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SECTION A

QUESTION 1 [11 MARKS]

(a) Solve for x :

$$(1) \quad 2x^2 - x + 1 = x + 1$$

$$2x^2 - 2x = 0$$

$$x(x - 1) = 0$$

$$x = 0 \text{ or } x = 1$$

[3]

$$(2) \quad \left(\frac{x}{2}\right)^2 - 5x \leq -24$$

$$x^2 - 20x + 96 \leq 0$$

$$(x - 8)(x - 12) \leq 0$$

$$8 \leq x \leq 12$$



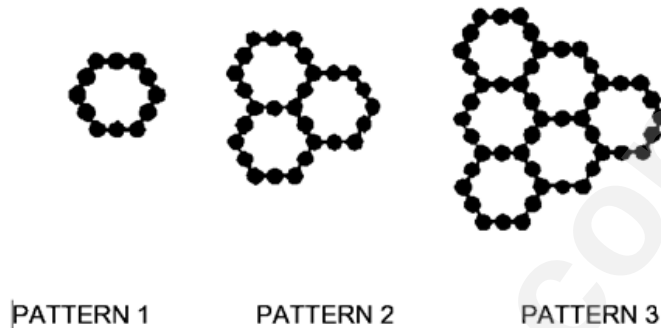
[4]

$$\begin{aligned} (b) \quad & \frac{a^{3x} + 2a^{2x}}{a^{3x} - 4a^x} \cdot (a^x - 2) \\ &= \frac{a^{2x}(a^x + 2)}{a^x(a^{2x} - 4)} \cdot (a^x - 2) \\ &= \frac{a^{2x}(a^x + 2)}{a^x(a^x + 2)(a^x - 2)} \cdot (a^x - 2) \\ &= a^x \end{aligned}$$

[4]

QUESTION 2 [12 MARKS]

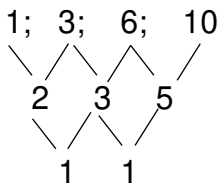
(a) Consider the following patterns:



(1) Write down the number of hexagons in the fourth pattern.

10

(2) Determine the number of hexagons in the n -th pattern.



$$\therefore 2a = 1$$

$$\therefore a = \frac{1}{2}$$

$$3a + b = 2 \text{ and } a + b + c = 1$$

$$b = \frac{1}{2}$$

$$c = 0$$

$$T_n = \frac{n^2}{2} + \frac{n}{2}$$

[5]

(b) Consider the series: $1 + 2 + 4 + 8 + \dots$

(1) Calculate S_n , the sum to n terms.

$$\begin{aligned} S_n &= \frac{1(2^n - 1)}{2 - 1} \\ &= 2^n - 1 \end{aligned} \quad [3]$$

(2) Prove that S_n , the sum to n terms is one less than $T_{(n+1)}$, the $(n+1)$ th term.

$$\begin{aligned} T_n &= 2^{(n-1)} \\ T_{(n+1)} &= 2^{(n+1-1)} = 2^n \\ \therefore S_n &= T_{(n+1)} - 1 \end{aligned} \quad [3]$$

QUESTION 3 [18 MARKS]

(a) The equation $3mx^2 - px + 5 = 0$; $p \neq 0$ has equal roots.

(1) Deduce that $f(x) = 3mx^2 - px + 5$; $p \neq 0$ has a minimum value.

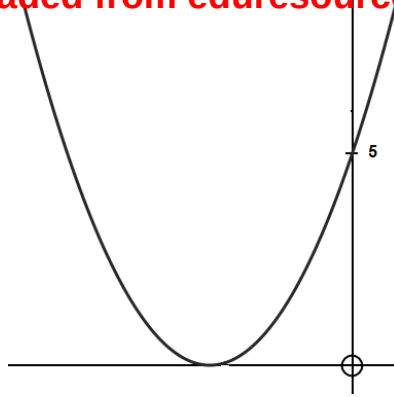
$$\begin{aligned} \Delta &= p^2 - 60m \\ 0 &= p^2 - 60m \\ p^2 &= 60m \\ \therefore m &> 0 \quad (p \neq 0) \\ \therefore 3m &> 0 \quad \therefore f(x) \text{ has a minimum value} \end{aligned} \quad [4]$$

(2) It is further given that $p < 0$.

Using the given information, sketch the graph with equation

$$f(x) = 3mx^2 - px + 5; \quad p \neq 0$$

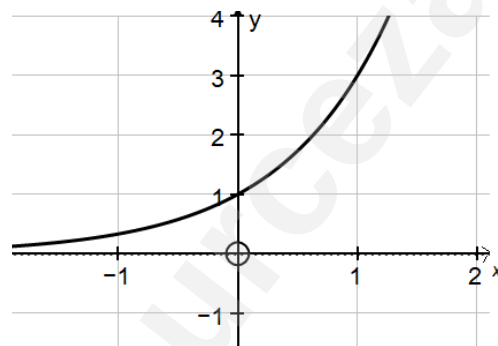
[3]



(b) The graph of f with equation $f(x) = \left(\frac{1}{3}\right)^{-x}$, is given.

(1) Sketch the graph of f on the set of axes provided below.

Indicate the intercept(s) with the axes, as well as the asymptote(s).



[3]

(2) State the range of f .
(0; ∞)

[1]

(3) Determine the equation of f^{-1} , the inverse function of f in the form
 $f^{-1}(x) = \dots$

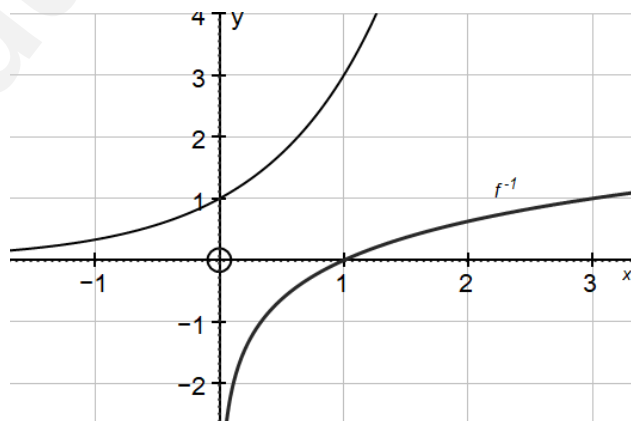
[2]

$$x = 3^y$$

$$f^{-1}(x) = \log_3 x$$

$$\text{or } f^{-1}(x) = -\log_{\frac{1}{3}} x$$

(4) Sketch the graph of f^{-1} on the same set of axes above.



[3]

- (5) Determine the values of x for which $f^{-1}(x) \geq 1$.

$$x \geq 3$$

[2]

QUESTION 4 [16 MARKS]

- (a) Alice compares investment options at two banks.

- Bank A offers 6,5% per annum compounded monthly.
- Bank B offers an effective rate of 6,75% annually.

Determine which Bank offers the best investment option.

(Substantiate your answer with the necessary calculation.)

$$1 + i_{\text{eff}} = \left(1 + \frac{0,065}{12}\right)^{12}$$

[3]

$$i_{\text{eff}} = 0,0669 \approx 6,69\%$$

Bank B offers the best option

- (b) Aleena secures a home loan for R1 500 000. The loan will be repaid over a period of 20 years at an interest rate of 11,5% per annum, compounded monthly.

- (1) Show that the monthly installments are R15996,44.

$$x = \frac{1500000 \left(\frac{0,115}{12} \right)}{1 - \left(1 + \frac{0,115}{12} \right)^{-240}}$$

[4]

$$x = 15996,44$$

- (2) Calculate the outstanding balance immediately after twelve years.

$$1500000 \left(1 + \frac{0,115}{12} \right)^{144} - \frac{15996,44 \left(\left(1 + \frac{0,115}{12} \right)^{144} - 1 \right)}{\frac{0,115}{12}}$$

$$= R 1001069,67$$

[4]

- (3) After 12 years Aleena deposits a lump sum of R200000 into her home loan account. She then continues making the same monthly installments of R 15996,44.

Calculate how many months it will take her to settle the loan, if the interest rate of 11,5% p.a. stays unchanged.

$$1001069,67 - 200000 = 801069,67$$

[5]

$$801069,67 = \frac{15996,44 \left(1 - \left(1 + \frac{0,115}{12} \right)^{-n} \right)}{\frac{0,115}{12}}$$

$$n = 68.544$$

∴ 69 months

QUESTION 5 [18 MARKS]

- (a) Given $f(x) = -3x^2 + 2x$, determine $f'(x)$ by first principles.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-3(x+h)^2 + 2(x+h) - (-3x^2 + 2x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-3(x^2 + 2xh + h^2) + 2x + 2h + 3x^2 - 2x}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-3x^2 - 6xh - 3h^2 + 2x + 2h + 3x^2 - 2x}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-6xh - 3h^2 + 2h}{h}$$

[5]

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(-6x - 3h + 2)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} (-6x - 3h + 2)$$

$$f'(x) = -6x + 2$$

(b) Determine $\frac{dy}{dx}$ for the following if:

(1) $y = (2x - 3)(x + 4)$

$$y = 2x^2 + 5x - 12$$

$$\frac{dy}{dx} = 4x + 5$$

[3]

(2) $xy = 5\sqrt{x} - \frac{2}{x^2} + 6x^3$

$$y = \frac{5x^{\frac{1}{2}}}{x} - \frac{2x^{-2}}{x} + \frac{6x^3}{x}$$

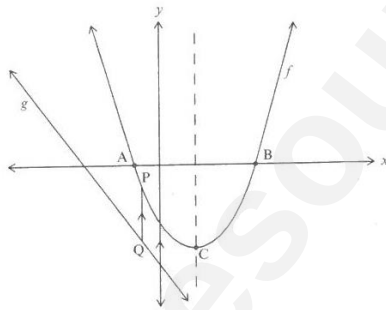
$$y = 5x^{-\frac{1}{2}} - 2x^{-3} + 6x^2$$

[5]

$$\frac{dy}{dx} = -\frac{5}{2}x^{-\frac{3}{2}} + 6x^{-4} + 12x$$

(c) The accompanying sketch shows the curves of f and g with equations

$$f(x) = x^2 - 2x - 3 \quad \text{and} \quad g(x) = -4x - 6 \quad \text{respectively.}$$



The graph of g is translated k units to the right to touch the graph of f ,

Showing all calculations, determine the value of k .

$$g(x - k) = f(x)$$

$$-4(x - k) - 6 = x^2 - 2x - 3$$

$$x^2 + 2x + (3 - 4k) = 0 \quad \text{has equal roots}$$

$$\Delta = 4 - 4(1)(3 - 4k) = 0$$

$$k = \frac{1}{2}$$

[5]

OR

$$f'(x) = -4$$

$$\therefore 2x - 2 = -4$$

$$x = -1$$

point of contact: $(-1; 0)$

x - intercept of $g: \left(-\frac{3}{2}; 0\right)$

$$\therefore k = \frac{1}{2}$$

75 marks

SECTION B

QUESTION 6 [17 MARKS]

- (a) The sport coordinator at Crawford La Lucia has reported the following data about sport and gender in her campus.

	Play sport	Do not play sport	TOTAL
Female	65	42	107
Male	36	68	104
TOTAL	101	110	211

Are the events 'male' and 'play sport' independent?

Show all calculations to support your answer.

$$P(\text{Male and Play Sport}) = \frac{36}{211} = 0,1706$$

[5]

$$P(\text{Male}) \times P(\text{Play Sport})$$

$$= \frac{104}{211} \times \frac{101}{211}$$

$$= 0,2359$$

$$P(\text{Male and Play Sport}) \neq P(\text{Male}) \times P(\text{Play Sport})$$

Events are NOT independent.

- (b) Number plates in Gauteng are created from a combination of the letters of the alphabet and numbers (0-9). The letter O is not used since it can be easily confused with the number 0. The letters and numbers may be repeated.

- (1) Calculate the number of registration numbers that can be created using two letters at the beginning, two digits in the middle and two letters at the end.

$$25 \times 25 \times 10 \times 10 \times 25 \times 25 \\ = 39062500$$

[3]

- (2) The provincial government projections indicate that the province will need to create nine million different registrations in the coming decade. It has been decided that registration numbers will begin with two letters and that the rest of the characters must be numbers. Determine the least number of digits that will be needed in each number plate.

$$25 \times 25 \times 10^n > 9000000000 \\ 10^n > 14400 \\ n > \log 14400 \\ \therefore n > 4,15 \\ \therefore 5 \text{ digits are needed}$$

[3]

- (c) Consider the single digits: 1 ; 2 ; 3 ; 4 ; 5 :

- (1) How many five-digit numbers can be created using the digits above, if 4 and 5 must be next to each other. Each digit can only be used once.

$$2 \times 4! = 48$$

[3]

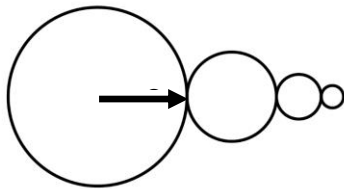
- (2) How many different numbers greater than 2100 can be created using the digits above and each digit can be used once only.

$$4 \times 4! + 5! = 216$$

[3]

QUESTION 7 [12 marks]

- (a) The radius of the first circle of an infinite sequence of circles is 3 cm. The radius of each of the following circles is half of the radius of the previous circle. Determine the total area (in terms of π) of the circles formed in this way.



$$\text{Radii: } 3; 3\left(\frac{1}{2}\right); 3\left(\frac{1}{2}\right)^2; \dots$$

$$\text{Areas: } \pi(3)^2; \pi\left(3\left(\frac{1}{2}\right)\right)^2; \pi\left(3\left(\frac{1}{2}\right)^2\right)^2; \dots$$

[6]

$$S_{\infty} = \frac{9\pi}{1 - \frac{1}{4}}$$

$$= 12\pi \text{ cm}^2$$

- (b) The first two terms of a geometric progression are the same as the first two terms of an arithmetic progression. The first term is 8 and is greater than the second term.
The sum of the first three terms of the arithmetic progression is 1,125 less than the sum of the first three terms of the geometric progression.
Determine the first three terms of each sequence.

$$\text{Geometric: } 8; 8r; 8r^2; \dots$$

$$\text{Arithmetic: } 8; 8+d; 8+2d; \dots$$

$$8r = 8 + d$$

$$\therefore d = 8r - 8$$

$$8 + (8 + d) + 8 + 2d = 8 + 8r + 8r^2 - \frac{9}{8}$$

$$24 + 3(8r - 8) = 8r^2 + 8r + \frac{55}{8}$$

[6]

$$192r = 64r^2 + 64r + 55$$

$$64r^2 - 128r + 55 = 0$$

$$r = \frac{5}{8}$$

$$\text{Geom: } 8; 5; \frac{25}{8}; \dots$$

QUESTION 8 [11 MARKS]

- (a) Solve for
- x
- if
- $\log_3(3x+1) - \log_3(2x-1) = 2$

$$\log_3\left(\frac{3x+1}{2x-1}\right) = 2$$

$$9 = \frac{3x+1}{2x-1}$$

$$18x - 9 = 3x + 1$$

$$15x = 10$$

$$x = \frac{2}{3}$$

[4]

- (b) Given:
- $mx^2 - x = m + 1$

Determine the value(s) of m if the sum of the squares of the roots is equal to 5.

$$mx^2 + x - m - 1 = 0$$

$$\text{Roots: } x = \frac{-1 \pm \sqrt{1 - 4(m)(-m-1)}}{2m}$$

$$x = \frac{-1 \pm \sqrt{(2m+1)^2}}{2m}$$

$$x = \frac{-1 \pm (2m+1)}{2m}$$

$$x = 1 \text{ or } x = \frac{-1-m}{m}$$

$$\therefore 1^2 + \left(\frac{-1-m}{m}\right)^2 = 5$$

$$1 + \frac{m^2 + 2m + 1}{m^2} = 5$$

$$m^2 + m^2 + 2m + 1 = 5m^2$$

$$3m^2 - 2m - 1 = 0$$

$$(3m+1)(m-1) = 0$$

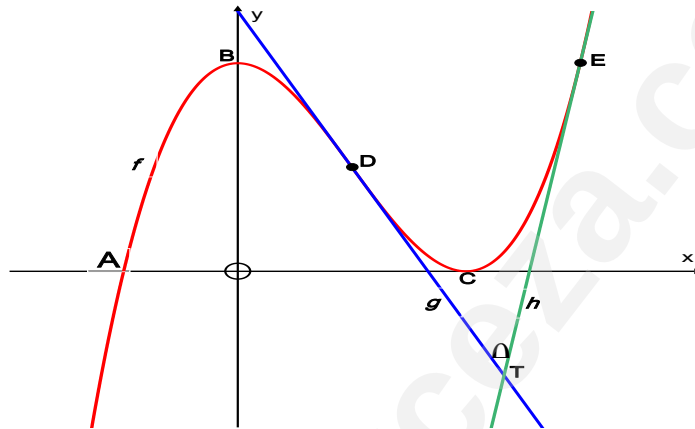
$$\therefore m = -\frac{1}{3} \text{ or } m = 1$$

[7]

QUESTION 9 [20 MARKS]

The function below represents the graph of f with equation $y = f(x) = x^3 - 3x^2 + 4$.

- A, B and C are intercepts with the axes and B and C are also stationary points of f .
- D is the point of inflection of f . E is a point on f .
- DT is a tangent to f at D.
- Straight lines DT and ET intersect at T and form an angle θ as indicated in the diagram.



- (a) Calculate the length of AC.

$$0 = x^3 - 3x^2 + 4$$

$$0 = (x+1)(x^2 - 4x + 4)$$

$$0 = (x+1)(x-2)^2$$

$$x = -1 \text{ or } x = 2$$

$$AC = 3 \text{ units}$$

[3]

- (b) Calculate the coordinates of D.

$$f(x) = x^3 - 3x^2 + 4$$

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6 = 0$$

$$x = 1$$

$$y = (1)^3 - 3(1)^2 + 4 = 2 \quad \therefore D(1; 2)$$

[4]

- (c) Determine the equation of the tangent to f at the point D. (i.e. line DT)

$$f'(1) = 3(1)^2 - 6(1) = -3$$

$$y - 2 = -3(x - 1)$$

$$g(x) = -3x + 5$$

[3]

- (d) The gradient of line ET is 9. Calculate the coordinates of point E.

$$f'(x) = 3x^2 - 6x = 9$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3$$

$$f(3) = (3)^3 - 3(3)^2 + 4 = 4 \quad E(3; 4)$$

[4]

- (e) Calculate the value of θ , correct to two decimal places.

$$\tan \alpha = -3$$

$$\tan \beta = 9$$

$$\alpha = 180^\circ - 71,57^\circ$$

$$\beta = 83,66^\circ$$

$$\alpha = 108,43^\circ$$

[4]

$$\theta = \alpha - \beta$$

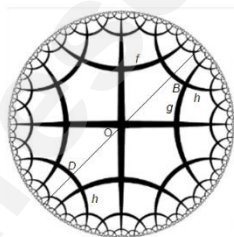
$$\theta = 108,43^\circ - 83,66^\circ = 24,77^\circ$$

- (f) $x \leq -1$ or $x \geq 1$

[2]

QUESTION 10 [15 MARKS]

- (a) Anna created a symmetrical design for an art project, consisting of parabolic and hyperbolic shapes as shown below. The design is placed on the Cartesian plane and the shapes are symmetrical about the origin.



- (1) Determine the equation of f

Points: $(0; 4)$ and $(3; 6)$

[3]

$$y = ax^2 + 4$$

$$6 = a(3)^2 + 4$$

$$y = \frac{2}{9}x^2 + 4$$

- (2) Calculate the shortest distance from the origin to the hyperbola h (distance OD).
Leave your answer in simplest surd form.

sub $(6; 3)$ into $y = \frac{m}{x}$

$$3 = \frac{m}{6}$$

$$\therefore m = 18$$

At D :

$$x^2 = 18$$

$$x = -3\sqrt{2} \text{ and } y = -3\sqrt{2}$$

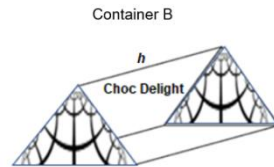
$$OD^2 = \sqrt{(-3\sqrt{2})^2 + (-3\sqrt{2})^2}$$

$$OD = 6$$

[4]

- (b) The company DELIGHT manufactures various containers for chocolate biscuits.

- Container A is a cylindrical shape with a surface area of 600 square units
- Container B is a triangular prism with an equilateral triangle as base.
- The sides of the triangles in container B are equal to the radius of the circular base in Container A.
- The height of the two containers are equal.



- (1) Show that the height of the containers can be expressed as

$$h = \frac{300}{\pi r} - r$$

$$2\pi r^2 + 2\pi r h = 600$$

$$2\pi r h = 600 - 2\pi r^2$$

$$h = \frac{300}{\pi r} - r$$

[3]

- (2) Calculate the value of r for which the volume of Container B is a maximum..

$$V = \frac{1}{2} x^2 \sin 60^\circ \times h$$

$$= \frac{\sqrt{3}}{4} r^2 \left(\frac{300}{\pi r} - r \right)$$

$$= \frac{\sqrt{3} \times 300}{4\pi} r - \frac{\sqrt{3}}{4} r^3$$

$$V'(x) = \frac{\sqrt{3} \times 300}{4\pi} - \frac{3\sqrt{3}}{4} r^2 = 0$$

$$\Rightarrow r^2 = \frac{100}{\pi}$$

$$r = 5,64$$

[5]

74 marks

Total: 150 marks