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**CRAWFORD INTERNATIONAL PRELIMINARY EXAMINATION
SEPTEMBER 2023
MATHEMATICS: PAPER I**

Time: 3 hours

150 marks

NAME OF CANDIDATE:	
NAME OF EDUCATOR:	

MARKER TO ENTER MARKS:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
11	12	18	16	18	17	12	11	20	15	150

1. This question paper consists of 22 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
Spare pages have been left for rough/extra working.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand this in at the end of the examination.
Remember to write your name in the space provided.
3. Diagrams are not necessarily drawn to scale.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Ensure that your calculator is in **DEGREE** mode.
6. All the necessary working details must be clearly shown.
Answers only will not necessarily be awarded full marks.
7. It is in your own interest to write legibly and to present your work neatly.
8. Round off to two decimal places unless otherwise stated.

SECTION A

QUESTION 1

(a) Solve for x :

(1) $2x^2 - x + 1 = x + 1$ (3)

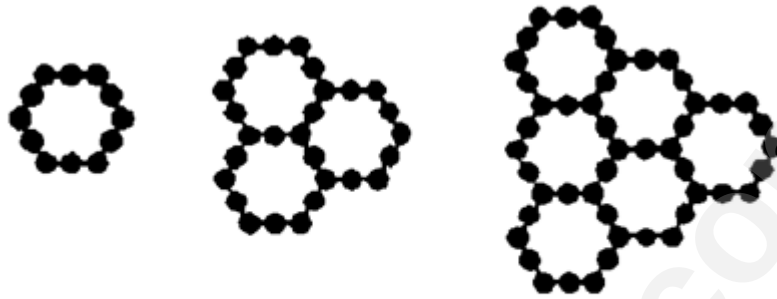
(2) $\left(\frac{x}{2}\right)^2 - 5x \leq -24$ (4)

(b) Simplify as far as possible:

$\frac{a^{3x} + 2a^{2x}}{a^{3x} - 4a^x} \cdot (a^x - 2)$ (4)

QUESTION 2

(a) Consider the following patterns:



PATTERN 1

PATTERN 2

PATTERN 3

- (1) Write down the number of hexagons in the fourth pattern. (1)
- (2) Determine the number of hexagons in the n -th pattern. (5)

(b) Consider the series: $1 + 2 + 4 + 8 + \dots$

(1) Calculate S_n , the sum to n terms.

(3)

(2) Prove that S_n , the sum to n terms is one less than $T_{(n+1)}$, the $(n+1)$ th term.

(3)

[12]

QUESTION 3

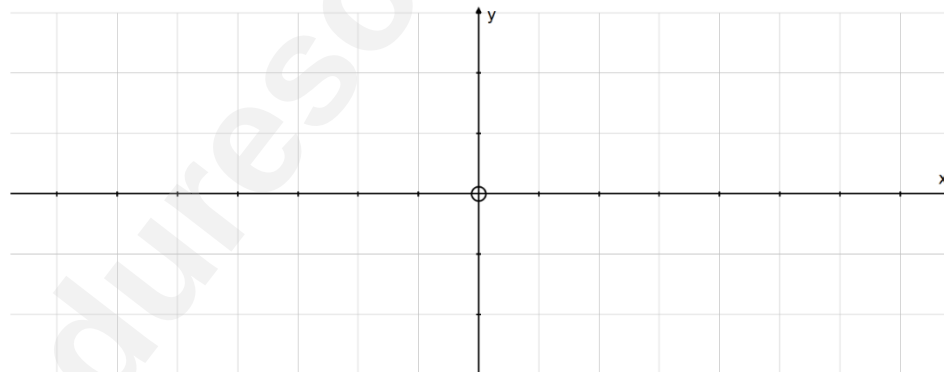
(a) The equation $3mx^2 - px + 5 = 0$; $p \neq 0$ has equal roots.

(1) Deduce that the expression $f(x) = 3mx^2 - px + 5$; $p \neq 0$ has a minimum value. (4)

(2) It is further given that $p < 0$.

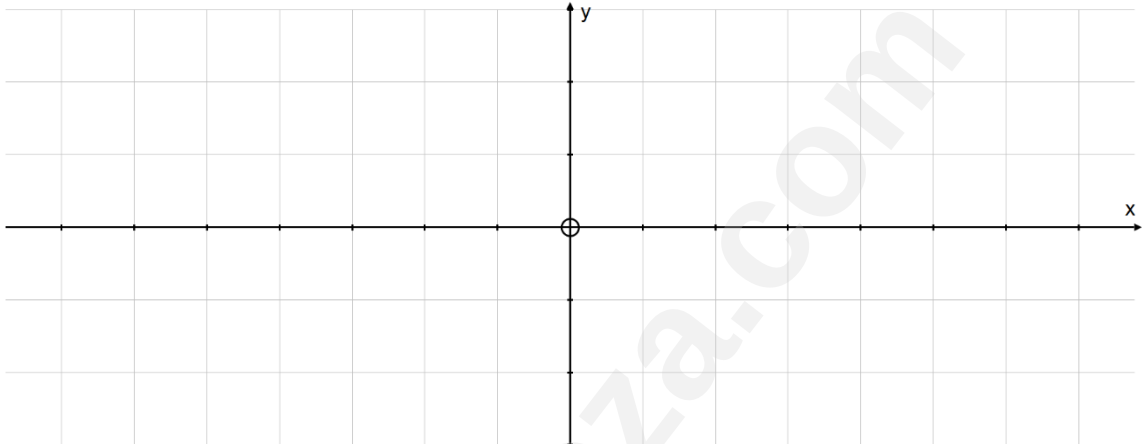
Using all the given information, sketch the graph with equation

$f(x) = 3mx^2 - px + 5$; $p \neq 0$. (3)



(b) Given that $f(x) = \left(\frac{1}{3}\right)^{-x}$.

- (1) Sketch the graph of f on the set of axes provided below. Indicate the intercept(s) with the axes, as well as the asymptote(s). (3)



- (2) State the range of f . (1)

- (3) Determine the equation of f^{-1} , the inverse function of f in the form $f^{-1}(x) = \dots$ (2)

- (4) Sketch the graph of f^{-1} on the same set of axes above. (3)

- (5) Determine the values of x for which $f^{-1}(x) \geq 1$. (2)

[18]

QUESTION 4

(a) Alice compares investment options at two banks.

- Bank A offers 6,5% per annum compounded monthly.
- Bank B offers an effective rate of 6,75% annually.

Determine which Bank offers the best investment option.

(Substantiate your answer with the necessary calculation.)

(3)

(b) Aleena secures a home loan for R1 500 000. The loan will be repaid over a period of 20 years at an interest rate of 11,5% per annum, compounded monthly.

(1) Show that the monthly installments are R15996,44.

(4)

- (2) Calculate the outstanding balance immediately after twelve years. (4)
- (3) After 12 years Aleena deposits a lump sum of R200 000 into her home loan account. She then continues making the same monthly installments of R 15996,44. Calculate how many months it will take her to settle the loan, if the interest rate of 11,5% p.a. compounded monthly stays unchanged. (5)

[16]

QUESTION 5

(a) Given $f(x) = -3x^2 + 2x$, determine $f'(x)$ by first principles.

(5)

(b) Determine $\frac{dy}{dx}$ for the following if:

(1) $y = (2x - 3)(x + 4)$

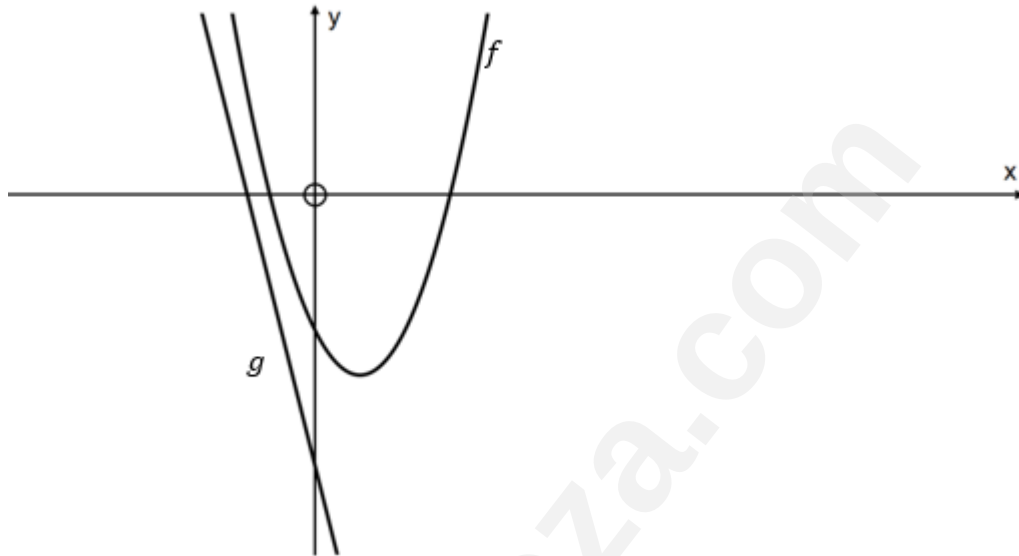
(3)

(2) $xy = 5\sqrt{x} - \frac{2}{x^2} + 6x^3$

(5)

(c) The accompanying sketch shows the curves of f and g with equations

$$f(x) = x^2 - 2x - 3 \quad \text{and} \quad g(x) = -4x - 6 \quad \text{respectively.}$$



The graph of g is translated k units to the right to touch the graph of f .

Showing all calculations, determine the value of k .

(5)

[18]

75 marks

SECTION B

QUESTION 6

- (a) The sport coordinator at Crawford La Lucia has reported the following data about sport and gender in her campus.

	Play sport	Do not play sport	TOTAL
Female	65	42	107
Male	36	68	104
TOTAL	101	110	211

Are the events 'male' and 'play sport' independent?

Show all calculations to support your answer.

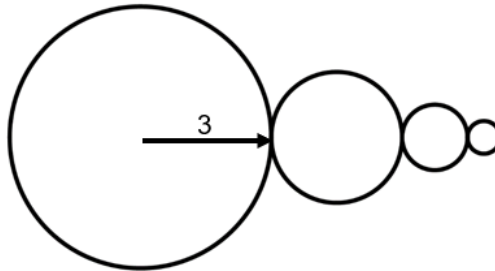
(5)

- (b) Number plates in a district in a certain country are created from a combination of the letters of the alphabet and numbers (0-9). The letter O is not used since it can be easily confused with the number 0. The letters and numbers may be repeated.
- (1) Calculate the number of registration numbers that can be created using two letters at the beginning, two digits in the middle and two letters at the end. (3)
- (2) The provincial government projections indicate that the province will need to create nine million different registrations in the coming decade. It has been decided that registration numbers will begin with two letters and that the rest of the characters must be numbers. Determine the least number of digits that will be needed in each number plate. (3)
- (c) Consider the single digits: 1 ; 2 ; 3 ; 4 ; 5 :
- ((1) How many five-digit numbers can be created using the digits above, if 4 and 5 must be next to each other. Each digit can only be used once. (3)
- (2) How many different numbers greater than 2100 can be created using the digits above and each digit can be used once only. (3)

[17]

QUESTION 7

- (a) The radius of the first circle of an infinite sequence of circles is 3 cm. The radius of each of the following circles is half of the radius of the previous circle. Determine the total area (in terms of π) of the circles formed in this way.



(6)

(b)

The first two terms of a geometric progression are the same as the first two terms of an arithmetic progression. The first term for both sequences is 8 and the first term of the geometric sequence is greater than its second term.

The sum of the first three terms of the arithmetic progression is 1,125 less than the sum of the first three terms of the geometric progression.

Determine the first three terms of the geometric sequence.

(6)

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QUESTION 8

(a) Solve for x if $\log_3(3x+1) - \log_3(2x-1) = 2$ (4)

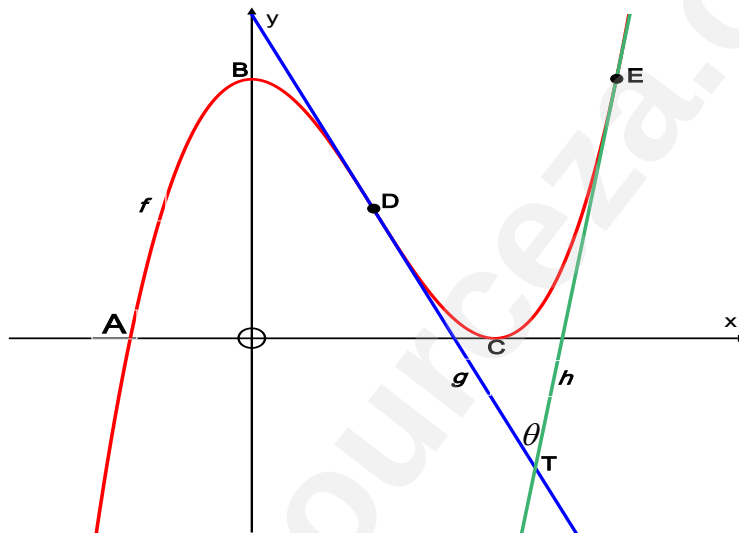
(b) Given: $mx^2 + x = m+1$
Determine the value(s) of m if the sum of the squares of the roots is equal to 5. (7)

QUESTION 9

The function below represents the graph of f with equation

$$y = f(x) = x^3 - 3x^2 + 4.$$

- A, B and C are intercepts with the axes and B and C are also stationary points of f .
- D is the point of inflection of f . E is a point on f .
- DT is a tangent to f at D.
- Straight lines DT and ET intersect at T and form an angle θ as indicated in the diagram.



(a) Calculate the length of AC. (3)

(b) Calculate the coordinates of D. (4)

- (c) Determine the equation of the tangent to f at point D. (i.e. line DT) (3)
- (d) The gradient of line ET is 9. Calculate the coordinates of point E. (4)
- (e) Calculate the value of θ , correct to two decimal places. (4)
- (f) For which values of x is $f(x) \cdot f''(x) \geq 0$? (2)

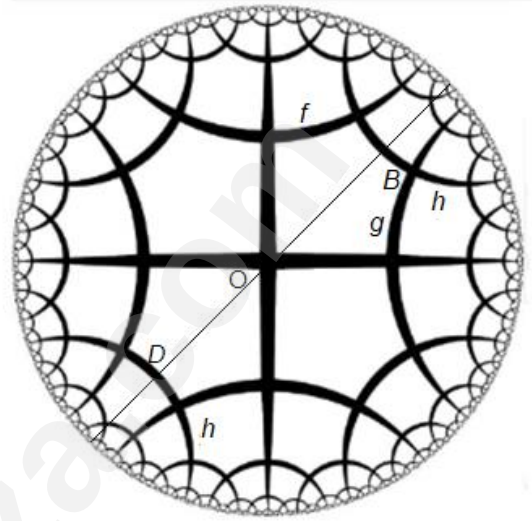
[20]

QUESTION 10

- (a) Anna created a symmetrical design for an art project, consisting of parabolic and hyperbolic shapes as shown below.

The design is placed on the Cartesian plane and the shapes are symmetrical about the origin.

- f is a parabola with equation $f(x) = ax^2 + q$.
- g is the reflection of f about the line $y = x$.
- The hyperbola labelled h has equation $h(x) = \frac{m}{x}$.
- The graph of g cuts the x -axis at $(4; 0)$.
- h and g intersect at $B(6; 3)$

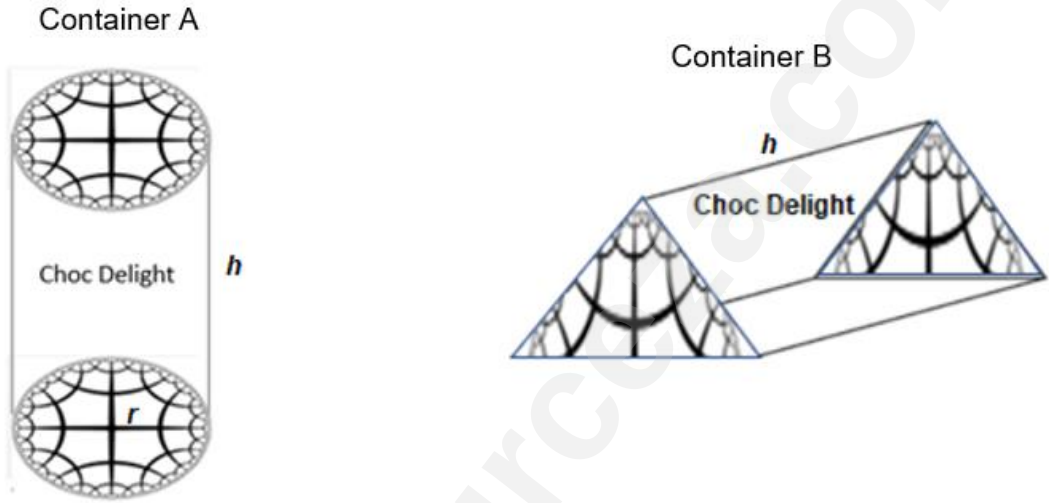


Study the diagram and answer the following questions:

- (1) Determine the equation of f . (3)
- (2) Calculate the shortest distance from the origin to the hyperbola h (distance OD). Leave your answer in simplest surd form. (4)

(b) The company DELIGHT manufactures various containers for chocolate biscuits.

- Container A is a cylindrical shape with a surface area of 600 square units.
- Container B is a triangular prism with an equilateral triangle as base.
- The sides of the triangles in container B are equal to the radius of the circular base in Container A.
- The height of the two containers are equal.
- $V = (\text{area of base}) \times h$



(1) Show that the height of the containers can be expressed as

$$h = \frac{300}{\pi r} - r \quad (3)$$

- (2) Calculate the value of r for which the volume of Container B is a maximum.

(5)

[15]

75 marks

Total: 150 marks

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