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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2024

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

(a) (1) $x^2 - 7x + 12 = 0$
 $x = 3$ or $x = 4$

(2) $(5 - x) = 27^{\frac{2}{3}}$
 $5 - x = 9$
 $-x = 4$
 $x = -4$

(3) $3x^2 - 11x + 6 \geq 0$
 Factorise
 Method
 $x \leq \frac{2}{3}$ or $x \geq 3$

(4) $\log_b \left(\frac{2x+7}{x} \right) = \log_b 3^2$
 $\frac{2x+7}{x} = 9$
 $2x+7 = 9x$
 $-7x = -7$
 $x = 1$

(5) $2^{x^2-6x} = 2^7$
 $x^2 - 6x - 7 = 0$
 $x = -1$ or $x = 7$

(b) $\Delta = 0$
 $(k+2)^2 - 4(1)(4k-7) = 0$
 $k^2 + 4k + 4 - 16k + 28 = 0$
 $k^2 - 12k + 32 = 0$
 $k = 4$ or $k = 8$

QUESTION 2

(a) (1) $T_7 = 28$ $T_{12} = 73$

$$28 = a + 6d$$

$$73 = a + 11d$$

$$a = 28 - 6d \rightarrow 73 = 28 - 6d + 11d$$

$$45 = 5d$$

$$\rightarrow d = 9$$

$$a = 28 - 6d$$

$$a = -26$$

(2) $S_{50} = \frac{50}{2} [2(-26) + 49(9)]$

$$S_{50} = 9725$$

(b) $T_8 = S_8 - S_7$

$$T_8 = \frac{8}{2} [8(8)^2 + 7(8)] - \frac{7}{2} [8(7)^2 + 7(7)]$$

$$T_8 = 728,5$$

(c) (1) $20 - a - a + 4 = 17 - 20 + a$

$$24 - 2a = -3 + a$$

$$-3a = -27$$

$$a = 9$$

(2) $4 ; 9 ; 20 ; 37$

$$5 ; 11 ; 17$$

$$6 ; 6$$

$$6 = 2a \quad 3a + b = 5 \quad a + b + c = 4$$

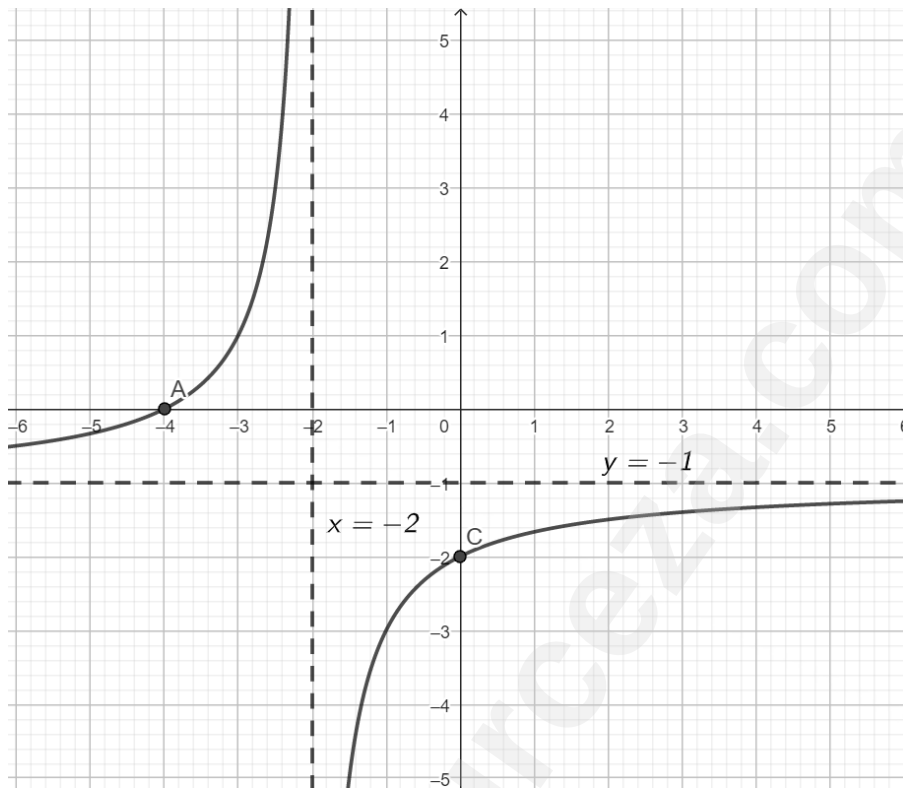
$$a = 3 \quad 3(3) + b = 5 \quad 3 - 4 + c = 4$$

$$b = 5 \quad c = 5$$

$$T_n = 3n^2 - 4n + 5$$

QUESTION 3

(a)



(b) (1) $x = \frac{-2}{y+1} - 2$

$$x + 2 = \frac{-2}{y+1}$$

$$y + 1 = \frac{-2}{x+2}$$

$$y = \frac{-2}{x+2} - 1$$

(2) Domain: $x \in \mathbb{R}, x \neq -2$

QUESTION 4

(a) *Domain* : $x > -1$

Range : $y \geq -3$

(b)
$$\frac{\text{sum of } x\text{-int}}{2} = 1$$

$$\frac{x_B + 4}{2} = 1$$

$$x_B = -2 \quad B(-2;0)$$

(c) *Use* (4;0) *or* (-2;0)

$$0 = p(4-1)^2 - 3$$

$$3 = 9p$$

$$p = \frac{1}{3}$$

(d)
$$h(x) = -\left[\frac{1}{3}(x-1-2)^2 - 3 - 3\right]$$

$$h(x) = -\frac{1}{3}(x-3)^2 + 6$$

(e) *Vertical asymptote* : $x = -1 \quad \therefore b = 1$

y – intercept : $y = 1$

$$f(x) = \log_a(x+1) - 2 \quad \text{sub (1;-3)}$$

$$-3 = \log_a(1+1) - 2$$

$$-1 = \log_a(2)$$

$$a^{-1} = 2$$

$$a = \frac{1}{2}$$

(f) at $f(x) \leq 0$:

$$0 = \log_{\frac{1}{2}}(x+1) - 2$$

$$2 = \log_{\frac{1}{2}}(x+1)$$

$$\left(\frac{1}{2}\right)^2 = x+1$$

$$x = -\frac{3}{4}$$

$$\therefore x \geq -\frac{3}{4}$$

QUESTION 5

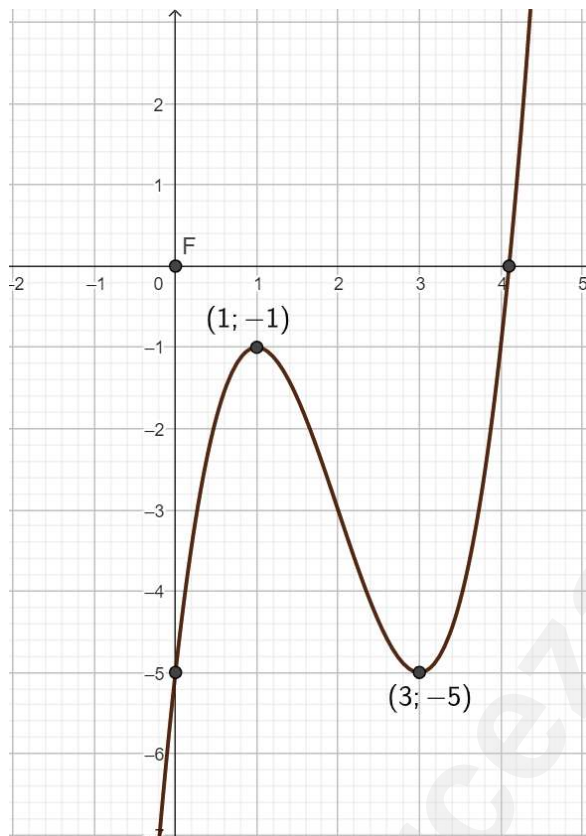
$$\begin{aligned}
 \text{(a)} \quad &= \lim_{h \rightarrow 0} \left(\frac{1}{x^2 + 2xh + h^2} - \frac{1}{x^2} \right) \div h \\
 &= \lim_{h \rightarrow 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2(x^2 + 2xh + h^2)} \times \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(-2x - h)}{x^4 + 2x^3h + x^2h^2} \times \frac{1}{h} \\
 &= \frac{-2x - (0)}{x^4 + 2x^3(0) + x^2(0)^2} \\
 &= \frac{-2x}{x^4} \\
 &= \frac{-2}{x^3}
 \end{aligned}$$

$$\text{(b)} \quad y = \left(2x - 3x^{\frac{1}{2}} \right)^2$$

$$y = 4x^2 - 12x^{\frac{3}{2}} + 9x$$

$$\frac{dy}{dx} = 8x - 18x^{\frac{1}{2}} + 9 \quad \text{or} \quad \frac{dy}{dx} = 8x - 18\sqrt{x} + 9$$

(c) (1)



(2) Point of inflection is halfway between turning points:

$$\frac{1+3}{2} = 2$$

Concave up at $x > 2$.

QUESTION 6

$$(a) \quad F = 1600 \left[\frac{\left(1 + \frac{9\%}{12}\right)^{48} - 1}{\frac{9\%}{12}} \right]$$

$$F = 92033,14$$

$$(b) \quad 131\,616,68 = 1\,600 \left[\frac{\left(1 + \frac{9\%}{12}\right)^n - 1}{\frac{9\%}{12}} \right] + 10\,000 \left(1 + \frac{9\%}{12}\right)^{12}$$

$$n = 60, \dots$$

$\therefore 61 \text{ months}$

QUESTION 7

- (a) 95% of R716 900
= R681 055

(b)
$$681\ 055 = x \left[\frac{1 - \left(1 + \frac{14,5\%}{12}\right)^{-72}}{\frac{14,5\%}{12}} \right]$$

$$x = 14\ 216,63$$

- (c) OB immediately after the 54th payment:

$$OB = 681\ 055 \left(1 + \frac{14,5\%}{12}\right)^{54} - 14\ 216,63 \left[\frac{\left(1 + \frac{14,5\%}{12}\right)^{54} - 1}{\frac{14,5\%}{12}} \right]$$

$$OB = 228\ 748,11$$

- (d) $228\ 748,11 \times \frac{0,145}{12}$
= R2 764,04

QUESTION 8

(a) $y \leq 7$

$$y \geq -\frac{1}{3}x + 4 \quad \text{or} \quad 3y + x \geq 12$$

$$y \leq -\frac{5}{6}x + 10 \quad \text{or} \quad 6y + 5x \leq 60$$

(b) $4 \leq a \leq 7 \quad \text{OR} \quad a \in [4;7]$

$$0 \leq b < 3 \text{ or } b > 6 \quad \text{OR} \quad b \in [0;3) \cup (6;\infty)$$

(c) (1) $S = 8x + 4y$

$$y = -2x + \frac{S}{4}$$

$$S_{\min \text{ at } L} = 8(1) + 4(5)$$

$$S_{\min \text{ at } L} = 28$$

(2) $T = 3y + 4x$

$$y = -\frac{4}{3}x + \frac{T}{3}$$

$$T_{\max \text{ at } O} = 3(5) + 4(6)$$

$$T_{\max \text{ at } O} = 39$$

(d) $S = cx + dy \text{ at } M(3;3) \quad \text{Maximises at gradients}$

$$y = -\frac{c}{d}x + \frac{s}{d}$$

$$-1 < -\frac{c}{d} < -\frac{1}{3}$$

$$\frac{1}{3} < \frac{c}{d} < 1$$

QUESTION 9

$$\frac{4^q}{2^p} = \frac{1}{2}$$

$$\log_5(p^2 + q^2 - 4) = 2$$

$$2^{2q} = \frac{2^p}{2}$$

$$2^{2q+1} = 2^p$$

$$p = 2q + 1 \text{ ----- } > \quad \log_5((2q+1)^2 + q^2 - 4) = 2$$

$$\log_5(4q^2 + 4q + 1 + q^2 - 4) = 2\log_5 5$$

$$\log_5(5q^2 + 4q - 3) = \log_5 5^2$$

$$5q^2 + 4q - 3 = 25$$

$$5q^2 + 4q - 28 = 0$$

$$< \text{-----} q = 2 \text{ or } q = -\frac{14}{5}$$

$$\text{for } q = 2 \quad ; \quad \text{for } q = -\frac{14}{5}$$

$$p = 2(2) + 1 \quad ; \quad p = 2\left(-\frac{14}{5}\right) + 1$$

$$p = 5 \quad ; \quad p = -\frac{23}{5}$$

QUESTION 10

- (a) Using $T_n = ar^{n-1}$; $a=5$, for $n=5$, 5th circle

$$195,3125 = 5r^4$$

$$r^4 = \frac{625}{16}$$

$$r = \frac{5}{2}$$

- (b) Using the sum formula for $n = 10$ and sum, noting that it needs to be doubled (diameter sum is required)

$$K_{new} = \frac{a(r^n - 1)}{r - 1}$$

$$K_{new} = \frac{10 \left(\left(\frac{5}{2} \right)^{10} - 1 \right)}{\frac{5}{2} - 1}$$

$$K_{new} =$$

- (c) Pattern expression for area.

$$Area = \pi \cdot 5^2 + \pi \cdot \left[\left(\frac{5}{2} \right) \cdot 5 \right]^2 + \pi \cdot \left[\left(\frac{5}{2} \right)^2 \cdot 5 \right]^2 + \dots + \pi \cdot \left[\left(\frac{5}{2} \right)^9 \cdot 5 \right]^2$$

$$Area = \pi \cdot 5^2 \left[1 + \left(\frac{5}{2} \right)^2 + \left(\frac{5}{2} \right)^4 + \left(\frac{5}{2} \right)^6 + \dots + \left(\frac{5}{2} \right)^{18} \right]$$

$$\Rightarrow [] \text{ 10 terms making geometric series with } r = \left(\frac{5}{2} \right)^2 = \frac{25}{4}$$

$$Area = 25\pi \times \frac{1 \left[\left(\frac{25}{4} \right)^{10} - 1 \right]}{\frac{25}{4} - 1}$$

$$Area = 433\,092\,710\pi \text{ units}^2$$

QUESTION 11

$$(a) \quad (1) \quad f(x) = \frac{bx^{-1}}{3} + \frac{2x^{\frac{1}{2}}}{3}$$

$$f'(x) = -\frac{bx^{-2}}{3} + \frac{x^{-\frac{1}{2}}}{3}$$

$$f'\left(\frac{1}{4}\right) = -\frac{b\left(\frac{1}{4}\right)^{-2}}{3} + \frac{\left(\frac{1}{4}\right)^{-\frac{1}{2}}}{3}$$

$$f'\left(\frac{1}{4}\right) = \frac{-16b+2}{3}$$

$$(2) \quad y = -10x + \frac{4}{3}$$

$$\text{gradient} = -10$$

$$\frac{-16b+2}{3} = -10$$

$$b = 2$$

$$f(x) = \frac{2+2x\sqrt{x}}{3x}$$

$$f\left(\frac{1}{4}\right) = \frac{2+2\left(\frac{1}{4}\right)\sqrt{\left(\frac{1}{4}\right)}}{3\left(\frac{1}{4}\right)} = 3$$

$$y = -10x + c \quad \text{sub}\left(\frac{1}{4}; 3\right)$$

$$3 = -10\left(\frac{1}{4}\right) + c$$

$$c = \frac{11}{2}$$

$$\therefore y = -10x + \frac{11}{2}$$

(b) $y^2 + y^2 = 4x^2$ *pythag*

$$2y^2 = 4x^2$$

$$y = \sqrt{2}x$$

$$\text{Area of } \Delta = \frac{1}{2}y^2 = \frac{1}{2}(\sqrt{2}x)^2 = \frac{1}{2} \cdot 2x^2 = x^2$$

for Volume

$$x^2h = 64$$

$$h = \frac{64}{x^2}$$

for surface area of prism

$$A = 2x^2 + 2yh$$

$$A(x) = 2x^2 + 2 \cdot \sqrt{2}x \cdot \frac{64}{x^2}$$

$$A(x) = 2x^2 + \frac{128\sqrt{2}}{x}$$

$$\text{at Min } A'(x) = 0$$

$$0 = x - 128\sqrt{2}x^{-2}$$

$$x^3 = 128\sqrt{2}$$

$$x = 5,7 \text{ units}$$

Minimum area

$$A(5,7) = 2(5,7)^2 + \frac{128\sqrt{2}}{5,7} = 96,7 \text{ units}^2$$

QUESTION 12

In 1 hour: X and Y fill $\frac{1}{12}$ of the tank

X and Z fill $\frac{1}{15}$ of the tank

Y and Z fill $\frac{1}{20}$ of the tank

Add equations: $X + Y + X + Z + Y + Z = \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$

$$2(X + Y + Z) = \frac{1}{5}$$

X, Y and Z together fill $\frac{1}{10}$ of the tank.

Therefore, Y alone fills $\frac{1}{10} - \frac{1}{15} = \frac{1}{30}$ of the tank in 1 hour

Therefore, 30 hours to fill the tank.

Total: 150 marks