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INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
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WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

(a) (1) $\frac{1}{4}x^2 - 3x = 0$

$$x\left(\frac{1}{4}x - 3\right) = 0$$

$$x = 0 \quad \text{of} \quad \frac{1}{4}x = 3$$

$$x = 12$$

(2) $4 \cdot 3^x - 3^{x+1} = 9$

$$3^x(4 - 3) = 9$$

$$3^x = 3^2$$

$$x = 2$$

(3) $2\log 3 + \log 2x - \log(3x - 1) = 1$

$$\log 9 + \log 2x - \log(3x - 1) = 1$$

$$\log\left(\frac{9 \cdot 2x}{3x - 1}\right) = \log 10$$

$$18x = 10(3x - 1)$$

$$18x - 30x = -10$$

$$-12x = -10$$

$$x = \frac{5}{6}$$

(b) Indien $\log 3 = p$ en $\log 7 = q$, bereken die waarde van $\log 4\frac{2}{7}$ in terme van p en q .

$$= \log \frac{30}{7}$$

$$= \log 30 - \log 7$$

$$= \log 10 + \log 3 - \log 7$$

$$= 1 + p - q$$

$$(c) \quad \frac{2^{x+1} \cdot 18^{x-2}}{4^x \cdot 3^{2x-5}}$$

$$= \frac{2^{x+1} \cdot 2^{x-2} \cdot 3^{2x-4}}{2^{2x} \cdot 3^{2x-5}}$$

$$= 2^{2x-1} \cdot 2^{2x} \cdot 3^{2x-4-2x+5}$$

$$= 2^{-1} \cdot 3$$

$$= \frac{3}{2}$$

VRAAG 2

(a) (1) $T_5 = 18$

$$6 + 4d = 18$$

$$4d = 12$$

$$d = 3$$

(2) $S_{20} = \frac{20}{2}[2(6) + 19(3)]$

$$S_{20} = 690$$

(b) (1) $T_2 = 12$

$$T_5 = 1,5$$

$$ar = 12$$

$$ar^4 = 1,5$$

$$a = \frac{12}{r} \quad \dots >$$

$$\frac{12r^4}{r} = 1,5$$

$$r^3 = \frac{1,5}{12} = \frac{1}{8}$$

$$< \dots r = \frac{1}{2}$$

$$a = \frac{12}{\left(\frac{1}{2}\right)}$$

$$a = 24$$

$$a = 24 ; r = \frac{1}{2}$$

(2) $S_{\infty} = \frac{24}{1 - \frac{1}{2}}$

$$S_{\infty} = 48$$

VRAAG 3

$$(a) \quad f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{2}(x+h)^2 + x + h - \left(\frac{1}{2}x^2 + x\right)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 + x + h - \frac{1}{2}x^2 - x}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h\left(x + \frac{1}{2}h + 1\right)}{h}$$

$$f'(x) = x + \frac{0}{2} + 1$$

$$f'(x) = x + 1$$

$$(b) \quad f(x) = 2x^5 - 4x^{-3} + x^{\frac{2}{5}}$$

$$f'(x) = 10x^4 + 12x^{-4} + \frac{2}{5}x^{-\frac{3}{5}}$$

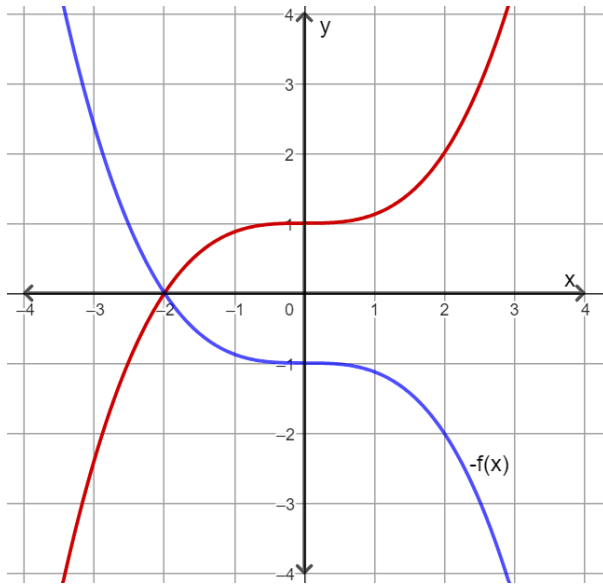
$$(c) \quad f(3) = 10 \therefore (3; 10) \text{ lê op } f.$$

$$f^{-1}(4) = 2 \therefore (2; 4) \text{ lê op } f.$$

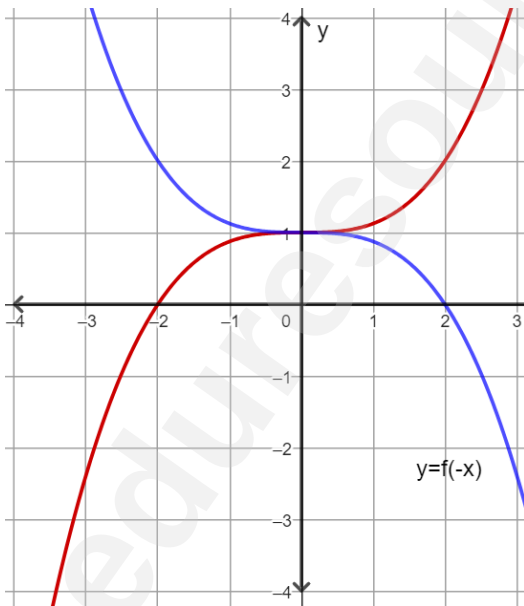
$$\text{gradiënt} = \frac{10 - 4}{3 - 2} = 6$$

VRAAG 4

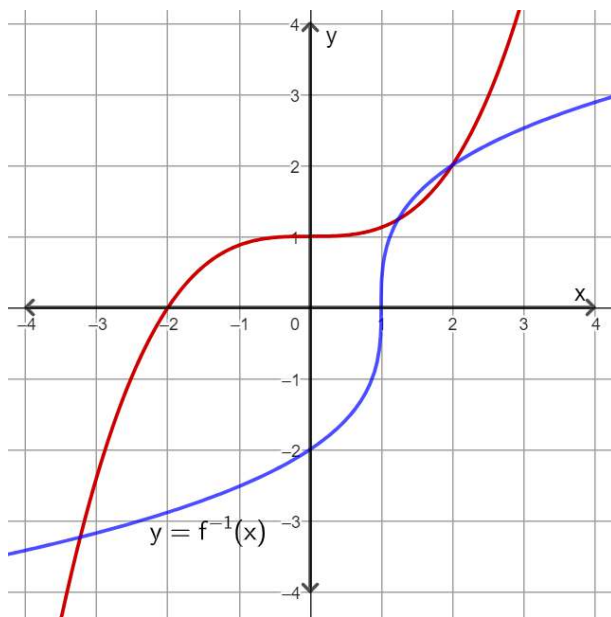
(a) $y = -f(x)$



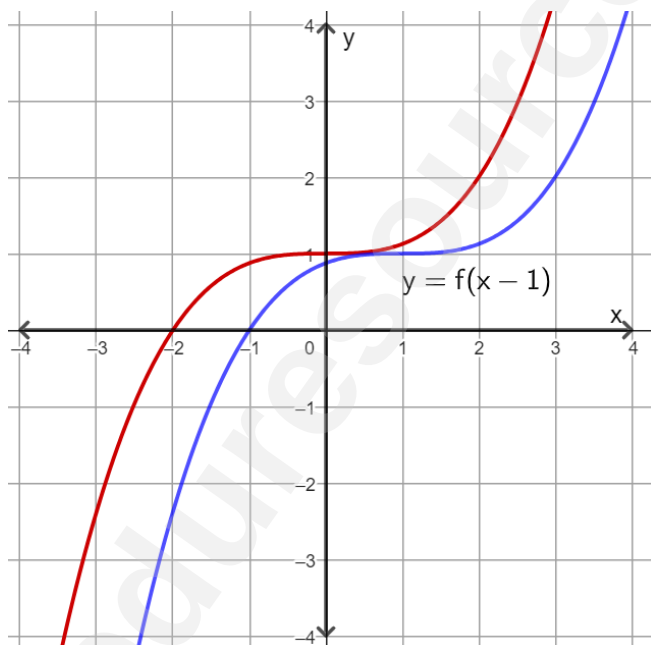
(b) $y = f(-x)$



(c) $y = f^{-1}(x)$



(d) $y = f(x - 1)$



VRAAG 5

(a) $A = 350000(1 + 0,069)^5$

$A = 488\,603,4964$ (akkuraat) of $A = 488\,603,50$

(b) $0,08 = \left(1 + \frac{r}{12}\right)^{12} - 1$

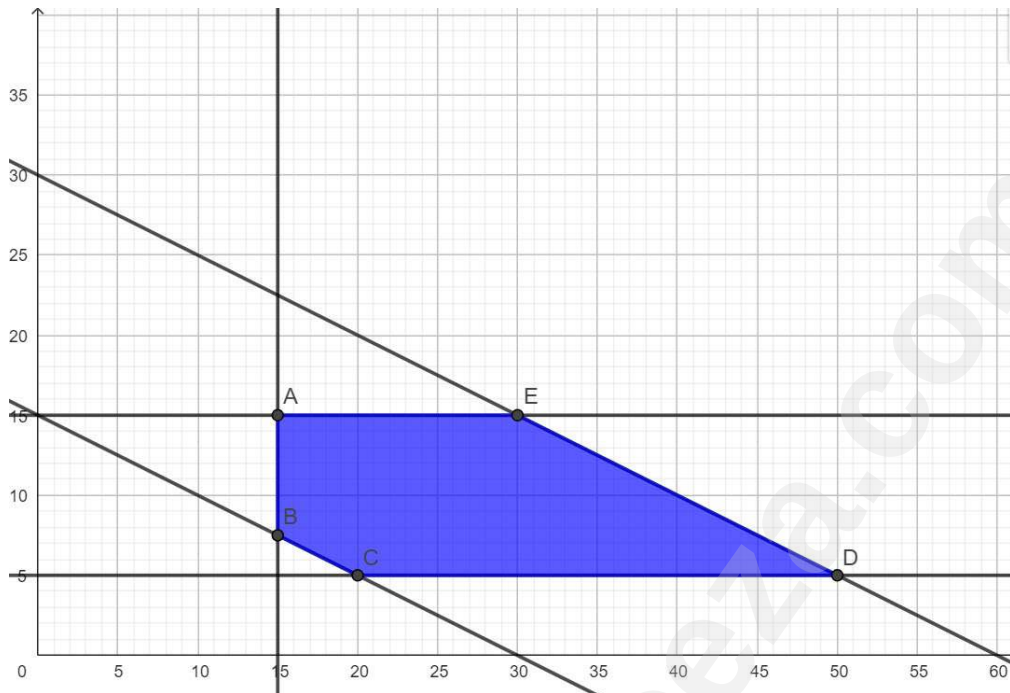
$r = 0,0772....$

(c) $488603,50 = x \frac{\left[\left(1 + \frac{0,0772..}{12}\right)^{61} - 1 \right]}{\frac{0,0772..}{12}}$

$x = 6566,02$ (akkuraat) of $6566,016(4dp)$

VRAAG 6

(a)



Elke lyn uitvoerbaarheidsgebied

(b) (1) $P = 15x + 60y$
 $y = -\frac{15}{60}x + \frac{P}{60}$
 $\therefore m = -\frac{1}{4} \rightarrow \text{gradiënt}$
 $\therefore x = 30 \text{ en } y = 15 \quad (\text{gebruik seeklyn})$

(2) $P_{\text{maks}} = 15(30) + 60(15)$
 $P_{\text{maks}} = 1350$

VRAAG 7

(a) $N = 10 - 3 = 7$ *jaar*

$\therefore 7 \times 12 = 84$ *betalings oor*

$$289\,985,16 = x \left[\frac{1 - \left(1 + \frac{14\%}{12}\right)^{-84}}{\frac{14\%}{12}} \right]$$

$x = 5434,33$ *maandeliks*

(b) $n = 3 \times 12 = 36$

$$289\,985,16 = P \left(1 + \frac{14\%}{12}\right)^{36} - 5\,434,33 \left[\frac{\left(1 + \frac{14\%}{12}\right)^{36} - 1}{\frac{14\%}{12}} \right]$$

$P = 350\,000$ *Oorspronklike lening*

VRAAG 8

(a) 26 ; p ; p^2 ; 34

$$p + 26; \quad p^2 - p; \quad 34 - p^2$$

$$p^2 - p - p - 26 = 34 - p^2 - p^2 + p$$

$$\therefore p^2 - 2p - 26 = -2p^2 + p + 34$$

$$\therefore 3p^2 - 3p - 60 = 0$$

$$\therefore p = 5 \quad \text{of} \quad p = -4$$

(b) $\sum_{k=1}^m \frac{1}{25} (5)^{k-1} < 6$

$$\frac{1}{25} + \frac{1}{5} + 1 + 5 + \dots$$

Meetkundig $r = 5$ en $a = \frac{1}{25}$

$$S_n = \frac{\frac{1}{25}(5^m - 1)}{5 - 1}$$

$$\frac{1}{100}(5^m - 1) < 6$$

$$(5^m - 1) < 600$$

$$5^m < 601$$

$$m < \log_5 601$$

$$m < 3,97$$

$$\therefore m = 3$$

VRAAG 9

(a) $x > 2$

(b) *vervang* (2;1)

$$1 = \sqrt{a(2)} - 1$$

$$2 = \sqrt{a(2)}$$

$$4 = 2a$$

$$\therefore a = 2$$

$$c = 3$$

$$g(x) = b \cdot 2^x + 3 \text{ *vervang nou* (2;1)}$$

$$1 = b \cdot 2^2 + 3$$

$$b = -\frac{1}{2}$$

(c) $(x) = -\frac{1}{2} \cdot 2^x + 3$

$$g^{-1}(x): \text{ruil } x \text{ en } y \text{ om}$$

$$x = -\frac{1}{2} \cdot 2^y + 3$$

$$x - 3 = -\frac{1}{2} \cdot 2^y$$

$$6 - 2x = 2^y$$

$$\therefore y = \log_2(6 - 2x)$$

VRAAG 10

- (a) Hellings gelyk by dieselfde
- x

$$\frac{dy}{dx} = 4x - 3$$

$$\frac{dy}{dx} = -2x + 3$$

$$4x - 3 = -2x + 3$$

$$6x = 6$$

$$x = 1$$

- (b)
- $f'(1) = 0$
- ,
- $f(1) = 5$

$$3ax^2 + 2bx + 9 = 0 \quad , \quad a(1)^3 + b(1)^2 + 9(1) + 1 = 5$$

$$3a(1)^2 + 2b(1) + 9 = 0 \quad , \quad a + b = -5$$

$$3a + 2b = -9,$$

$$\text{---} a = -b - 5$$

$$3(-b - 5) + 2b = -9$$

$$-3b - 15 + 2b = -9$$

$$-b = 6$$

$$b = -6 \text{ ---} > \quad , \quad a = -(-6) - 5$$

$$a = 1$$

$$\therefore y = x^3 - 6x^2 + 9x + 1$$

Aangesien $a > 0$, lokale minimum by $x = 3$

$$\therefore y = (3)^3 - 6(3)^2 + 9(3) + 1$$

$$\therefore y = 1$$

$\therefore$ Lokale minimum $(3;1)$

(c) (1) $A(x) = A_{\text{reghoek}} - A_{\text{groot sirkel}} - A_{\text{klein sirkel}}$

$$A(x) = 2.4x - (2x)^2 \pi - \left(\frac{4}{3}x\right)^2 \pi$$

$$A(x) = 8x - 4x^2 \pi - \frac{16}{9}x^2 \pi$$

$$A(x) = 8x - \frac{52}{9} \pi x^2$$

(2) By maks $A'(x) = 0$

$$0 = 8 - \frac{104}{9} \pi x$$

$$\frac{104}{9} \pi x = 8$$

$$\therefore x = \frac{8.9}{104\pi}$$

$$\therefore x = 0,22m$$

VRAAG 11

$$x^2 + 2x = k(x^2 - 2x + 2) + 2$$

$$x^2 + 2x - k(x^2 - 2x + 2) - 2 = 0$$

$$(1 - k)x^2 + (2 + 2k)x + (-2k - 2) = 0$$

vir afsonderlike reële wortels $\Delta > 0$, $b^2 - 4ac > 0$

$$(2 + 2k)^2 - 4(1 - k)(-2k - 2) > 0$$

$$4 + 8k + 4k^2 - [(4 - 4k)(-2k - 2)] > 0$$

$$4 + 8k + 4k^2 - [-8k - 8 + 8k^2 + 8k] > 0$$

$$4 + 8k + 4k^2 - 8k^2 + 8 > 0$$

$$-4k^2 + 8k + 12 > 0$$

$$k^2 - 2k - 3 < 0$$

$$\therefore -1 < k < 3, m \neq 1 \text{ of } \{-1 < k < 1\} \cup \{-1 < k < 3\}$$

VRAAG 12

$$(a) \quad \prod_{k=2}^{100} \left(1 - \frac{1}{k}\right) = \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{6}\right) \dots \left(1 - \frac{1}{100}\right)$$

$$\prod_{k=2}^{100} \left(1 - \frac{1}{k}\right) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \dots \frac{1}{89} \cdot \frac{89}{99} \cdot \frac{99}{100}$$

Let op hoe opeenvolgende noemer en teller mekaar kanselleer.

$$\prod_{k=2}^{100} \left(1 - \frac{1}{k}\right) = \frac{1}{100}$$

(b) Laat a , b , c en d die vier getalle voorstel.

$$\text{gemiddelde} = \frac{a + b + c + d}{4}$$

Wanneer die eerste getal by die gemiddelde van die ander drie getalle bygetel word, is die resultaat 25.

$$\text{Dus: } a + \frac{b + c + d}{3} = 25$$

$$\text{Kan geskryf word as } 3a + b + c + d = 75$$

Wanneer die tweede getal by die gemiddelde van die ander drie getalle bygetel word, is die resultaat 37.

$$\text{Dus: } b + \frac{a + c + d}{3} = 37$$

$$\text{Kan geskryf word as } a + 3b + c + d = 111$$

Wanneer die derde getal by die gemiddelde van die ander drie getalle bygetel word, is die resultaat 43.

$$\text{Dus: } c + \frac{a + b + d}{3} = 43$$

$$\text{Kan geskryf word as } a + b + 3c + d = 129$$

Wanneer die vierde getal by die gemiddelde van die ander drie getalle bygetel word, is die resultaat 51.

$$\text{Dus: } d + \frac{a + b + c}{3} = 51$$

$$\text{Kan geskryf word as } a + b + c + 3d = 153$$

Wanneer (1), (2), (3) en (4) bymekaargetel word, verkry ons:

$$6a + 6b + 6c + 6d = 468$$

$$\text{Deling deur 6 gee } a + b + c + d = 78$$

$$\therefore \text{gemiddelde} = \frac{a + b + c + d}{4} = \frac{78}{4} = 19,5$$

Totaal: 150 punte