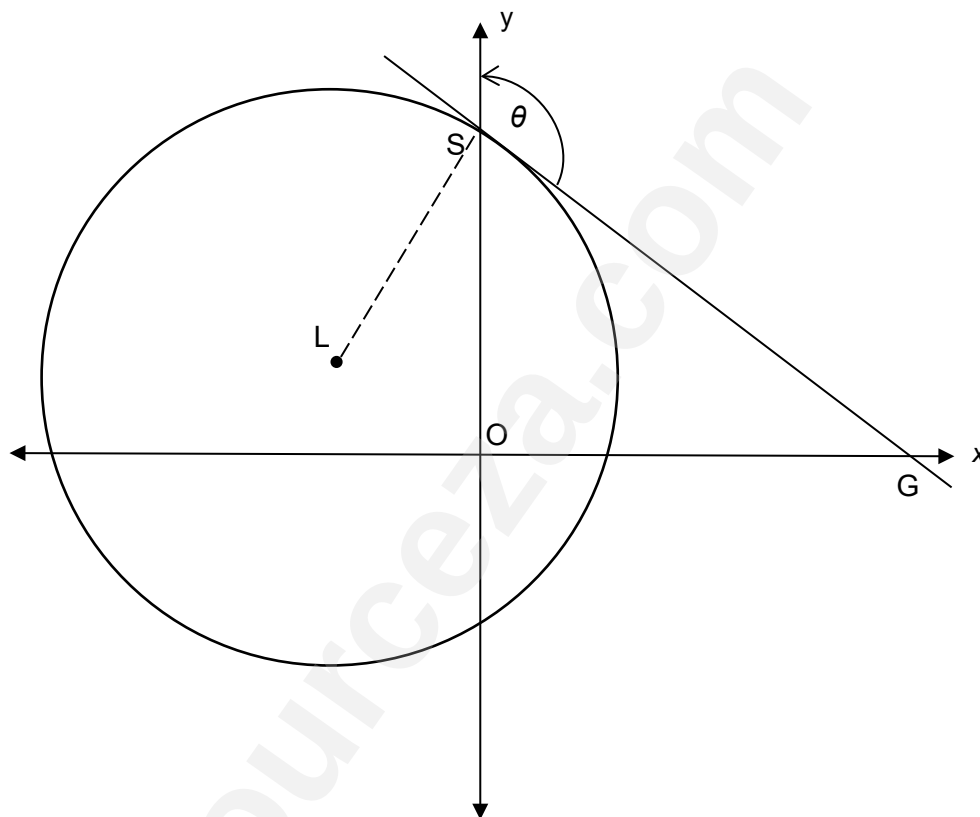
$$\therefore 2AD^2 = OA \cdot BC$$

$$\therefore AD^2 = \frac{OA \cdot BC}{2} \quad \checkmark \text{ A simplifying}$$

$$\therefore AD = \sqrt{\frac{OA \cdot BC}{2}}$$

QUESTION 9**[17]**

- (a) The circle sketched below has centre L and equation $x^2 + y^2 + 6x - 4y - 12 = 0$. The circle cuts the y -axis at point S. GS is a tangent to the circle with G on the x -axis and the angle θ lies between the y -axis and GS.



- (1) Determine the co-ordinates of L and the length of the radius of the circle. (5)

$$x^2 + 6x + 3^2 + y^2 - 4y + 2^2 = 12 + 9 + 4$$

$$(x + 3)^2 + (y - 2)^2 = 25$$

$$\therefore L(-3; 2) \text{ and } r = 5$$

$$\checkmark A \checkmark A \quad \checkmark A$$

✓M complete square ✓A

- (2) Determine the co-ordinates of point S. (4)

S is a y-intercept of circle

$$\therefore x = 0 \quad \checkmark M$$

$$(0+3)^2 + (y-2)^2 = 25$$

$$y^2 - 4y + 4 + 9 - 25 = 0 \quad \checkmark A \text{ equation}$$

$$y^2 - 4y - 12 = 0$$

$$(y-6)(y+2) = 0$$

$$\therefore y = 6 \quad \text{or} \quad y = -2 \quad \checkmark A \text{ solutions}$$

$$\therefore S(0; 6) \quad \checkmark A \text{ final co-ordinates}$$

- (3) If it is given that the gradient of the radius LS is $\frac{4}{3}$, calculate the size of θ . (5)

$$m_{SG} \times m_{LS} = -1 \quad \text{rad} \perp \text{tangent} \quad \checkmark s \quad \checkmark r$$

$$\therefore m_{SG} = \frac{-3}{4} \quad \checkmark A$$

$$\therefore \hat{G}_1 = 36,869...^\circ$$

$$\therefore \theta = 90^\circ + 36,869...^\circ \quad \text{ext} < \Delta \quad \checkmark s/r$$

$$= 126,9^\circ \quad \checkmark CA \text{ with } G$$

- (b) Consider the circle with equation $(x-2+t)^2 + (y+t^2-4) = 25^2$ where t is any real number. Determine the value(s) of t for which the circle has its centre on the x-axis but not on the y-axis. (3)

if centre on x-axis then $t^2 - 4 = 0 \quad \checkmark A \text{ deduction} \quad \checkmark A \text{ equation}$

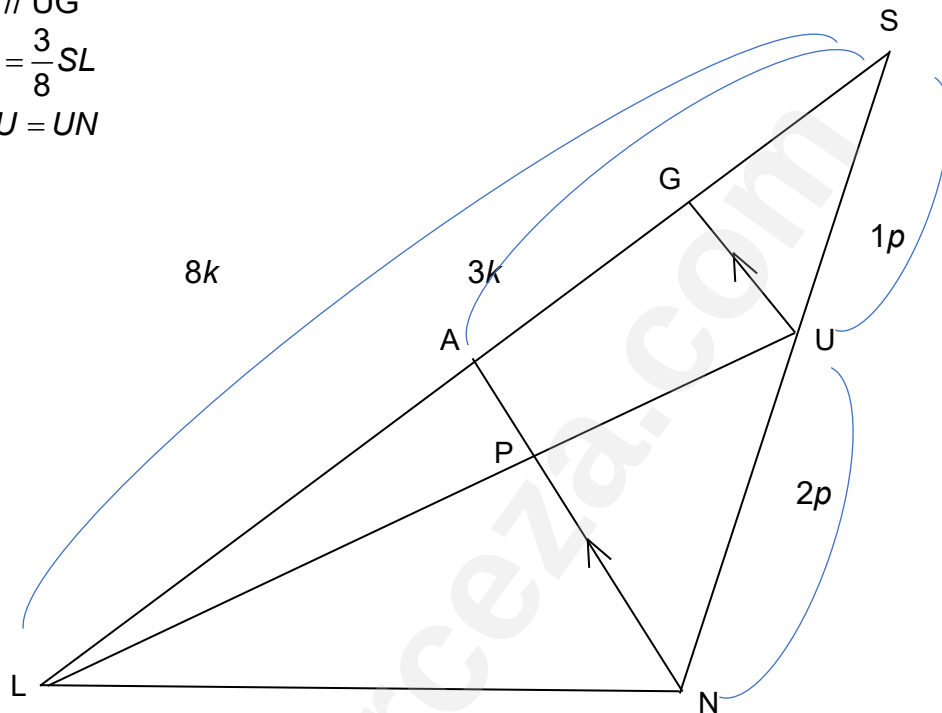
$$\therefore t = 2 \quad \text{or} \quad t = -2$$

but if $t = 2$ then centre also on x-axis

$$\therefore t = -2 \quad \checkmark A \text{ only solution}$$

[10]

- LAGS and SUN are two straight lines.
- $NA \parallel UG$
- $SA = \frac{3}{8}SL$
- $2SU = UN$



- $$\frac{GA}{SG} = \frac{UN}{SN} \quad \text{prop int them UG//NA} \quad \checkmark r$$
- $$\therefore \frac{GA}{SG} = \frac{2}{1} \quad \checkmark A$$

- $$GA = 2k \quad \checkmark A$$
- $$\frac{UP}{UL} = \frac{GA}{GL} \quad \text{prop int thm UG//NA} \quad \checkmark s/r$$
- $$\therefore \frac{UP}{UL} = \frac{2}{7} \quad \checkmark A$$

(c) $\frac{\text{Area} \triangle SGU}{\text{Area} \triangle SLN}$ (5)

$$\text{Area } \triangle SGU = \frac{1}{2}(SG)(SU)\sin S \quad \checkmark \text{M area rule}$$

$$\text{Area } \triangle SLN = \frac{1}{2}(SL)(SN)\sin S \quad \checkmark \text{A area rule subst}$$

$$\begin{aligned} & \frac{\frac{1}{2}.SG.SU.\sin S}{\frac{1}{2}.SL.SN.\sin S} \\ \therefore &= \frac{SG}{SL} \cdot \frac{SU}{SN} \\ &= \frac{1}{8} \times \frac{1}{3} \\ & \checkmark \text{A} \quad \checkmark \text{A values} \end{aligned}$$

$$= \frac{1}{24} \quad \checkmark \text{CA with step before}$$

QUESTION 11**[8]**

(a) The following information is given:

$$\sum_{k=1}^{20} (x_k - \bar{x})^2 = 156$$

$$\sum_{k=1}^{20} x_k = 1220$$

Determine:

(1) the number of pupils in the class. (1)
20 ✓A

(2) the mean mark of the data. (1)

$$1220 \div 20 = 610 \quad \checkmark A$$

(3) the standard deviation. (2)

$$156 \div 20 = 7,8$$

$$\sqrt{7,8} = 2,8$$

✓M sq root variance and divide by 20 ✓A

(b) Suppose the data set a, b, c, d and e has a mean of 8. (4)
Find the mean of $10 - a$; $10 - b$; $20 - c$; $20 - d$ and $50 - e$.

$$\frac{a + b + c + d + e}{5} = 8$$

$$\therefore a + b + c + d + e = 40 \quad \checkmark A$$

$$\frac{(10 - a) + (10 - b) + (20 - c) + (20 - d) + (50 - e)}{5} \quad \checkmark M \text{ mean of new values}$$

$$\therefore \frac{110 - (a + b + c + d + e)}{5}$$

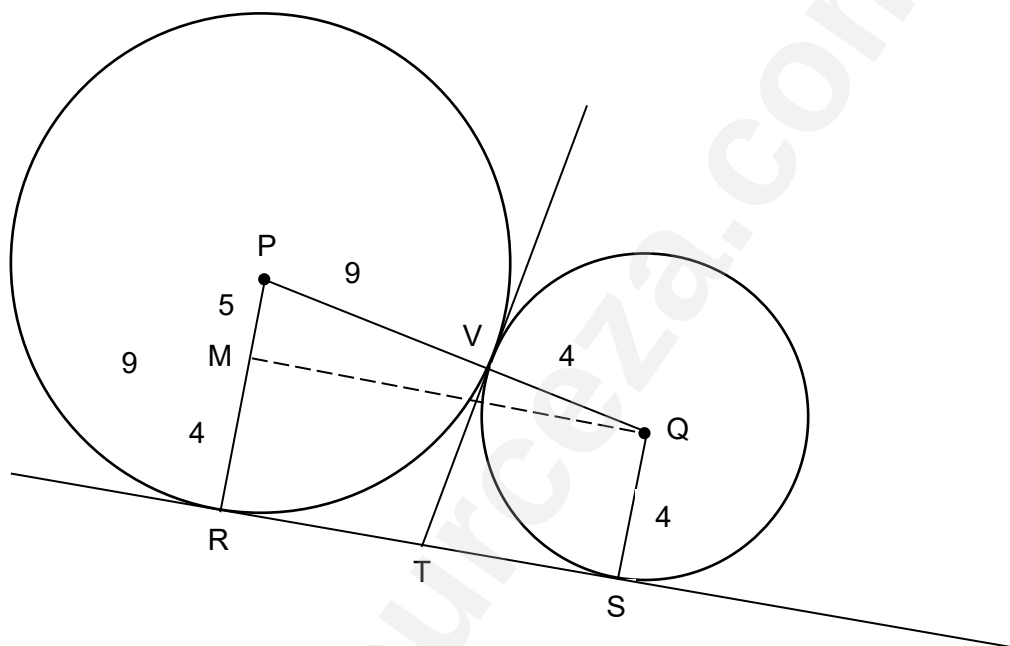
$$= \frac{110 - 40}{5} \quad \checkmark A \text{ subst 40}$$

$$= 14$$

✓CA with simplifying

QUESTION 12

- (a) A circle with centre P and radius 9 cm is a tangent to a smaller circle with centre Q and radius 4 cm. Common tangents VT and RS are drawn to the circles making points of contact at V, R and S.



Determine the length of VT .

Constr: join MQ such that MQ//RS ✓M constr

$$\hat{R} = \hat{S} = 90^\circ \quad \text{rad} \perp \text{tangent} \quad \checkmark \text{ s/r}$$

∴ RMQS is a rectangle

$$\therefore PM = 5 \text{ cm} \quad \checkmark A$$
$$PQ = 9 + 4$$
$$= 13 \text{ cm} \quad \checkmark \text{CA with PM}$$

RT = VT = TS tangents from common point ✓s/r

$$RS^2 = 13^2 - 5^2$$
$$\therefore RS = 12 \text{ cm} \quad \checkmark \text{ CA with PQ}$$

- $\therefore VT = 6 \text{ cm}$ ✓CA with step before
 (b) Determine the maximum radius of the circle with equation (7)
 $x^2 + y^2 - 2x \cos \theta - 4y \cos \theta = -2$ for any value of θ .

completing the square:

$$x^2 - 2 \cos \theta \cdot x + y^2 - 4 \cos \theta \cdot y = -2 \quad \checkmark \text{M coefficients to complete the square}$$

$$x^2 - 2 \cos \theta \cdot x + \left(\frac{2 \cos \theta}{2}\right)^2 + y^2 - 4 \cos \theta \cdot y + \left(\frac{4 \cos \theta}{2}\right)^2 = -2 + \cos^2 \theta + (2 \cos \theta)^2$$

$\checkmark \text{A compl square}$
 $\checkmark \text{A RHS}$

$$r = -2 + 5 \cos^2 \theta \quad \checkmark \text{A radius}$$

max value of $\cos \theta$ is 1

$$\therefore \text{max value of } \cos^2 \theta \text{ is } 1 \quad \checkmark \text{A}$$

$$\therefore r^2 = -2 + 5(1)$$

$$= 3 \quad \checkmark \text{M sub in } r$$

$$\therefore \text{max value of } r \text{ is } \sqrt{3} \quad \checkmark \text{CA with step before}$$