



eduresourceza.com



INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
MEI 2023

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A**VRAAG 1**

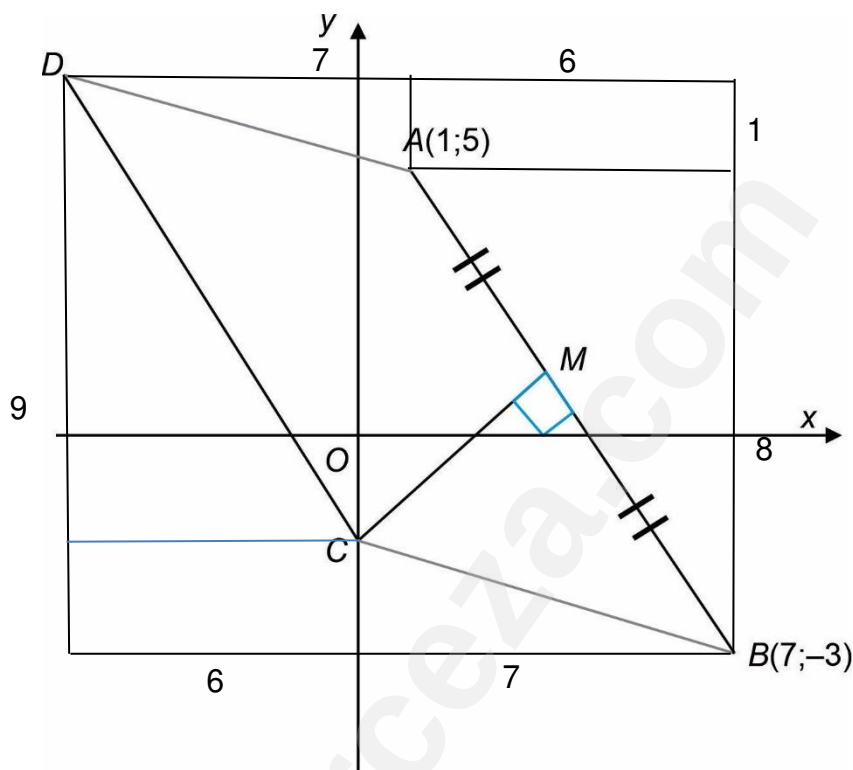
$$\begin{aligned}
 \text{(a)} \quad m_{AB} &= \frac{5+3}{1-7} & m_{CM} &= \frac{3}{4} \\
 &= \frac{8}{-6} & CM &\perp AB \\
 &= \frac{-4}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad M(4; 1) \quad & y - 1 = \frac{3}{4}(x - 4) \\
 & y - 1 = \frac{3}{4}x - 3 \\
 & y = \frac{3}{4}x - 2 \\
 \therefore C(0, -2)
 \end{aligned}$$

$$\text{(c)} \quad D(-6; 6)$$

$$\begin{aligned}
 \text{(d)} \quad CM^2 &= (0 - 4)^2 + (-2 - 1)^2 & AB^2 &= (7 - 1)^2 + (-3 - 5)^2 \\
 &= 16 + 9 & &= 36 + 64 \\
 &= 25 & &= 100 \\
 CM &= 5 & AB &= 10 \\
 \text{Oppervlakte van parm} &= 10 \times 5 \\
 &= 50 \text{ eenhede}^2
 \end{aligned}$$

OF ALTERNATIEWE OPLOSSING



$$\begin{aligned}
 \text{Oppervlakte van parm} &= 9 \times 13 - 2(0,5 \times 7 \times 1) - 2(6 \times 1) - 2(0,5 \times 6 \times 8) \\
 &= 117 - 7 - 12 - 48 \\
 &= 50 \text{ eenhede}^2
 \end{aligned}$$

$$(e) \quad m_{BC} = \frac{-2+3}{0-7}$$

$$= \frac{1}{-7}$$

$$\tan \theta_2 = \frac{-1}{7}$$

$$\theta_2 = 171,9^\circ$$

$$\begin{aligned}
 \therefore \hat{ABC} &= 171,9^\circ - 126,9^\circ \\
 &= 45^\circ
 \end{aligned}$$

$$\tan \theta_1 = \frac{-4}{3}$$

$$\theta_1 = 126,9^\circ$$

OF ALTERNATIEWE OPLOSSING

$$CM = 5$$

$$AB = 10$$

$$BM = 5 \quad M \text{ is middelpunt}$$

$$\text{Dus } BM = CM$$

Driehoek MCB gelykbenig

$$\therefore \hat{ABC} = 45^\circ \quad (\angle \text{ teenoor gelyke sye})$$

VRAAG 2

$$(a) \quad 2x + x + 60^\circ = 180^\circ \quad \text{teenoorstaande } \angle \text{ e van koordevierhoek}$$

$$3x = 120^\circ$$

$$x = 40^\circ$$

$$4(40^\circ) - 87^\circ + y = 180^\circ \quad \text{teenoorstaande } \angle \text{ e van koordevierhoek}$$

$$y = 107^\circ$$

$$(b) \quad (1) \quad \hat{C} = 56^\circ \quad \angle \text{ e in dieselfde segment}$$

$$(2) \quad \hat{O} = 112^\circ \quad \angle \text{ by middelpunt} = 2 \text{ maal } \angle \text{ by omtrek}$$

$$(3) \quad \hat{D}_1 + 56^\circ = 90^\circ \quad \angle \text{ in halfsirkel}$$

$$\hat{D}_1 = 34^\circ$$

$$(4) \quad \hat{B}_1 = 34^\circ \quad \angle \text{ e in dieselfde segment}$$

$$\hat{B}_1 = \hat{B}_2 \quad \text{gegee}$$

$$\therefore \hat{AED} + 68^\circ = 180^\circ \quad \text{teenoorstaande } \angle \text{ e van koordevierhoek}$$

$$\hat{AED} = 112^\circ$$

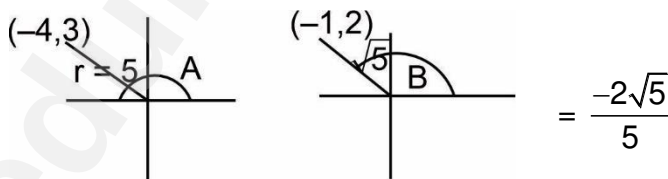
VRAAG 3

$$\begin{aligned}
 (a) \quad (1) \quad & \tan(180^\circ + \theta) \cdot \sin(90^\circ - \theta) \cdot \sin \theta + \cos^2(360^\circ - \theta) \\
 &= (\tan \theta)(\cos \theta) \sin \theta + (+\cos \theta)^2 \\
 &= \frac{\sin \theta}{\cos \theta} \times \cos \theta \times \sin \theta + \cos^2 \theta \\
 &= \sin^2 \theta + \cos^2 \theta \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad &= \sin(x + 64^\circ) \cdot \cos(x + 379^\circ - 360^\circ) + \sin(x + 19^\circ) \cos(180^\circ + (64^\circ + x)) \\
 &= \sin(x + 64^\circ) \cos(x + 19^\circ) + \sin(x + 19^\circ)(-\cos(64^\circ + x)) \\
 &= \sin(x + 64^\circ) \cos(x + 19^\circ) - \sin(x + 19^\circ) \cos(x + 64^\circ) \\
 &= \sin[x + 64^\circ - (x + 19^\circ)] \\
 &= \sin 45^\circ \\
 &= \frac{\sqrt{2}}{2}
 \end{aligned}$$

(b) (1) Kwadrant II

$$(2) \quad \frac{\cos A}{\sin B} = \frac{\frac{-4}{5}}{\frac{2}{\sqrt{5}}} = \frac{-4}{5} \times \frac{\sqrt{5}}{2}$$



$$5 = 1 + y^2$$

$$4 = y^2$$

$$y = 2$$

$$\begin{aligned}
 \text{(c)} \quad (1) \quad LK &= \frac{2(\cos^2 x - 1)}{\cos^2 x(1 - \cos^2 x)} \\
 &= \frac{-2}{\cos^2 x} \\
 &= RK
 \end{aligned}$$

$$(2) \quad \frac{-2}{\cos^2 x} = \frac{-4 \sin x}{\cos x}$$

$$-2 \cos x = 4 \cos^2 x \sin x$$

$$0 = 4 \cos^2 x \sin x + 2 \cos x$$

$$0 = 2 \cos x(2 \cos x \sin x + 1)$$

$$2 \cos x = 0 \quad \text{OF} \quad \sin 2x + 1 = 0$$

$$\cos x = 0 \quad \sin 2x = -1$$

$$x = 90^\circ + k180^\circ \quad 2x = -90^\circ + k360^\circ$$

$$(k \in \mathbb{Z}) \quad x = -45^\circ + k180^\circ$$

OF

$$2x = 270^\circ + k360^\circ$$

$$x = 135^\circ + k180^\circ$$

$$(k \in \mathbb{Z})$$

VRAAG 4

(a) $V = \pi r^2 h$

$$75,4 = \pi r^2 (6)$$

$$r^2 = 4,0$$

$$r = 2 \text{ m}$$

(b) $\triangle ABC$ is gelyksydig

$$\text{Oppervlakte van } \triangle ABC = \frac{1}{2} (6)(6) \sin 60^\circ$$

$$= 15,5884 \text{ m}^2$$

$$\text{Oppervlakte van } \odot = \pi (2)^2$$

$$= 12,5663 \text{ m}^2$$

$$\therefore \text{Vlakoppervlakte} = 15,5884 - 12,5663$$

$$= 3,022$$

$$= 3$$

(c) Oppervlakte $\triangle$ prisma $= 3(6 \times 6) + 2(3)$

$$= 108 + 6$$

$$= 114 \text{ m}^2$$

$$\text{Oppervlakte binnesilinder} = 2 \pi r h$$

$$= 2 \pi (2)(6)$$

$$= 75,3982 \text{ m}^2$$

$$\text{Totale oppervlakte} = 189,4 \text{ m}^2$$

(d) Getal blikke $= \frac{189,4}{9}$

$$= 21,044$$

$\therefore$ 21 blikke verf

VRAAG 5

(a) (1) $p = \underline{\quad 12 \quad}$

$$q = \underline{\quad 10 \quad}$$

(2) (i) $\bar{x} = 132,6$

(ii) $\sigma = 19,0$

(3) Dit sal steiler wees, data is nader aan mekaar.

(b) (1)
$$\begin{aligned}\text{Gemiddelde} &= \frac{p + 6 + 2p}{3} \\ &= \frac{3p + 6}{3} \\ &= p + 2\end{aligned}$$

(2) Variansie sal nie verander nie.

AFDELING B**VRAAG 6**

(a) 360°

(b) 1

(c) 4 oplossings

(d) $g(x) = \cos 2x$ $\cos^2 x - \sin^2 x = 0$
 $\cos 2x = 0$
 $\therefore g(x) = 0$ Dus die x-afsnitte
 $\therefore x = 45^\circ$ of $x = 135^\circ$

(e) $g(x) \cdot f(x) \geq 0$
 $x \in [30^\circ, 45^\circ]$ OF $[135^\circ, 180^\circ]$

VRAAG 7

- (a) Konstrueer middellyn SOA en verbind A met R

$$\hat{S}_1 + \hat{S}_2 = 90^\circ \quad \text{radius} \perp \text{raaklyn}$$

$$\hat{R}_1 + \hat{R}_2 = 90^\circ \quad \angle \text{ in halfsirkel}$$

$$\text{Maar } \hat{S}_2 = \hat{R}_1 \quad \angle \text{e in dieselfde segment}$$

$$\therefore \hat{S}_1 = \hat{R}_2$$

$$\text{OF } \hat{QST} = \hat{R}$$

- (b) (1)
- $\hat{X}_1 = \hat{Q}_1$
- raaklyn-koord-stelling

$$\hat{Q}_1 = \hat{Y}_1 + \hat{Y}_2 \quad \text{buite } \angle \text{ van koordevierhoek}$$

$$\therefore \hat{X}_1 = \hat{Y}_1 + \hat{Y}_2 \quad \text{maar hulle is verw. } \angle \text{e}$$

- (2)
- $\hat{Y}_2 = 30^\circ$
- (
- $\angle \text{e in dieselfde segment}$
-): TX II y

$$\text{Maar } \hat{Y}_1 + \hat{Y}_2 = 60^\circ \quad (\text{verw. } \angle \text{e op II lyne})$$

$$\therefore \hat{Y}_1 = 30^\circ \quad \therefore \text{QY halveer } \hat{X}\hat{Y}\hat{Z}$$

- (3) PQ II YZ (middelpuntstelling)

$$\hat{Y}_1 + \hat{Y}_2 = 60^\circ \quad (\text{verw. } \angle \text{e QP II ZY})$$

$$\hat{Q}_3 = 180^\circ - (60^\circ + 30^\circ) \quad \text{som van } \angle \text{e in } \Delta$$

$$\therefore \hat{Q}_3 = 90^\circ$$

$$\therefore \text{YZ is 'n middellyn} \quad (\text{onderspan } 90^\circ)$$

VRAAG 8

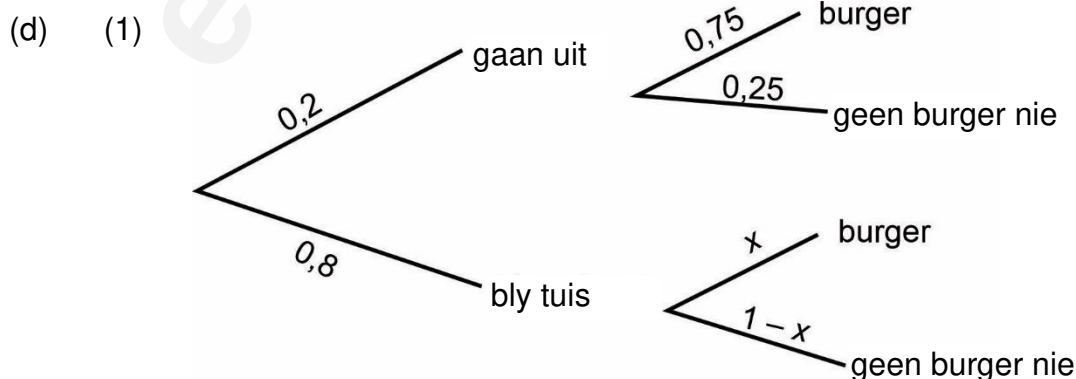
(a) (1) $a = -17,1$ $b = 0,86$
 $y = -17,1 + 0,9x$

(2) $r = 0,9908$
 $r = 1$
 byna perfekte korrelasie (positief)

(b) (1) $5 \times 8 \times 7 \times 6 \times 5 = 8\,400$

(2) Indien eerste syfer onewe is
 $3 \times 7 \times 6 \times 5 \times 4 = 2\,520$
 Metode indien eerste syfer ewe is
 $2 \times 7 \times 6 \times 5 \times 5 = 2\,100$
 $\therefore$ totaal = 4 620

(c) $p(A \cap B) = p(A) \times p(B)$
 $0,2 = x \times 0,3$
 $x = \frac{2}{3}$
 $p(A \text{ OF } B) = p(A) + p(B) - p(A \cap B)$
 $= 0,67 + 0,3 - 0,2$
 $= 0,77$
 $\therefore y = 1 - 0,77$
 $y = 0,23$



$$\begin{aligned}(2) \quad 0,2 \times 0,75 + 0,8x &= 0,5 \\ 0,15 + 0,8x &= 0,5 \\ 0,8x &= 0,35 \\ x &= \frac{3,5}{8} \\ &= 0,4375\end{aligned}$$

VRAAG 9(a) (1) In $\triangle ABC$ en $\triangle ADO$

$$\hat{B} = 90^\circ (\angle \text{ in halfsirkel}) \quad \hat{D}_1 = 90^\circ (\text{radius} \perp \text{raaklyn})$$

$$(1) \quad \hat{B} = \hat{D}_1 \text{ bewys}$$

$$(2) \quad \hat{A} = \hat{A}$$

$$(3) \quad \hat{C} = \hat{Q}$$

$$\therefore \triangle ABC \parallel \triangle DAO \quad (H,H,H)$$

$$(2) \quad \hat{A} = 30^\circ \quad (\angle \text{ by middelpunt} = 2 \text{ maal } \angle \text{ by omtrek})$$

$$\frac{DO}{AO} = \sin 30^\circ$$

$$\frac{5}{\frac{1}{2}} = AO$$

$$\therefore AO = 10$$

(3) Oppervlakte $\triangle ADO = \frac{1}{2} 5 \times a = \frac{5}{2} a$	$= \frac{10a}{15}$
Oppervlakte $\triangle ABC = \frac{1}{2} (2a) 20 \sin 30^\circ$	$= 10 \times \frac{2}{15}$
Oppervlakte vierhoek DOCB = $10a - \frac{5}{2} a$	$= \frac{4}{3}$
$= \frac{15}{2} a$	

ALTERNATIEF

$$\frac{\text{Oppervlakte van } \triangle ABC}{\text{Oppervlakte van } \perp \text{ Trapesium}} = \frac{\frac{1}{2} \times 10 \times 20a}{\frac{1}{2} \times 10a(10+5)} = \frac{4}{3}$$

(b) (1) In $\triangle CDT$ en $\triangle BTA$

$$\frac{DT}{TA} = \frac{5}{16} = \frac{1}{2} \quad \frac{CD}{AB} = \frac{9}{18} = \frac{1}{2} \quad \frac{CT}{TB} = \frac{7}{14} = \frac{1}{2}$$

$\therefore \triangle CDT \parallel \triangle BTA$ (sy eweredig)

$$\therefore \hat{CDB} = \hat{CAB}$$

(2) Dus is CBAD koordevierhoek

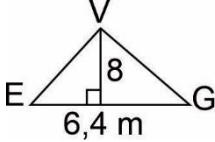
CB onderspan gelyke $\angle e$

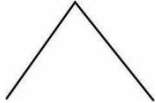
$$\therefore \hat{BDA} = \hat{BCA} \quad (\angle e \text{ in dieselfde segment})$$

VRAAG 10

(a) $AC^2 = 10^2 + 8^2$ Pythagoras
 $= 100 + 64$
 $= 164$
 $\therefore AC = 12,8$

(b) $VM = 8$ $EM = 6,4$
 $\tan \hat{VEM} = \frac{8}{6,4}$
 $\hat{VEM} = 51,3^\circ$

(c) VE  $VE^2 = 8^2 + 6,4^2$ Pythagoras
 $VE^2 = 104,96$
 $VE = 10,2$

(d) $\hat{E} \hat{V} F$  $EF^2 = VE^2 + FV^2 - 2(VE)(VF) \cos \hat{E} \hat{V} F$
 $100 = 2(10,2)^2 - 2(10,2)^2 \cos \hat{E} \hat{V} F$
 $208,08 \cos \hat{E} \hat{V} F = 108,08$
 $\cos \hat{E} \hat{V} F = 0,5194$
 $\hat{E} \hat{V} F = 58,7^\circ$

VRAAG 11

(a) $(-12, -15) \quad (24, 12)$

$$AC^2 = (24 + 12)^2 + (12 + 15)^2$$

$$= 36^2 + 27^2$$

$$= 2\,025$$

$$AC = 45$$

$$5 + \text{middellyn} + 10 = 45$$

$$\text{middellyn} = 30$$

$$r = 15$$

(b) (1) $x^2 - kx + \left(\frac{-k}{2}\right)^2 = -c + \frac{k^2}{4} + \frac{(k+2)^2}{4}$

$$\left(x - \frac{k}{2}\right)^2 + \left(y + \frac{k+2}{2}\right)^2 = \frac{-4c + k^2 + k^2 + 4k + 4}{4}$$

$$\text{Middelpunt} \left(\frac{k}{2}; \frac{-(k+2)}{2}\right)$$

$$\text{Vervang in } 2x + 5y + 14 = 0$$

$$2\left(\frac{k}{2}\right) + 5\left(\frac{-k-2}{2}\right) + 14 = 0$$

$$2k + 5(-k - 2) + 28 = 0$$

$$-3k = -18$$

$$k = 6$$

(2) $r^2 = \frac{2k^2 + 4k + 4 - 4c}{4}$

$$\therefore r = \sqrt{25 - c} \quad = \frac{72 + 24 + 4 - 4c}{4}$$

$$\text{Dus } 25 - c > 0 \quad = \frac{96 + 4 - 4c}{4}$$

$$25 > c \quad = 24 + 1 - c$$

$$= 25 - c$$

Totaal: 150 punte