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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2023

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A**QUESTION 1**

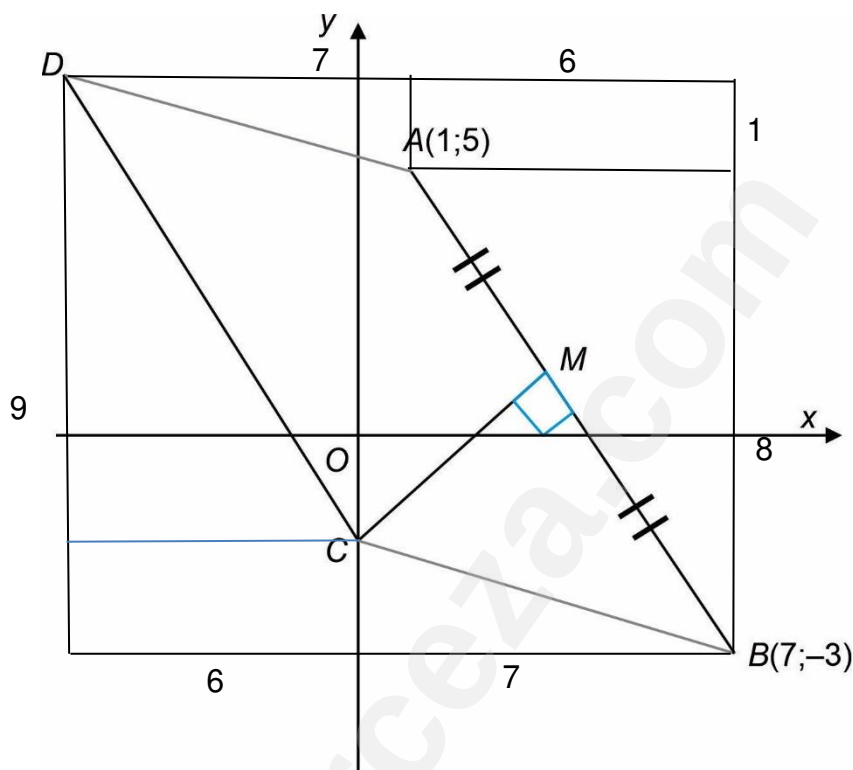
$$\begin{aligned}
 \text{(a)} \quad m_{AB} &= \frac{5+3}{1-7} & m_{CM} &= \frac{3}{4} \\
 &= \frac{8}{-6} & CM &\perp AB \\
 &= \frac{-4}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad M(4; 1) \quad & y - 1 = \frac{3}{4}(x - 4) \\
 & y - 1 = \frac{3}{4}x - 3 \\
 & y = \frac{3}{4}x - 2 \\
 \therefore C(0, -2) \quad (4)
 \end{aligned}$$

$$\text{(c)} \quad D(-6; 6)$$

$$\begin{aligned}
 \text{(d)} \quad CM^2 &= (0 - 4)^2 + (-2 - 1)^2 & AB^2 &= (7 - 1)^2 + (-3 - 5)^2 \\
 &= 16 + 9 & &= 36 + 64 \\
 &= 25 & &= 100 \\
 CM &= 5 & AB &= 10 \\
 \text{area of Parm} &= 10 \times 5 \\
 &= 50 \text{ units}^2
 \end{aligned}$$

OR ALTERNATE SOLUTION



$$\begin{aligned}\text{area of Parm} &= 9 \times 13 - 2(0,5 \times 7 \times 1) - 2(6 \times 1) - 2(0,5 \times 6 \times 8) \\ &= 117 - 7 - 12 - 48 \\ &= 50 \text{ units}^2\end{aligned}$$

(e) $m_{BC} = \frac{-2+3}{0-7}$

$$= \frac{1}{-7}$$

$$\tan \theta_2 = \frac{-1}{7}$$

$$\theta_2 = 171,9^\circ$$

$$\therefore \hat{A\hat{B}C} = 171,9^\circ - 126,9^\circ = 45^\circ$$

$$\tan \theta_1 = \frac{-4}{3}$$

$$\theta_1 = 126,9^\circ$$

OR ALTERNATE SOLUTION

$$CM = 5$$

$$AB = 10$$

$$BM = 5 \quad M \text{ is midpoint}$$

$$\text{Thus } BM = CM$$

Triangle MCB isosceles

$$\therefore \hat{ABC} = 45^\circ \quad (\angle \text{ opposite equal sides})$$

QUESTION 2

$$(a) \quad 2x + x + 60^\circ = 180^\circ \quad \text{opp } \angle \text{ of cyclic quad}$$

$$3x = 120^\circ$$

$$x = 40^\circ$$

$$4(40^\circ) - 87^\circ + y = 180^\circ \quad \text{opp } \angle \text{ of cyclic quad}$$

$$y = 107^\circ$$

$$(b) \quad (1) \quad \hat{C} = 56^\circ \quad \angle\text{'s in same segm}$$

$$(2) \quad \hat{O} = 112^\circ \quad \angle \text{ at centre} = 2 \angle \text{ at circumference}$$

$$(3) \quad \hat{D}_1 + 56^\circ = 90^\circ \quad \angle \text{ in a semi-circle}$$

$$\hat{D}_1 = 34^\circ$$

$$(4) \quad \hat{B}_1 = 34^\circ \quad \angle\text{'s in same segm}$$

$$\hat{B}_1 = \hat{B}_2 \quad \text{given}$$

$$\therefore \hat{AED} + 68^\circ = 180^\circ \quad \text{opp of cyclic quad}$$

$$\hat{AED} = 112^\circ$$

QUESTION 3

(a) (1) $\tan(180^\circ + \theta) \cdot \sin(90^\circ - \theta) \cdot \sin \theta + \cos^2(360^\circ - \theta)$

$$= (\tan \theta)(\cos \theta) \sin \theta + (\cos \theta)^2$$

$$= \frac{\sin \theta}{\cos \theta} \times \cos \theta \times \sin \theta + \cos^2 \theta$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1$$

(2) $= \sin(x + 64^\circ) \cdot \cos(x + 379^\circ - 360^\circ) + \sin(x + 19^\circ) \cos(180^\circ + (64^\circ + x))$

$$= \sin(x + 64^\circ) \cos(x + 19^\circ) + \sin(x + 19^\circ)(-\cos(64^\circ + x))$$

$$= \sin(x + 64^\circ) \cos(x + 19^\circ) - \sin(x + 19^\circ) \cos(x + 64^\circ)$$

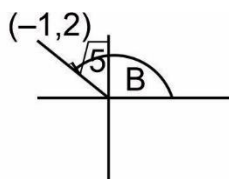
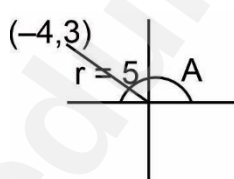
$$= \sin[x + 64^\circ - (x + 19^\circ)]$$

$$= \sin 45^\circ$$

$$= \frac{\sqrt{2}}{2}$$

(b) (1) II Quad

(2) $\frac{\cos A}{\sin B} = \frac{\frac{-4}{5}}{\frac{2}{\sqrt{5}}} = \frac{-4}{5} \times \frac{\sqrt{5}}{2}$



$$= \frac{-2\sqrt{5}}{5}$$

$$5 = 1 + y^2$$

$$4 = y^2$$

$$y = 2$$

$$\begin{aligned}
 \text{(c) (1) LHS} &= \frac{2(\cos^2 x - 1)}{\cos^2 x(1 - \cos^2 x)} \\
 &= \frac{-2}{\cos^2 x} \\
 &= \text{RHS}
 \end{aligned}$$

$$(2) \quad \frac{-2}{\cos^2 x} = \frac{-4 \sin x}{\cos x}$$

$$-2 \cos x = 4 \cos^2 x \sin x$$

$$0 = 4 \cos^2 x \sin x + 2 \cos x$$

$$0 = 2 \cos x (2 \cos x \sin x + 1)$$

$$2 \cos x = 0 \quad \text{OR} \quad \sin 2x + 1 = 0$$

$$\cos x = 0$$

$$x = 90^\circ + k180^\circ$$

$$(k \in \mathbb{Z})$$

$$\sin 2x = -1$$

$$2x = -90^\circ + k360^\circ$$

$$x = -45^\circ + k180^\circ$$

OR

$$2x = 270^\circ + k360^\circ$$

$$x = 135^\circ + k180^\circ$$

$$(k \in \mathbb{Z})$$

QUESTION 4

(a) $V = \pi r^2 h$

$$75,4 = \pi r^2 (6)$$

$$r^2 = 4,0$$

$$r = 2 \text{ m}$$

(b) $\triangle ABC$ is equilateral

$$\text{Area of } \triangle ABC = \frac{1}{2} (6)(6) \sin 60^\circ$$

$$= 15,5884 \text{ m}^2$$

$$\text{Area of } \odot = \pi (2)^2$$

$$= 12,5663 \text{ m}^2$$

$$\therefore \text{Face Area} = 15,5884 - 12,5663$$

$$= 3,022$$

$$= 3$$

(c) $\text{Area of } \triangle \text{ prism} = 3(6 \times 6) + 2(3)$

$$= 108 + 6$$

$$= 114 \text{ m}^2$$

$$\text{Area of inner cylinder} = 2 \pi r h$$

$$= 2 \pi (2)(6)$$

$$= 75,3982 \text{ m}^2$$

$$\text{Total Area} = 189,4 \text{ m}^2$$

(d) $\text{No of tins} = \frac{189,4}{9}$

$$= 21,044$$

$\therefore$ 21 tins of paint

QUESTION 5

(a) (1) $p = \underline{12}$

$$q = \underline{10}$$

(2) (i) $\bar{x} = 132,6$

(ii) $\sigma = 19,0$

(3) It will be steeper, data is closer together

(b) (1)
$$\begin{aligned}\text{Mean} &= \frac{p + 6 + 2p}{3} \\ &= \frac{3p + 6}{3} \\ &= p + 2\end{aligned}$$

(2) Variance would not change

SECTION B**QUESTION 6**

(a) 360°

(b) 1

(c) 4 solution

(d) $g(x) = \cos 2x$ $\cos^2 x - \sin^2 x = 0$
 $\cos 2x = 0$
 $\therefore g(x) = 0$ So the x-intercepts
 $\therefore x = 45^\circ$ or $x = 135^\circ$

(e) $g(x) \cdot f(x) \geq 0$
 $x \in [30^\circ, 45^\circ]$ OR $[135^\circ, 180^\circ]$

QUESTION 7

- (a) Constr diam SOA and join A to R

$$\hat{S}_1 + \hat{S}_2 = 90^\circ \quad \text{rad} \perp \text{tan}$$

$$\hat{R}_1 + \hat{R}_2 = 90^\circ \quad \text{in a semi-circle}$$

$$\text{But } \hat{S}_2 = \hat{R}_1 \quad \angle\text{'s in same segm}$$

$$\therefore \hat{S}_1 = \hat{R}_2$$

$$\text{OR } \hat{QST} = \hat{R}$$

- (b) (1)
- $\hat{X}_1 = \hat{Q}_1$
- tan chd Th

$$\hat{Q}_1 = \hat{Y}_1 + \hat{Y}_2 \quad \text{ext } \angle \text{ of cyclic quad}$$

$$\therefore \hat{X}_1 = \hat{Y}_1 + \hat{Y}_2 \quad \text{but these are alt } \angle\text{'s}$$

- (2)
- $\hat{Y}_2 = 30^\circ$
- (
- $\angle\text{'s in same segm}$
-): TXlly

$$\text{but } \hat{Y}_1 + \hat{Y}_2 = 60^\circ \quad (\text{alt } \angle \text{ on ll lines})$$

$$\therefore \hat{Y}_1 = 30^\circ \quad \therefore \text{QY bisects } \hat{X}\hat{Y}\hat{Z}$$

- (3) PQ || YZ (midpoint theorem)

$$\hat{Y}_1 + \hat{Y}_2 = 60^\circ \quad (\text{alt } \angle \text{ QP ||ZY})$$

$$\hat{Q}_3 = 180^\circ - (60^\circ + 30^\circ) \quad \text{sum of } \angle \text{ in } \Delta$$

$$\therefore \hat{Q}_3 = 90^\circ$$

$$\therefore \text{YZ is a diameter} \quad (\text{subt } 90^\circ)$$

QUESTION 8

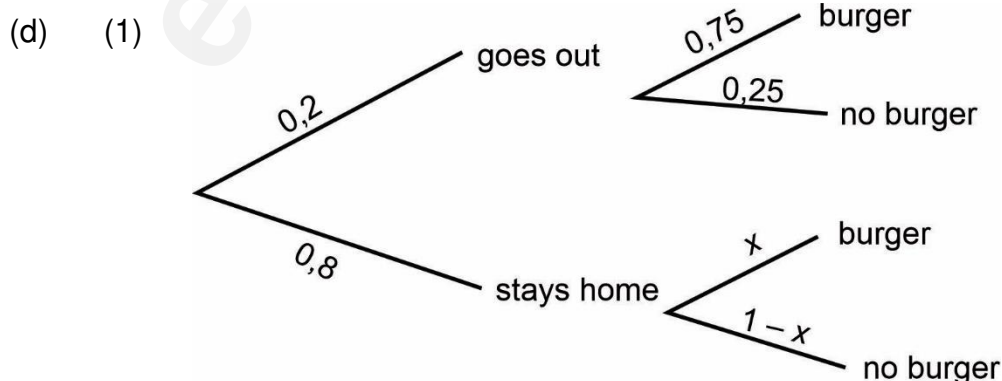
(a) (1) $a = -17,1$ $b = 0,86$
 $y = -17,1 + 0,9x$

(2) $r = 0,9908$
 $r = 1$
 near-perfect correlation (positive)

(b) (1) $5 \times 8 \times 7 \times 6 \times 5 = 8\,400$

(2) If first digit is odd
 $3 \times 7 \times 6 \times 5 \times 4 = 2\,520$
 Method if first digit is even
 $2 \times 7 \times 6 \times 5 \times 5 = 2\,100$
 $\therefore \text{total} = 4\,620$

(c) $p(A \cap B) = p(A) \times p(B)$
 $0,2 = x \times 0,3$
 $x = \frac{2}{3}$
 $p(A \text{ OR } B) = p(A) + p(B) - p(A \cap B)$
 $= 0,67 + 0,3 - 0,2$
 $= 0,77$
 $\therefore y = 1 - 0,77$
 $y = 0,23$



$$\begin{aligned}(2) \quad 0,2 \times 0,75 + 0,8x &= 0,5 \\ 0,15 + 0,8x &= 0,5 \\ 0,8x &= 0,35 \\ x &= \frac{3,5}{8} \\ &= 0,4375\end{aligned}$$

QUESTION 9(a) (1) In $\triangle ABC$ and $\triangle ADO$

$$\hat{B} = 90^\circ (\angle \text{ in a semi-circle}) \quad \hat{D}_1 = 90^\circ (\text{rad} \perp \text{tan})$$

$$(1) \quad \hat{B} = \hat{D}_1 \text{ proven}$$

$$(2) \quad \hat{A} = \hat{A}$$

$$(3) \quad \hat{C} = \hat{Q}$$

$$\therefore \triangle ABC \parallel \triangle DAO \quad (A,A,A)$$

$$(2) \quad \hat{A} = 30^\circ \quad (\angle \text{ at centre} = 2 \angle \text{ at circum})$$

$$\frac{DO}{AO} = \sin 30^\circ$$

$$\frac{5}{1} = AO$$

$$\therefore AO = 10$$

$(3) \quad \text{Area } \triangle ADO = \frac{1}{2} 5 \times a = \frac{5}{2} a$	$= \frac{10a}{15}$
$\text{Area } \triangle ABC = \frac{1}{2} (2a) 20 \sin 30^\circ$	$= 10 \times \frac{2}{15}$
$\text{Area Quad DOCB} = 10a - \frac{5}{2} a$	$= \frac{4}{3}$
$= \frac{15}{2} a$	

ALTERNATIVE

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \perp \text{ Trapezium}} = \frac{\frac{1}{2} \times 10 \times 20a}{\frac{1}{2} \times 10a(10+5)} = \frac{4}{3}$$

(b) (1) In $\triangle CDT$ and $\triangle BTA$

$$\frac{DT}{TA} = \frac{5}{16} = \frac{1}{2} \quad \frac{CD}{AB} = \frac{9}{18} = \frac{1}{2} \quad \frac{CT}{TB} = \frac{7}{14} = \frac{1}{2}$$

$\therefore \triangle CDT \parallel \triangle BTA$ (sides prop)

$$\therefore \hat{CDB} = \hat{CAB}$$

(2) Thus quad CBAD is cyclic

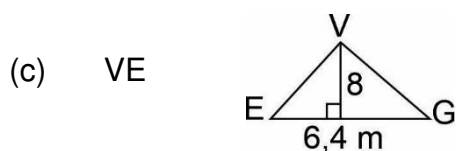
CB subt = $\angle$'s

$$\therefore \hat{BDA} = \hat{BCA} \quad (\angle\text{'s in same segm})$$

QUESTION 10

(a) $AC^2 = 10^2 + 8^2$ Pythagoras
 $= 100 + 64$
 $= 164$
 $\therefore AC = 12,8$

(b) $VM = 8$ $EM = 6,4$
 $\tan \hat{VEM} = \frac{8}{6,4}$
 $\hat{VEM} = 51,3^\circ$



$VE^2 = 8^2 + 6,4^2$ Pythagoras
 $VE^2 = 104,96$
 $VE = 10,2$



$EF^2 = VE^2 + FV^2 - 2(VE)(VF) \cos \hat{E} \hat{V} F$
 $100 = 2(10,2)^2 - 2(10,2)^2 \cos \hat{E} \hat{V} F$
 $208,08 \cos \hat{E} \hat{V} F = 108,08$
 $\cos \hat{E} \hat{V} F = 0,5194$
 $\hat{E} \hat{V} F = 58,7^\circ$

QUESTION 11

(a) $(-12, -15) \quad (24, 12)$

$$AC^2 = (24 + 12)^2 + (12 + 15)^2$$

$$= 36^2 + 27^2$$

$$= 2\,025$$

$$AC = 45$$

$$5 + \text{diam} + 10 = 45$$

$$\text{diam} = 30$$

$$r = 15 \quad (5)$$

(b) (1) $x^2 - kx + \left(\frac{-k}{2}\right)^2 = -c + \frac{k^2}{4} + \frac{(k+2)^2}{4}$

$$\left(x - \frac{k}{2}\right)^2 + \left(y + \frac{k+2}{2}\right)^2 = \frac{-4c + k^2 + k^2 + 4k + 4}{4}$$

$$\text{Centre} \left(\frac{k}{2}; \frac{-(k+2)}{2}\right)$$

$$\text{Sub in } 2x + 5y + 14 = 0$$

$$2\left(\frac{k}{2}\right) + 5\left(\frac{-k-2}{2}\right) + 14 = 0$$

$$2k + 5(-k-2) + 28 = 0$$

$$-3k = -18$$

$$k = 6$$

(2) $r^2 = \frac{2k^2 + 4k + 4 - 4c}{4}$

$$\therefore r = \sqrt{25 - c} \quad = \frac{72 + 24 + 4 - 4c}{4}$$

$$\text{So } 25 - c > 0 \quad = \frac{96 + 4 - 4c}{4}$$

$$25 > c \quad = 24 + 1 - c$$

$$= 25 - c$$

Total: 150 marks