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FINALS EXAM PRACTICE 2

MATHEMATICS PAPER 2 – GRADE 12

MARKS: 150

TIME: 3 Hours

- This question paper comprises 13 pages with 13 questions.
- Answer all questions, noting the three bullets below.
- This paper is in general DBE format, however it differs in that it uniquely contains a number of question choices.
- If you are assessing your typical time management in completing a paper, then in each of these cases, answer only ONE of the question choices.
- No Theorem has been included. Learn all your Theorems.
- Use 10 - 15 minutes reading time before commencement of the allocated time, and make use of this time to consider the various option questions.
- Number your answers exactly as described in the questions.
- Clearly show all workings and calculations, as applicable.
- Correct answers only will not necessarily generate full marks.
- A non-programmable calculator may be used.
- All answers should be rounded to TWO decimal places unless otherwise directed.
- Diagrams are not necessarily drawn to scale.
- A formula sheet is to be found on page 13.
- Once completed, finalise use of these revision questions:
 - Complete all of the questions choices which you did not complete in the original paper.
 - Allow yourself 1 hour for these questions.

NOTE: A Memorandum may be provided on request. It is certainly in your interest to attempt and apply your mind to all questions prior to receiving the memorandum. Use self-marking to reinforce understanding of all concepts covered.

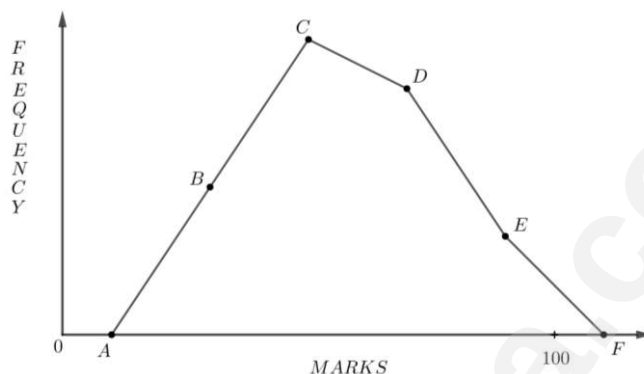
Think twice, write once, and most of all, ENJOY!

Alan Christison

alanchristison04@gmail.com

QUESTION 1

1.1 The diagram shows a frequency polygon $ABCDEF$ of student maths marks out of 100 in a test. The lowest mark was 20 and no student scored 100. The coordinates of $A - F$ are $A(10 ; 0), B(30 ; 15), C(50 ; 30), D(70 ; 25), E(90 ; 10), F(110 ; 0)$.



1.1.1 Determine the estimated mean mark for the test. (3)

1.1.2 Write down the modal class in the form $[a ; b)$. (2) [5]

1.2 A medium sized data set C is comprised of n values with a mean of $\bar{x}$. Another data set D is formed by adding a constant k to every value in data set C .

1.2.1 If $\bar{x}_D$ is the mean in data set D , mathematically complete the following equation using $\bar{x}$ and k :

$$\bar{x}_D = \dots \dots \dots (2)$$

1.2.2 Using the formula $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$, show algebraically why the standard deviation in the two sets will be identical. (3)

1.2.3 Data set C has a mathematical outlier at either end of the ordered set. This set has a standard deviation of σ and a median of $Q2$. The two outliers are removed and a new standard deviation and new median are calculated as σ_{new} and $Q2_{new}$.

Which two of the following statements are correct?

- | | |
|-------------------------------|----------------------|
| (i) $\sigma_{new} < \sigma$ | (iv) $Q2_{new} > Q2$ |
| (ii) $\sigma_{new} = \sigma$ | (v) $Q2_{new} = Q2$ |
| (iii) $\sigma_{new} > \sigma$ | (vi) $Q2_{new} < Q2$ |
- (2)

1.2.4 Complete the sentence: "Standard deviation is a measure of". (1) [8]

QUESTION 2

Eleven competitors A – K competed over two rounds in a memory skills competition. Each were shown ten different 4-digit numbers in each round, with the numbers varying in each round. In Round 1 they were given 12 seconds to memorise the numbers, and in Round 2 they were given 30 seconds. In recalling the numbers, their results out of 10 are reflected in the table below. The Round 1 score of competitor K was misplaced, and is shown as M.

Competitor	A	B	C	D	E	F	G	H	I	J	K
Round 1 (x)	6	3	5	8	4	2	7	7	4	5	M
Round 2 (y)	8	8	7	10	7	4	10	9	7	10	6

2.1 Using only the scores of competitors A – J, determine the equation of the least squares regression line in the form $y = bx + a$. (3)

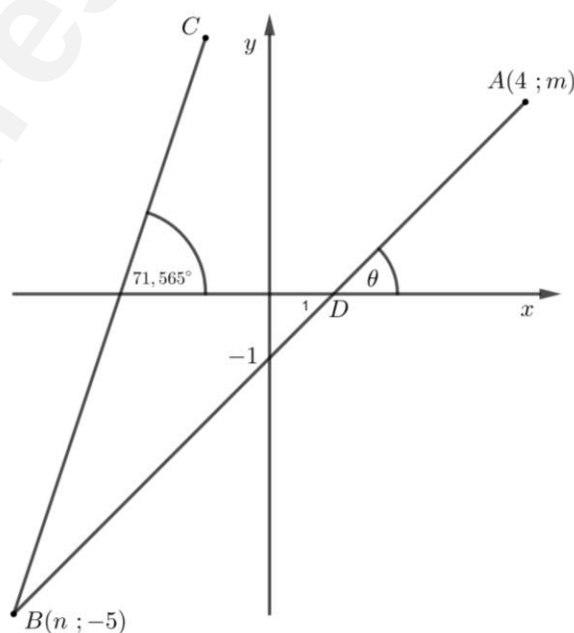
2.2 Use this equation to estimate the integer value of M. (2)

2.3 In this competition, comment on the level of correlation between having additional time and improving one's score. Back up your comments with appropriate mathematics. (2) [7]

[20]

QUESTION 3

In the diagram, line AB intersects the y – axis at its midpoint. $A(4; m)$; $B(n; -5)$. The angle of inclination of BC is $71,565^\circ$, and that of AB is θ , as shown.

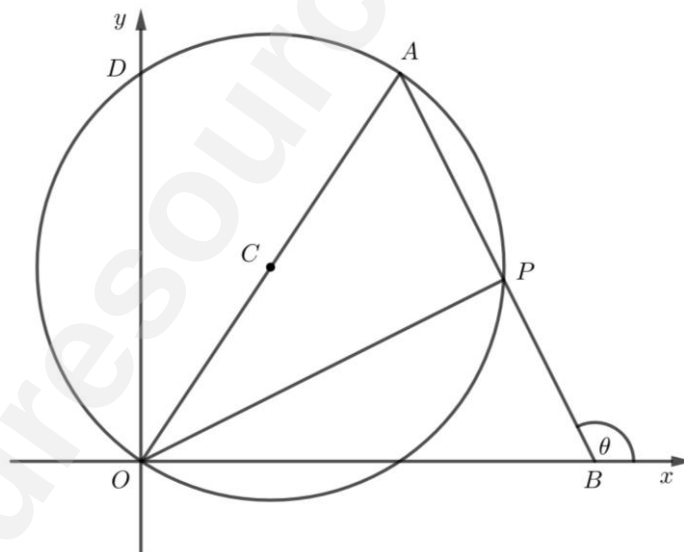


- 3.1 Determine the values of m and n . (2)
- 3.2 Calculate the value of $\widehat{ABC}$. (4)
- 3.3 If $n = -4$, show that the equation of BC is $y = 3x + 7$. (2)
- 3.4 Hence or otherwise, if $BC = 3\sqrt{10}$ units, determine the coordinates of C . (4) [12]

QUESTION 4

ANSWER 4.1 OR 4.2 ONLY

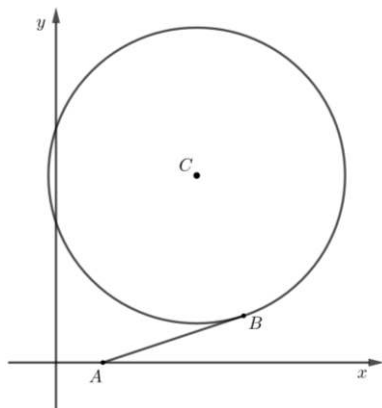
4.1 In the figure, circle centre C has the equation $(x - 2)^2 + (y - 3)^2 = 13$. Point P lies on the circle such that $\theta = 116,565^\circ$. D is a y - intercept of the circle, and B lies on the x - axis. OCA is a diameter.



- 4.1.1 Determine the equation of OP . (3)
- 4.1.2 Determine the coordinates of D . (2)
- 4.1.3 Hence or otherwise, determine the equation of the tangent at A . (5) [10]

OR

4.2 In the figure, AB is tangent to the circle centre $C(3; p)$ at $B(n; 1)$.
 A lies on the x – axis.



4.2.1 If $m_{AB} = \frac{1}{3}$, show that $p = 3n - 8$. (3)

4.2.2 If the coordinates of A are $(1; 0)$, determine the value of n . (2)

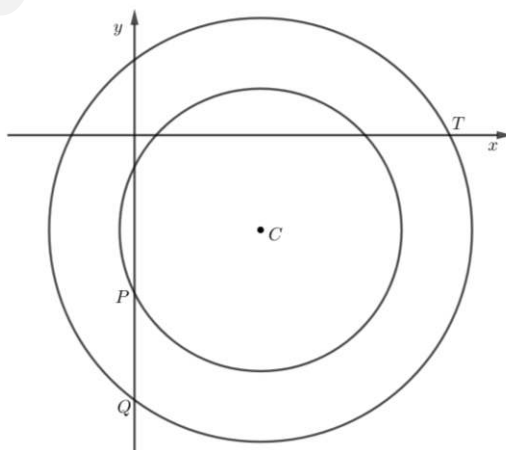
Hence determine:

4.2.3 The equation of AB . (2)

4.2.4 The equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3) [10]

QUESTION 5

Two concentric circles, centres C , are shown in the diagram. The equation of the smaller circle is given by: $x^2 + y^2 - 8x + 6y + 5 = 0$. P and Q are y – intercept's of the two circles respectively as shown, and T is an x – intercept of the larger circle. $PQ = \sqrt{29} - 2$ units.



5.1 Determine the coordinates of C , and the radius of the smaller circle. (3)

5.2 Calculate the area of the triangle with vertices P , Q , and C . (2)

5.3 Determine the equation of the larger circle in the form:

$$(x - a)^2 + (y - b)^2 = r^2. \quad (7)$$

5.4 $R(x; y)$ is the y – intercept of the tangent to the circle at T . Determine the area of ΔRPT . (6) [18]

[40]

QUESTION 6

ANSWER 6.1 OR 6.2 ONLY

6.1 Simplify the following expression to an integer without the use of a calculator:

$$\frac{\cos 260^\circ}{(\cos 310^\circ - \sin 230^\circ)(\sin 40^\circ - \sin 130^\circ)} \quad (7)$$

OR

6.2 Without the use of a calculator, fully simplify the following expression to a single term being a single trig ratio only:

$$\sin 153^\circ \cdot \cos 243^\circ + \frac{\tan 333^\circ}{\tan (-207^\circ)} \quad (7)$$

6.3 Prove the identity: $\frac{\sin 3x + \sin x}{2 \cdot \cos x} = \sin 2x$ (5)

6.4 Determine the maximum value of the following expression:

$$5 \cdot \sin(180^\circ - A) \cdot \cos(-A) - \tan(360^\circ - A) \cdot \sin^2(90^\circ + A) \quad (5)$$

6.5 If $\sin 23^\circ = m$, prove, without the use of a calculator, that:

$$\frac{\cos 23^\circ \cdot \cos 67^\circ}{2 \cdot \tan 157^\circ} = \frac{1}{2}(m^2 - 1) \quad (4)$$

ANSWER 6.6 OR 6.7 ONLY

6.6 If $\sin 2A = -\frac{4}{5}$ and $135^\circ < A < 180^\circ$, show, without the use of a calculator, that:

$$\tan A = -\frac{1}{2} \quad (7)$$

OR

6.7 If $\sin 44^\circ = m$ and $\cos 28^\circ = n$, without the use of a calculator:

6.7.1 Determine the value of $\cos 44^\circ$ in terms of m . (2)

6.7.2 Show that $\sin 16^\circ = m \cdot n - \sqrt{1 - n^2 - m^2 + m^2 \cdot n^2}$ (5) [7]

ANSWER 6.8 or 6.9 ONLY

6.8 Determine the general solution of: $\cos(30^\circ + \theta) = 3 \cdot \sin(60^\circ + \theta)$. (6)

OR

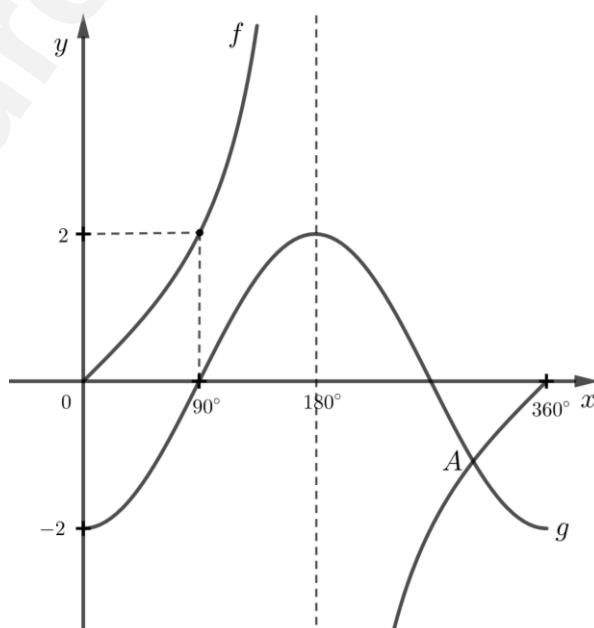
6.9 Given the identity, $\sin(E + F) - \sin(E - F) = 2 \cdot \cos E \cdot \sin F$, solve for x if:

$$\sin 7x + \sin(-x) = \cos 4x ; \quad x \in [0^\circ ; 90^\circ] \quad (6)$$

[34]

QUESTION 7

The graphs of $f(x) = m \cdot \tan\left(\frac{x}{2}\right)$ and $g(x) = n \cdot \cos x$, $x \in [0^\circ ; 360^\circ]$ are shown.
The two graphs intersect at A.

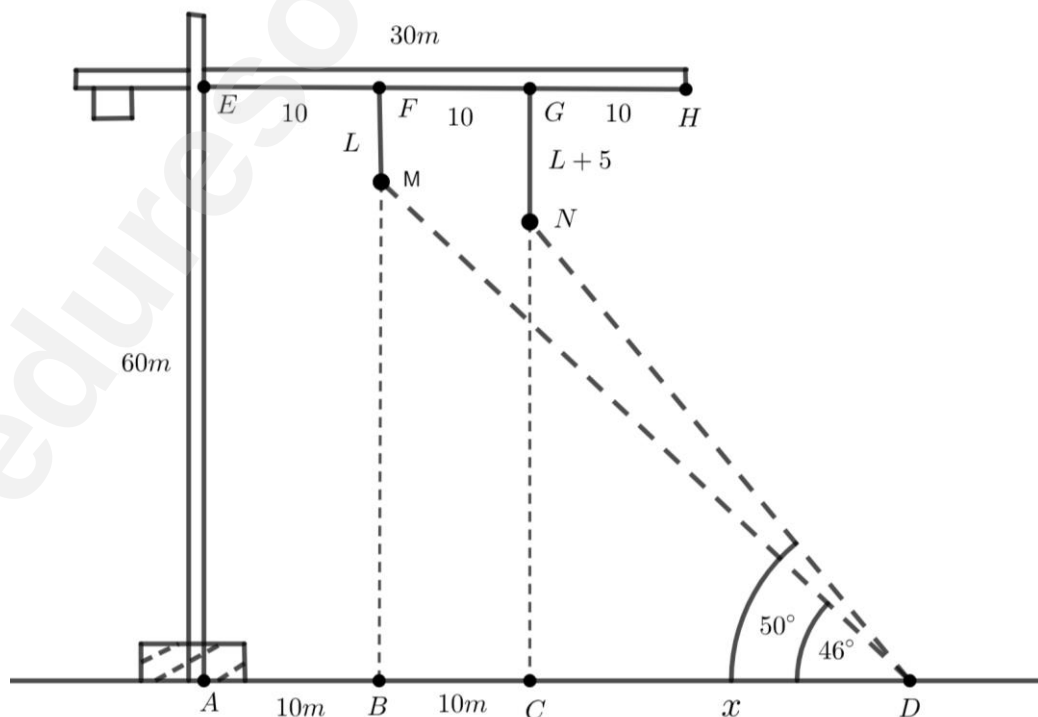


- 7.1 Determine the values of m and n . (2)
- 7.2 For which values of x in the given domain is $f(x) \cdot g(x) \leq 0$? (3)
- 7.3 Given $A(302, 94^\circ; -1, 09)$. For which values of x in the given domain is $f(x) > g(x)$? (2)
- 7.4 The graph of g is reflected about the x – axis, then translated 30° to the left to form graph h . Determine the coordinates of the y – intercept of h . (3) [10]

QUESTION 8**ANSWER 8.1 OR 8.2 ONLY**

8.1 In the figure, a large crane used in construction is shown. The crane is stabilised by heavy weights. The swing arm EH can move through 360° . Material/equipment is transported using cable FM from the ground to say a building floor under construction, or from that floor to the ground. Thus FM may move between E and H, and up/down as required. ABCD represents the horizontal ground level.

- EH $\parallel$ AD , AE $\perp$ AD , AE = 60 metres , EH = 30 metres.
- Two positions, FM and GN are shown for the cable. EF = FG = GH = 10 metres.
- Cable FM = L metres and cable GN = L+5 metres.
- B and C are vertically below F and G respectively. CD = x metres.
- The angle of elevation of the material at M and N from D is 46° and 50° respectively.



9

8.1.1 Show that:

$$x = \frac{10 \tan 46^\circ - 5}{\tan 50^\circ - \tan 46^\circ} \quad (5)$$

8.1.2 Hence calculate the value of L. (2)

[7]



OR

8.2 The time on a large analogue clock is eight minutes past ten in the morning. The minute hand is 12cm long and the hour hand is 8cm long.



Calculate the distance between the two arrow tips.

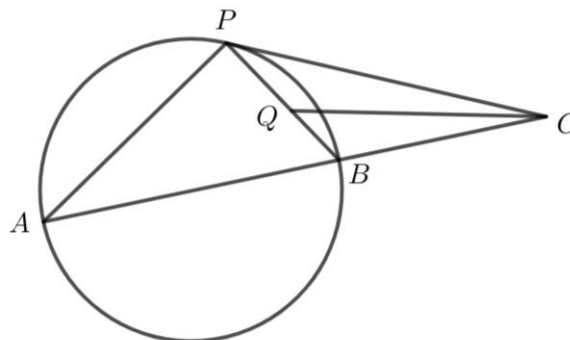
(7)

[51]

IN QUESTIONS 9 – 13, STATE ALL NECESSARY REASONING

QUESTION 9

In the diagram, AB is a diameter of the circle, and is extended to C such that CP is a tangent to the circle at P . CQ bisects $\angle PCB$. $\angle PCB = 2x$ and $\angle PAB = y$.



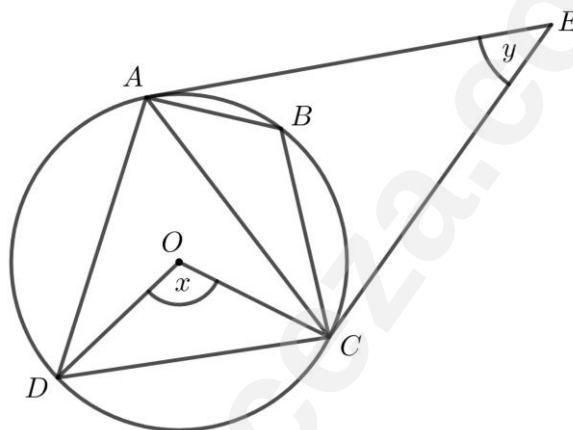
10

9.1 Determine the equation connecting x and y . (4)

9.2 Hence prove that $\angle CQB = 45^\circ$. (2) [6]

QUESTION 10

In the figure, A, B, C , and D lie on the circle centre O . EA and EC are tangents at A and C respectively. $\angle DOC = x$. $\angle E = y$.

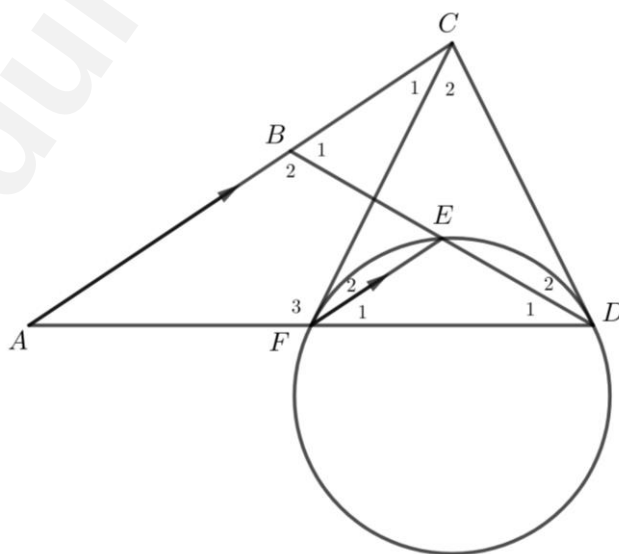


10.1 Show that $\angle ADO = \frac{x-y}{2}$. (6)

10.2 If $\angle B = 3y$, determine the value of y . (3) [9]

QUESTION 11

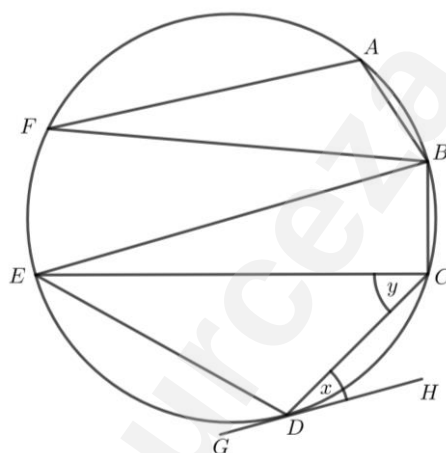
In the figure all lines are straight. D, E , and F lie on the circle such that CF and CD are tangents and $AC \parallel FE$.



- 11.1 Name two other angles equal to $\widehat{D}_2$. (4)
- 11.2 Prove that $CF^2 = BC \times AC$. (5)
- 11.3 If $\widehat{C}_1 = 28^\circ$ and $\widehat{D}_2 = 42^\circ$, determine the size of $\widehat{FED}$. (4) [13]

QUESTION 12

In the figure, A, B, C, D, E , and F lie on the circle. BE is a diameter. GDH is a tangent to the circle at D . $AB = BC$ $\widehat{CDH} = x$ $\widehat{DCE} = y$



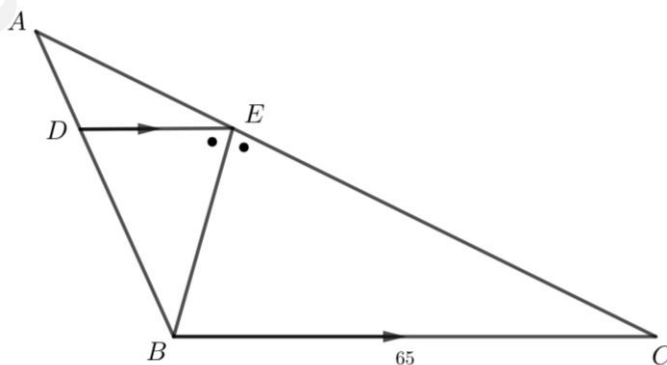
Determine the value of $\widehat{F}$ in terms of x and y . (6)

QUESTION 13

ANSWER 13.1 OR 13.2 ONLY

- 13.1 In the figure, D and E lie on the sides of $\triangle ABC$ such that $DE \parallel BC$.

$$AD : AB = 3 : 8 \quad \widehat{DEB} = \widehat{BEC} \quad BC = 65 \text{ units}$$

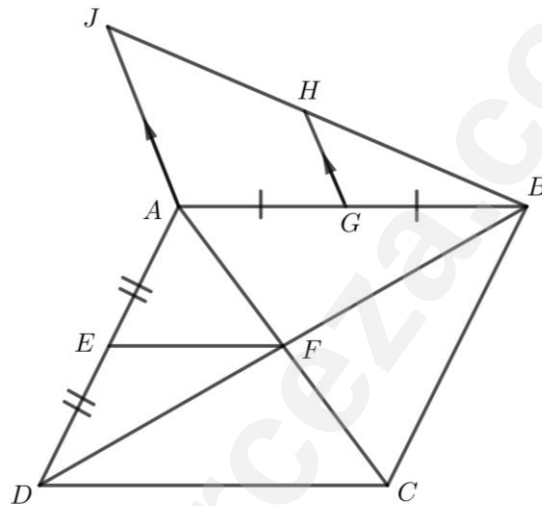


Determine the length of AE . (5)

12

OR

13.2 In the diagram, the perimeter of the rhombus ABCD is 28cm . $JH : BC = 5 : 7$.



Determine the ratio $EF : BH$.

(5)

[39]

[150]

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{Area } \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$