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**CRAWFORD INTERNATIONAL PRELIMINARY EXAMINATION
SEPTEMBER 2023
MATHEMATICS: PAPER II**

Time: 3 hours

150 marks

NAME OF CANDIDATE:	
NAME OF EDUCATOR:	

MARKER TO ENTER MARKS:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
16	13	8	14	12	13	6	6	8	14	17	12	11	150

INSTRUCTIONS

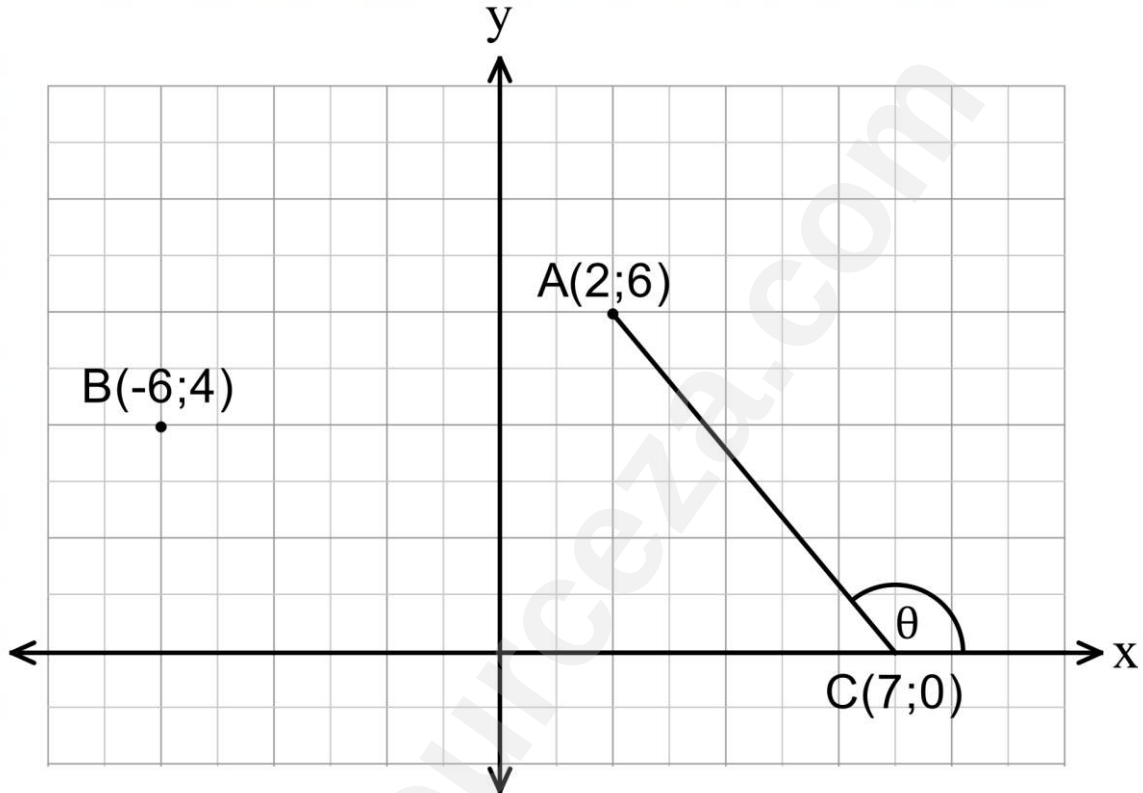
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages. Please check that your question paper is complete.
2. Read all the questions carefully. **Answer ALL the questions on the question paper and hand it in at the end of the session.**
3. Three blank pages (pages 26 to 28) are included at the end of the paper. If you run out of space for a question, use these pages. **Clearly indicate the question number of your answer should you use this extra space.**
4. Diagrams are not necessarily drawn to scale.
5. Ensure that your calculator is set to **DEGREE MODE**.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. It is in your own interest to write legibly and present your work neatly.
9. Round off to **one decimal place** unless otherwise stated.

SECTION A

QUESTION 1

Given the diagram below with straight line A(2; 6) and C(7; 0). B(-6; 4) is a point in the second quadrant and θ is the angle of inclination of AC.



(a) Determine:

(1) the co-ordinates of D, the midpoint of AB.

(2)

(2) the value of d if the points B, A and F (20; d) are collinear.

(4)

(3) the size of angle θ .

(3)

- (b) Determine the equation of the circle with centre at point B which has the x -axis as a tangent. You may leave your answer in the form $(x - a)^2 + (y - b)^2 = r^2$.

(3)

- (c) Calculate the possible value(s) of p if E $(6; p)$ is 5 units away from A.

(4)

[16]

QUESTION 2

Consider the following two circles: Circle 1: $(x + 4)^2 + (y - 2)^2 = 20$

$$\text{Circle 2: } (x - 12)^2 + (y - 10)^2 = p$$

- (a) Determine whether the point $(-2; -2)$ lies **on** or **inside** or **outside** Circle 1.

(2)

- (b) Calculate the distance between the centres of Circle 1 and Circle 2.

(3)

- (c) Determine p if it is further given that the two circles **touch** one another.

(3)

- (d) State the relationship between the radius of a circle and the tangent to the circle.

(1)

- (e) Determine the equation of the tangent to Circle 1 at the point $(-6; -2)$.

(4)
[13]

QUESTION 3

- (a) Prove the following identity:

$$\frac{\cos x - \cos 2x + 2}{3\sin x - \sin 2x} = \frac{1 + \cos x}{\sin x}$$

(5)

- (b) Determine the values of x for which the identity is undefined.

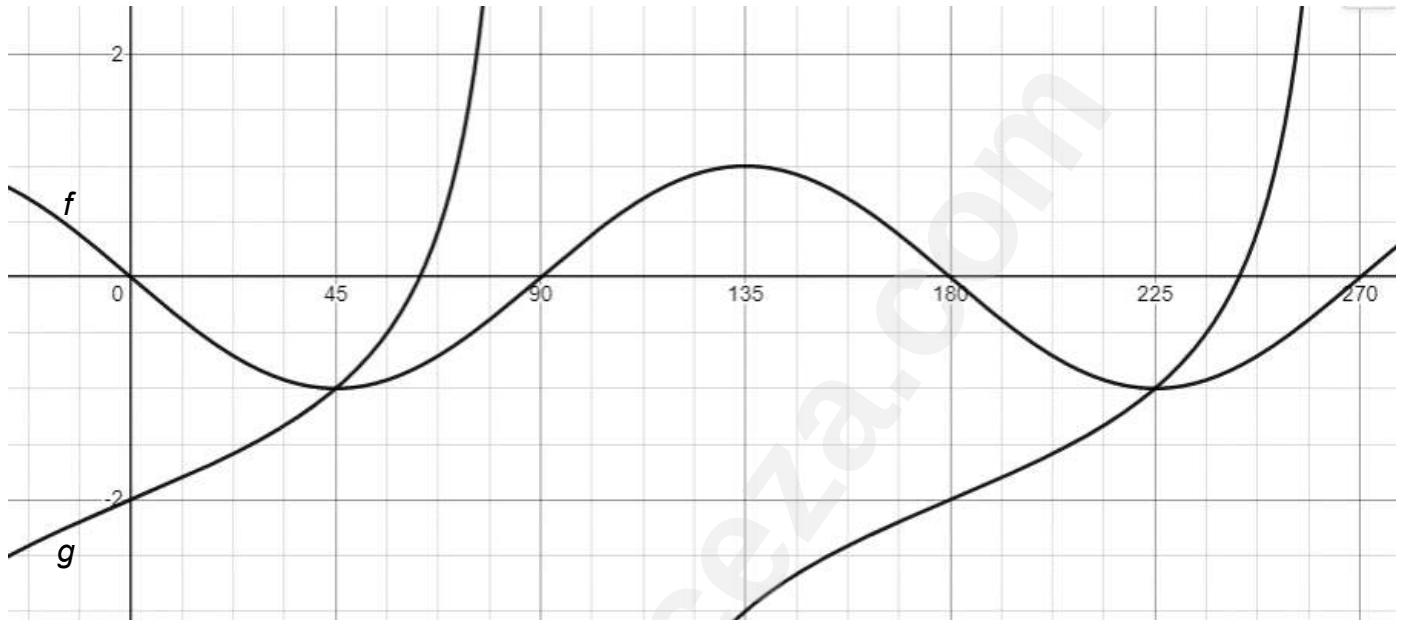
(3)

[8]

QUESTION 4

Given that.

- (a) On the diagram below, the graphs of $f(x) = -\sin 2x$ and $g(x) = \tan x - 2$ are sketched.



- (a) Write down the range of f .

(2)

- (c) Write down the period of f .

(1)

- (d) **Use the graph** to write down the solutions to the equation $\sin 2x + \tan x = 2$ for $x \in [0^\circ; 270^\circ]$.

(2)

(e) Determine the values of x for which $f(x) \leq g(x)$ if $x \in [0^\circ; 270^\circ]$.

(4)

(f) Determine the values of x for which $f(x) \cdot g(x) \leq 0$ if $x \in [0^\circ; 270^\circ]$.

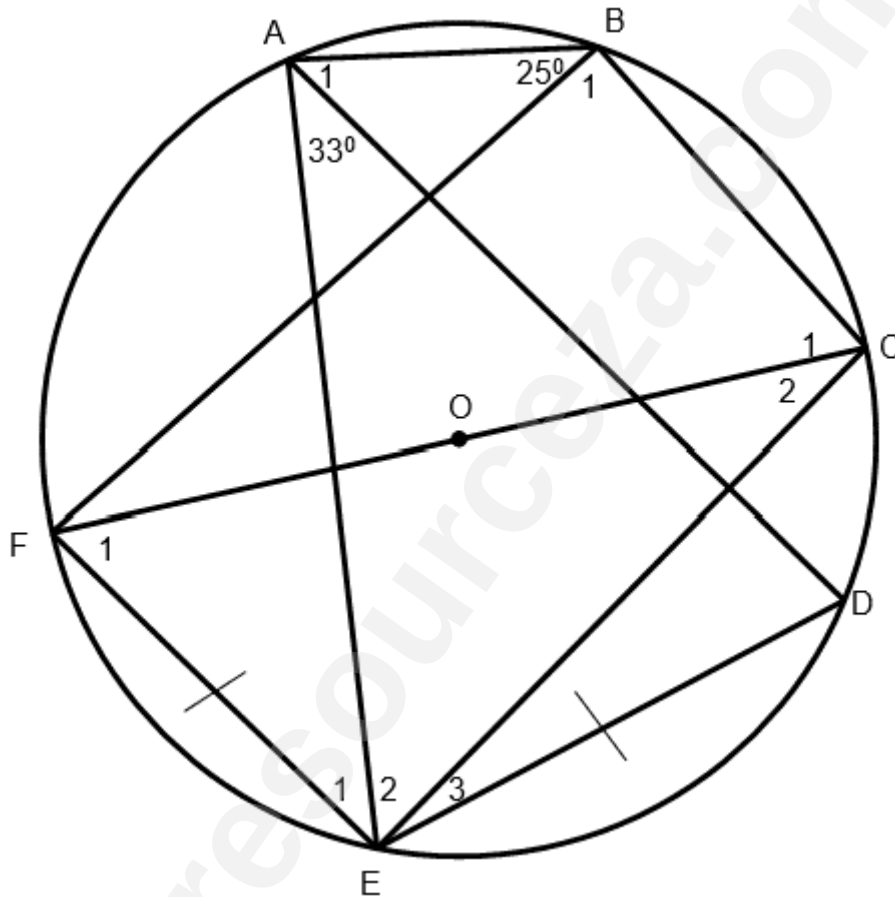
(5)

[14]

QUESTION 5

- (a) Complete the statement: The angle subtended by the diameter to the circumference of the circle is _____ (1)

- (b) In the diagram below, A, B, C, D and F are points on the circle. O is the centre of the circle. Chords EF = ED, $\angle ABF = 25^\circ$ and $\angle EAD = 33^\circ$.



Calculate, with reasons, the sizes of the following angles:

- (1) $\hat{B}_1$

(2)

(2) $\hat{C}_2$

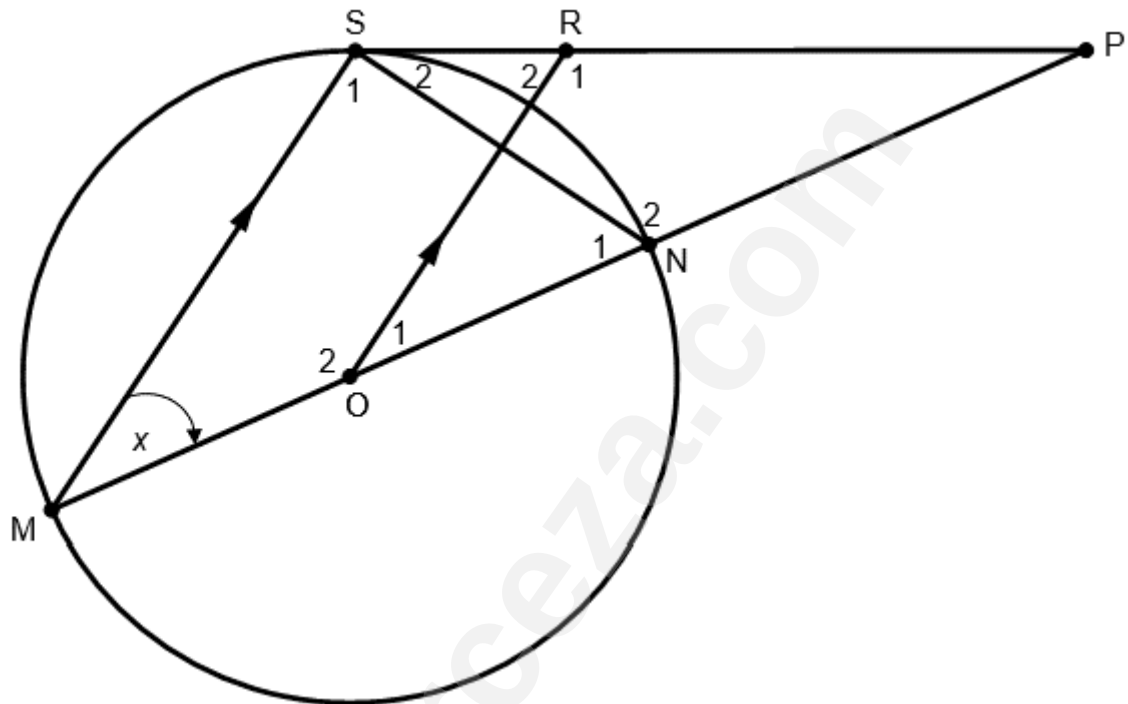
(2)

(3) $\hat{E}_2$

(3)

(c) In the diagram, MN is the diameter of the circle, centre O.

MN is produced to P. PS is the tangent to the circle at S. $MS \parallel OR$. $\hat{M} = x$.



(1) If $\hat{M} = x$, name with reasons, 2 other angles equal to x .

(2)

(2) Hence, write down the value of $\hat{P}$, in terms of x . Give reasons.

(2)

[12]

QUESTION 6

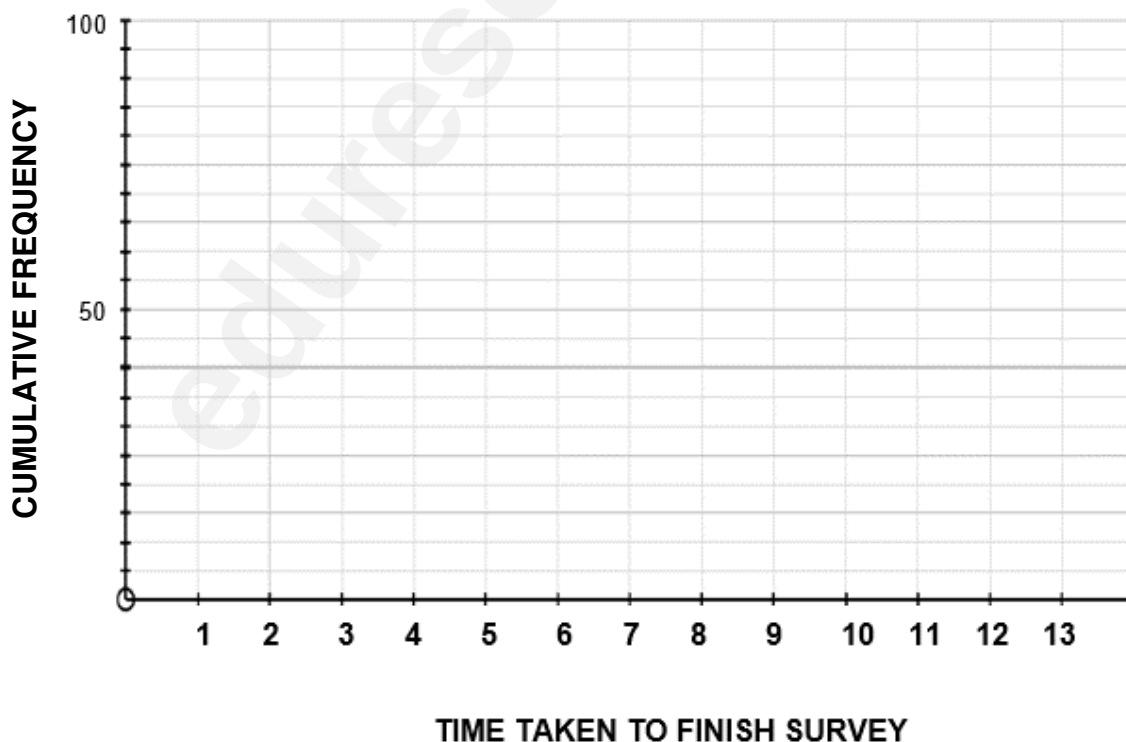
A research company, Cutting Edge Analytics, commissioned a survey to be done in various parts of Cape Town. The time taken by the different teams to complete the survey is summarised in the table below.

No. of HOURS	FREQUENCY	CUMULATIVE FREQUENCY
6 – 7	0	0
7 – 8	2	2
8 – 9	3	5
9 – 10	19	24
10 – 11	p	60
11 – 12	40	100

- (a) Write down the value of p.

(1)

- (b) Sketch an ogive (cumulative frequency curve) on the following axes:



(4)

- (c) Use your graph to find an estimate for the median for this set of data, and show where you read off this value using the letter M on your graph.

(2)

- (d) Use your graph to find an estimate of the 1st and 3rd quartiles. Use Q_1 and Q_3 to show where these values are read off your graph.

(2)

- (e) Calculate the estimated mean time for the survey.

(2)

- (f) What percentage of the survey times occurred within one standard deviation from the mean?

(2)

[13]

TOTAL SECTION A: 76 MARKS

SECTION B

QUESTION 7

Two friends, Tom and Teddy, recorded their pulse rates, P , and the time, t minutes after they had completed a workout in preparation for a competition. Tom's results are recorded in the table below.

Time (t)	0,5	1,0	1,5	2,0	3,0	4,0	5,0
Tom's pulse rate (P)	125	113	104	95	82	85	72

- (a) Determine the equation of the least squares regression line for Tom's pulse rate in the form $\hat{y} = a + bx$.

(2)

- (b) Use the equation of the least squares regression line to estimate Tom's pulse rate after 2,5 minutes. Give your answer to the nearest whole number.

(2)

- (c) The equation for the least squares regression line for Teddy's results is given by $\hat{y} = 116,79 - 6,61x$. The fitter a person, the quicker the return of the person's pulse rate to normal. Which of the two friends is fitter? Justify your answer.

(2)

[6]

QUESTION 8

Show that the equation of the line through the point $(a; b)$, perpendicular to the line $ax + by = 1$, is $bx - ay = 0$.

[6]

QUESTION 9

Determine the solutions of the following equation in the interval $\theta \in [-90^\circ; 270^\circ]$:

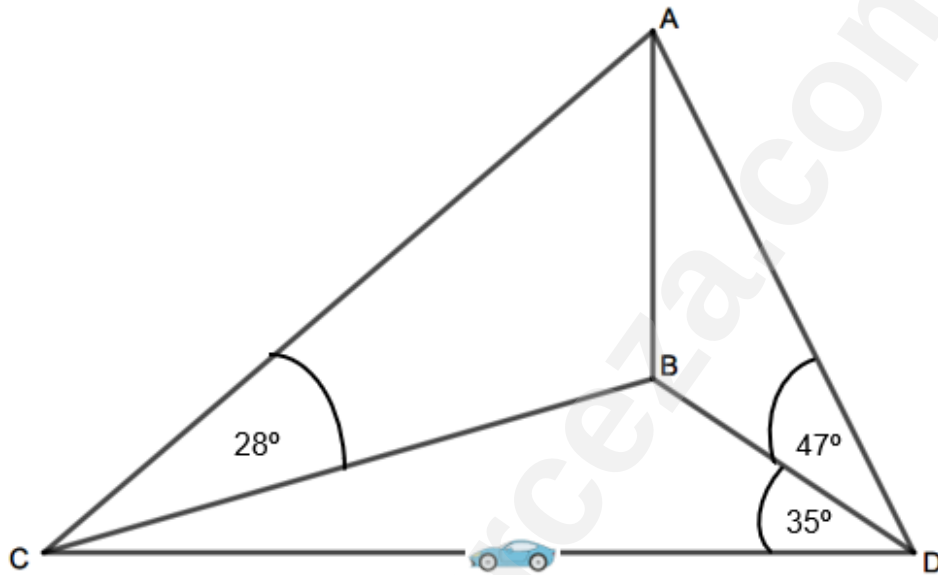
$$2 \cos 2\theta + \sin \theta = -1$$

[8]

QUESTION 10

In the diagram below, a car is travelling from D to C along a straight road DC.

- B, C and D are in the same horizontal plane.
- AB is the edge of a vertical building of height 108m metres.
- The angles of elevation of A from C and D respectively are 28° and 47° .
- $\widehat{BDC} = 35^\circ$.



- (a) Calculate the length of the road CD correct to the nearest metre.

(8)

- (b) Determine the range of the angles of elevation from the car to the top of the building as the car moves from D to C.

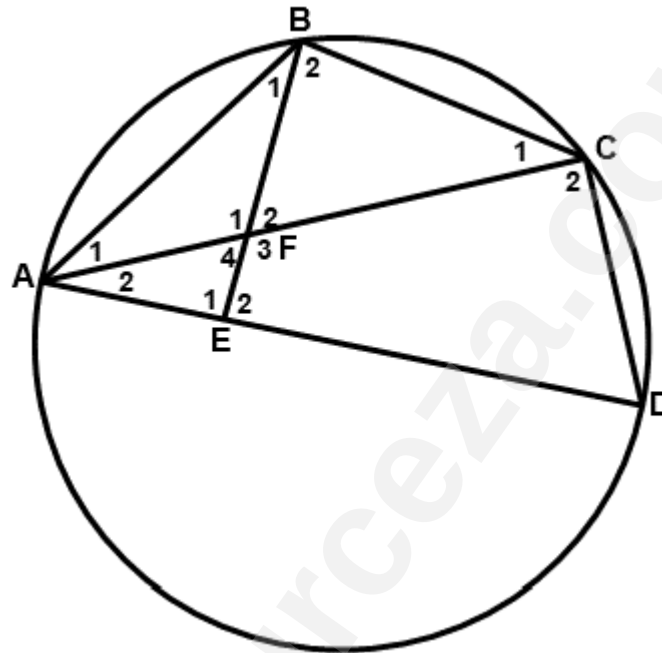
(6)

[14]

QUESTION 11

Refer to the figure below.

- AD is the diameter.
- B and C are points on the circumference of the circle.
- F is a point on AC such that $\hat{B}_1 = \hat{C}_1 = x$.
- BF is produced to meet AD at E.



- (a) Prove that $\triangle ABF \parallel \triangle ACB$.

(3)

(b) Prove that FCDE is a cyclic quadrilateral.

(c) Prove that $\triangle ACD \parallel \triangle AEF$.

(4)

(d) Hence, prove that $BE^2 = AE(AD - AE)$.

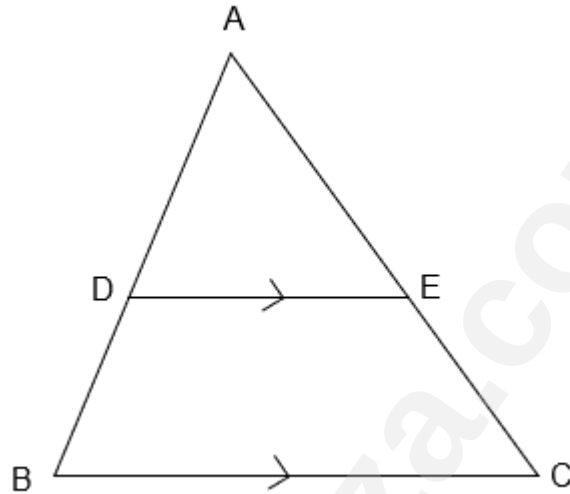
(3)

(7)

[17]

QUESTION 12

- (a) Use the diagram below to prove the theorem that states that **a line drawn parallel to one side of the triangle divides the other two sides proportionally.**



Required to prove: $\frac{AD}{DB} = \frac{AE}{EC}$

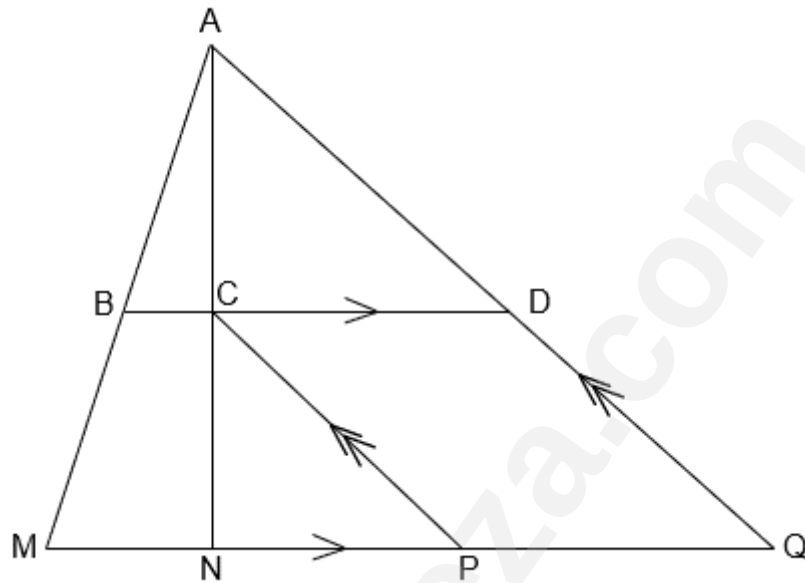
Construction:

Proof:

(6)

(b) In $\triangle AMQ$ below:

- $BD \parallel MQ$
- N and P are any points on MQ such that AN cuts BD in C
- $CP \parallel ADQ$



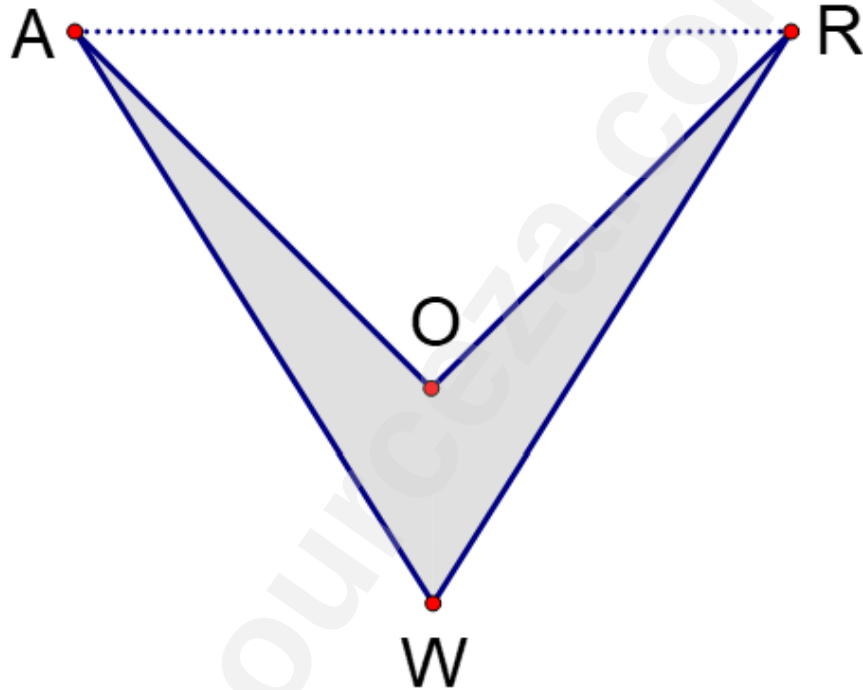
Prove that $AB \cdot NP = BM \cdot CD$.

(6)
[12]

QUESTION 13

The diagram below represents an arrow head. It is given that:

- $\hat{AOR} = 90^\circ$
- $AO = OR$
- $AW = WR$
- $\frac{OW}{RO} = \frac{\sqrt{2}}{2}$



- (a) A circle, defined by $(2x - 6)^2 + (2y - 10)(2y + 10) = 0$, passes through points A, R and O.

Find the centre and radius of the circle.

(4)

- (b) Find the area of the arrow head.

(7)
[11]

TOTAL SECTION B: 74 MARKS

REMEMBER TO INDICATE CLEARLY AT THE QUESTION THAT YOU HAVE USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

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