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## Heritage House

| MATHEMATICS       |             |          |            |
|-------------------|-------------|----------|------------|
| LONG TASK PAPER 1 |             |          |            |
| Examiner          | L Mudzamiri | Marks    | 120        |
| Moderator         | C Sam       | Time     | 3 Hours    |
| Date handed out   | 29/05/2023  | Due date | 29/05/2023 |

NAME &amp; CLASS:

MEMO

## INSTRUCTIONS:

1. Write your name, surname and class on this cover page.
2. This question paper consists of **3 tasks and 11 pages**.
3. Answer all the questions in the spaces provided on the question paper.
4. You may use an approved, non-programmable and non-graphical calculator, unless otherwise stated. Round off to 2 decimal places unless the context requires rounding up or down.
5. All the necessary working details must be clearly shown.
6. Write legibly and present your work neatly.
7. The **information sheet** is on page 11, and you're allowed to detach it.

## TASK ONE – ALGEBRA AND EQUATIONS

## Question 1

[40]

Jack is a grade 12 learner that attends Mudzamiri High School. He has measles which is a highly contagious disease and must therefore stay at home for a week until he is not infectious. Jack is forwarded work on Microsoft Teams; however, he does not fully understand Algebra and Equations and requires you to assist him in answering all the questions that follow:

a) Given that:  $2x^2 + px - 2 = 0$ .

1. Calculate the value of  $p$  if  $\frac{1}{2}$  is a root of the equation. (3)

$$2\left(\frac{1}{2}\right)^2 + p\left(\frac{1}{2}\right) - 2 = 0 \checkmark$$

$$\frac{1}{2} + \frac{p}{2} = 2 \checkmark$$

$$p = 3 \checkmark$$

2. Hence determine the other root. (2)

$$2x^2 + 3x - 2 = 0$$

$$(2x - 1)(x + 2) = 0$$

$$x = -2 \checkmark$$

- b) Show that the nature of roots of  $3x^2 + (k+2)x - 1$  are real and rational for all values of  $k$ . (4)

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (k+2)^2 - 4(3)(k-1) \checkmark \\ &= k^2 + 4k + 4 - 12 + 12 \checkmark \\ &= k^2 - 8k + 16 \\ &= (k-4)^2 \checkmark ; (k-4)^2 \geq 0 \therefore \text{roots are real and rational}\end{aligned}$$

- c) If the roots of a quadratic equation are given as  $3 - \sqrt{2}$  and  $3 + \sqrt{2}$ . Determine the equation in the form:  $ax^2 + bx + c = 0$ . (4)

$$\begin{aligned}[x - (3 - \sqrt{2})][x - (3 + \sqrt{2})] &= 0 \checkmark \\ x^2 - 3x - x\sqrt{2} - 3x + x\sqrt{2} + 9 - 2 &= 0 \checkmark \\ x^2 - 6x + 7 &= 0 \checkmark\end{aligned}$$

- d) Find the values of  $x$  for which  $\frac{\sqrt{2x-5}}{x+3}$  will be real? (2)

$$\begin{aligned}2x - 5 &\geq 0 \checkmark \\ x &\geq \frac{5}{2} \checkmark\end{aligned}$$

- e) The equations  $x^2 + ax + b = 0$  and  $x^2 + bx + a = 0$  both have equal and real roots. Solve for  $a$  and  $b$ , where  $a > 0$  and  $b > 0$ . (7)

$$\begin{aligned}\Delta &= b^2 - 4ac \\ \Delta &= 0 \text{ (equal and real roots)} \\ a^2 - 4b &= 0 \dots (1) \checkmark \\ b^2 - 4a &= 0 \dots (2) \checkmark \\ b &= \frac{a^2}{4} \dots (3) \checkmark \\ \text{Subst. (3) in (2)} &\rightarrow \left(\frac{a^2}{4}\right)^2 - 4a = 0 \\ a^4 - 64a &= 0 \\ a(a^3 - 64) &= 0 \checkmark \\ a(a-4)(a^2 + 4a + 16) &= 0 \checkmark \\ a &= 4 \checkmark \\ b &= \frac{4^2}{4} \\ b &= 4 \checkmark\end{aligned}$$

f) Jack was asked to solve  $x - 3\sqrt{x+2} = 2$  and his steps were as follows:

$$\begin{aligned} x^2 - 3^2(\sqrt{x+2})^2 &= 2^2 \\ x^2 - 9(x+2) &= 4 \\ x^2 - 9x - 22 &= 0 \\ (x-11)(x+2) &= 0 \\ x &= 11 \text{ or } x = -2 \end{aligned}$$

1. State (in words) the fundamental error/s that Jack made in the above steps. (2)

- Not separating the surd before squaring ✓
- Not combining the terms before squaring ✓

2. Help him to solve for  $x$  accurately in the equation  $x - 3\sqrt{x+2} = 2$  (5)

$$\begin{aligned} (x-2)^2 &= (3\sqrt{x+2})^2 \quad \checkmark \\ x^2 - 4x + 4 &= 9(x+2) \quad \checkmark \\ x^2 - 4x - 9x + 4 - 18 &= 0 \\ (x^2 - 13x - 14) &= 0 \\ (x-14)(x+1) &= 0 \quad \checkmark \\ x &= 14 \quad \checkmark, \quad x \neq -1 \quad \checkmark \end{aligned}$$

g) Solve for  $x$  and  $y$  simultaneously in the following equations:

$$y + 3x = 2 \text{ and } 5^{x^2+1} = 5^{x-y} \quad (7)$$

$$\begin{aligned} y + 3x &= 2 \dots (1) & (x-1)(x-3) &= 0 \quad \checkmark \\ x^2 + 1 &= x - y \dots (2) \quad \checkmark & x &= 1 \text{ or } x = 3 \\ y &= 2 - 3x \dots (3) \quad \checkmark & y &= -7 \text{ or } y = -1 \\ \text{subst. (3) in (2)} & & & \\ x^2 + 1 &= x - (2 - 3x) \quad \checkmark & (3, -7) & \text{ or } (1, -1) \quad \checkmark \\ x^2 + 1 &= x - 2 + 3x \\ x^2 - 4x + 3 &= 0 \quad \checkmark \end{aligned}$$

4. Hence, calculate the number of chairs in the 25<sup>th</sup> row.

(1)

$$T_{25} = 2(25) + 3$$

$$= 53$$

5. The theatre manager bought 2 000 chairs for valedictory ceremony for the matric learners.

Use relevant formula to determine how many complete rows can be placed.

(7)

$$S_n = 5 + 7 + 9 + \dots$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{n}{2} [2 \cdot 5 + (n-1)2]$$

$$S_n = \frac{n}{2} (10 + 2n - 2)$$

$$S_n = \frac{n}{2} (2n + 8)$$

$$S_n = n^2 + 4n$$

$$n^2 + 4n = 2\,000$$

$$n^2 + 4n - 2\,000 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-2\,000)}}{2(1)}$$

$$x \neq -46,77 \text{ or } x = 42,77$$

42 complete rows.

b) Due to load-shedding, the automated water pump in Mudzamiri High School malfunctioned causing the volume of water in (litres) to increase given by the following series:

$$2(5)^5 + 2(5)^4 + 2(5)^3 + \dots$$

1. Show that the series converges.

(2)

$$-1 < r < 1$$

$$r = \frac{2(5)^4}{2(5)^5} = \frac{1}{5}$$

2. Determine the value of  $k$  if:

(5)

$$\sum_{t=1}^{\infty} 4 \cdot k^{t-1} = 5$$

$$4 + 4k + 4k^2 + \dots = 5$$

$$\frac{4}{1-k} = 5$$

$$4 = 5 - 5k$$

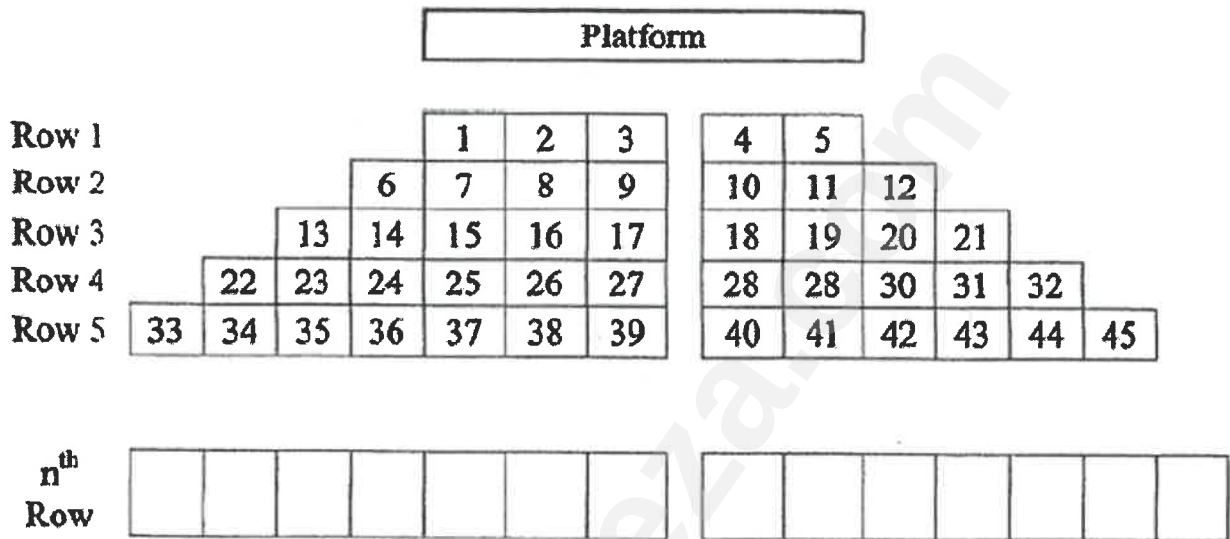
$$k = \frac{1}{5}$$

**Question 2**

[40]

Due to load-shedding, Mudzamiri High School decides to have all their gatherings and social activities in an open theatre.

a) The seats in the open-air theatre are numbered as in the sketch below.



1. If 1; 6; 13; 22; 33 forms a sequence of seat numbers of the first seat in each row, determine the  $n^{\text{th}}$  term of this row. (4)

|                |                        |
|----------------|------------------------|
| $2a = 2$       | $a + b + c = 1$        |
| $a = 1$ ✓      | $1 + 2 + c = 1$        |
| $3a + b = 5$   | $c = -2$ ✓             |
| $3(1) + b = 5$ | $T_n = n^2 + 2n - 2$ ✓ |
| $b = 2$ ✓      |                        |

2. Hence, find the seat number of the 7<sup>th</sup> seat in row 15. (2)

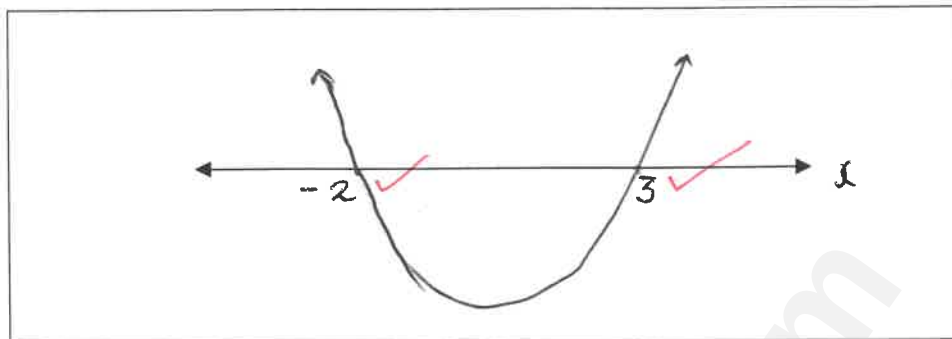
|                                     |  |
|-------------------------------------|--|
| $T_{15} = (15)^2 + 2(15) - 2 + 6$ ✓ |  |
| $= 259$ ✓                           |  |
|                                     |  |

3. Determine the number of chairs in the  $n^{\text{th}}$  row. (3)

|                      |  |
|----------------------|--|
| $T_n = a + (n-1)d$   |  |
| $T_n = 5 + (n-1)2$ ✓ |  |
| $T_n = 2n + 3$ ✓     |  |
|                      |  |

h) Jack's mathematics teacher taught him that to solve for quadratic inequalities, a rough sketch of a parabola must be drawn.

1. On the  $x$ -axis, construct a rough sketch of  $y = x^2 - x - 6$  indicating the  $x$ -intercepts. (2)



2. Hence, solve for  $x^2 - x > 6$ . (2)

$$x < -2 \text{ or } x > 3$$

- c) Because of load-shedding, the school closed the cleaning equipment at night and as a result, the number of micro-organisms in Mudzamiri High School's swimming pool grew each day according to the following sequence: 3 ; 9 ; 17 ; 27 ; ...

1. Give the number of micro-organisms in the next 3 sequences. (3)

39 ; 53 ; 69

2. Determine an equation for the  $n^{th}$  term of the sequence. (4)

$$T_n = an^2 + bn + c$$

$$2a = 2$$

$$a = 1$$

$$3a + b = 6$$

$$b = 3$$

$$a + b + c = 3$$

$$1 + 3 + c = 3$$

$$c = -1$$

$$T_n = n^2 + 3n - 1$$

3. Calculate the 1<sup>st</sup> day on which the micro-organisms in the trough will be greater than 269. (4)

$$n^2 + 3n - 1 > 269$$

$$n^2 + 3n - 270 > 0$$

$$(n - 15)(n + 18) > 0$$

$$n > 15$$

$$n = 16$$

4. The swimming pool has now started to leak water droplets every minute. The first three terms of the geometric series  $2 + 6 + 18 + \dots$  represent the number of water droplets leaked after every minute. Find the value of  $n$  if  $S_n = 177\ 146$ . (5)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\frac{2(3^n - 1)}{3 - 1} = 177\ 146$$

$$3^n - 1 = 177\ 146$$

$$3^n = 177\ 147$$

$$3^n = 3^{11}$$

$$n = 11$$

Question 3

[40]

a) Slondiwe, a grade 12 learner at Mudzamiri High School, would like to get into the medical field of study when she finishes school. As part of her university entry requirements, she needs to perform well in her National Benchmark Tests (NBTs). Help Slondiwe practice for the Cubic Polynomials and Differential Calculus section of her NBTs by completing the questions that follow.

1. The expression  $px^3 - 8x^2 + qx + 6$  is exactly divisible by  $x^2 - 2x - 3$ . Find the values of  $p$  and  $q$ . (6)

$$\begin{aligned}
 x^2 - 2x - 3 &= (x-3)(x+1) \\
 p(3)^3 - 8(3)^2 + q(3) + 6 &= 0 \quad \checkmark \\
 27p - 72 + 3q + 6 &= 0 \\
 27p + 3q &= 66 \\
 9p &= 22 - 9q \quad \dots (A) \quad \checkmark \\
 p(-1)^3 - 8(-1)^2 + q(-1) + 6 &= 0 \quad \checkmark \\
 -p - 8 - q + 6 &= 0 \\
 -p - q &= 2 \\
 q &= -p - 2 \quad \dots (B) \quad \checkmark \\
 (A) = (B) &: 22 - 9p = -p - 2 \\
 -8p &= -24 \\
 p &= 3 \quad \checkmark \\
 q &= -3 - 2 \\
 q &= -5 \quad \checkmark
 \end{aligned}$$

2. Show that the equation  $x^2(x + 3) = -9x - 7$  has only one real solution. (5)

$$\begin{aligned}
 x^3 + 3x^2 + 9x + 7 &= 0 \quad \checkmark \\
 (-1)^3 + 3(-1)^2 + 9(-1) + 7 &= -1 + 3 - 9 + 7 = 0 \quad \checkmark \\
 \text{Using Synthetic Division:} & \quad \begin{array}{r|rrrr} 1 & 1 & 3 & 9 & 7 \\ -1 & & -1 & -2 & -7 \\ \hline & 1 & 2 & 7 & 0 \end{array} \quad \checkmark \\
 (x + 1)(x^2 + 2x + 7) &= 0 \quad \checkmark \\
 \Delta &= (2)^2 - 4(1)(7) \\
 \Delta &= -24 \quad \checkmark
 \end{aligned}$$

function has only one real solution.

3. Evaluate the following limit. Downloaded from eduresourceza.com

$$\lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\pi - x}$$

(3)

$$\begin{aligned} &= \lim_{x \rightarrow \pi} \frac{(x + \pi)(x - \pi)}{-(x - \pi)} \\ &= \lim_{x \rightarrow \pi} -(x + \pi) \\ &= -(\pi + \pi) \\ &= -2\pi \end{aligned}$$

4. Determine  $f'$  from first principles if  $f(x) = -x^2 + 2x$

(5)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ f(x+h) &= -(x+h)^2 + 2(x+h) \\ &= -x^2 - 2xh - h^2 + 2x + 2h \\ f'(x) &= \lim_{h \rightarrow 0} \frac{-x^2 - 2xh - h^2 + 2x + 2h - (-x^2 + 2x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2xh - h^2 + 2h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-2x - h + 2)}{h} = -2x - (0) + 2 \\ &= -2x + 2 \end{aligned}$$

b) Determine:

1.  $D_x[(x+1)(3x-7)]$

(2)

$$\begin{aligned} &= D_x(3x^2 - 4x - 7) \\ &= 6x - 4 \end{aligned}$$

2.  $\frac{dy}{dx}$  if  $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$

(4)

$$\begin{aligned} y &= x^{\frac{3}{2}} - 5x^{-1} + \frac{1}{2}\pi \\ \frac{dy}{dx} &= \frac{3}{2}x^{\frac{1}{2}} + 5x^{-2} \end{aligned}$$

3.  $D_x \left( \frac{x^3-1}{x-1} \right) = D_x \left[ \frac{(x-1)(x^2+x+1)}{x-1} \right]$  (3)

$$= D_x (x^2+x+1)$$

$$= 2x+1$$

4.  $f''$  if  $f(x) = \sqrt{x}$  (3)

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$$

c) Prove that  $\frac{dy}{dx} = 12x^5\sqrt{y}$ , if  $y = (x^6 - 1)^2$  and  $x > 1$ . (3)

$$y = x^{12} - 2x^6 + 1$$

$$\frac{dy}{dx} = 12x^{11} - 12x^5$$

$$12x^5\sqrt{y}$$

$$= 12x^5(x^6-1)$$

$$= 12x^{11} - 12x^5$$

$$= \frac{dy}{dx}$$

d) Given  $h(x) = ax^2, x > 0$ .

Determine the value of  $a$  if it is given that  $h^{-1}(-8) = h'(4)$ . (6)

$$h(x) = ax^2$$

$$h^{-1}: x = ay^2$$

$$h^{-1}: y = \sqrt{\frac{x}{a}}$$

$$h^{-1}(-8) = \sqrt{\frac{-8}{a}}$$

$$h(x) = ax^2$$

$$h'(x) = 2ax$$

$$h'(4) = 2a(4)$$

$$h'(4) = 8a$$

$$\sqrt{\frac{-8}{a}} = 8a$$

$$-\frac{8}{a} = (8a)^2$$

$$-\frac{8}{a} = 64a^2$$

$$64a^3 + 8 = 0$$

$$8a^3 + 1 = 0$$

$$(2a+1)(4a^2-2a+1) = 0$$

$$a = -\frac{1}{2}$$

END OF PAPER

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_{\infty} = \frac{a}{1-r}; -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

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