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TOTAL
MARKS

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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER I

EXAMINATION NUMBER

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Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. FOUR blank pages (pages 21–24) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
11	14	11	11	7	15	9	12	5	15	15	9	16	/150

SECTION A**QUESTION 1**

(a) Simplify the expression: $2^x + 2^{x+1} + 2^{x+2}$ (2)

(b) Solve for all real value(s) of x , leaving in simplest surd form, if necessary:

$$(x+8)(x^2+8)(x^3+8)=0 \quad (3)$$

(c) By using the substitution $y = x(x+1)$, solve for x :

$$\sqrt{x^2 + x + 10} + 2 = x(x+1) \quad (6)$$

QUESTION 2

(a) It is given that $a = \log 2$ and $b = \log 3$. Find $\log 7,5$ in terms of a and b . (4)

(b) Solve for x : $\log_2(x+1) - \log_{\frac{1}{2}}(x-5) = \log_2 7$ (6)

(c) The following formula defines the relationship between two variables x and y :

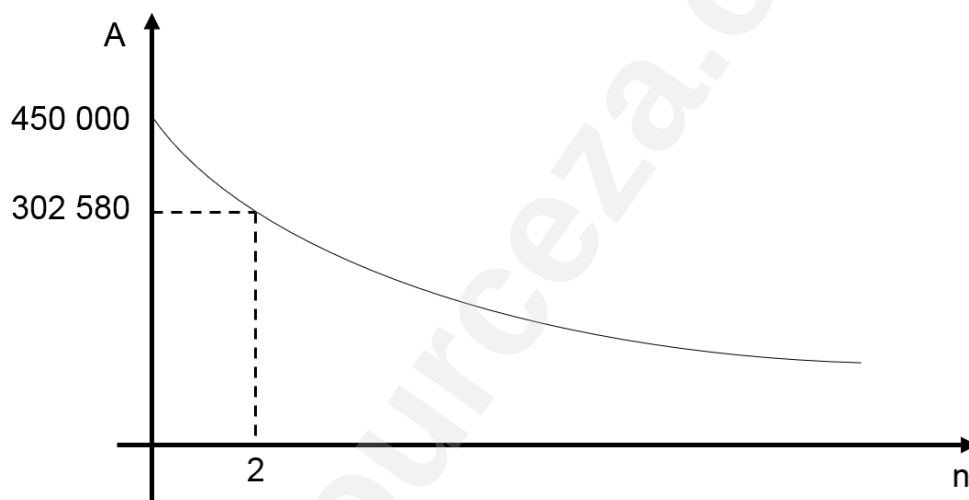
$$\log(y-1) = 2\log(x) + \log 4.$$

Express this in the form $y = \dots$ in fully simplified form. (4)

QUESTION 3

- (a) An investment grows with compound interest of 15% compounded monthly. After 7 years the amount is R41 167,14. Determine the initial investment. (3)

- (b) The graph below illustrates the trend of the depreciating value of a car, A (in rand) over a period of time, n (in years).



- (i) What is the cost of the car new? (1)
- (ii) What type of depreciation is illustrated? (1)
- (iii) Use the information on the graph to find the rate of depreciation. (4)

(iv) Calculate the value of the car after 7 years.

(2)

[11]

QUESTION 4

(a) Find $f'(x)$ from first principles if: $f(x) = -3x^2 + 4$. (5)

(b) (i) Find $f'(-1)$ if $f(x) = \frac{1}{2}x^2 - \frac{3}{x^3}$. (3)

(ii) Hence determine the equation of the tangent to f at the point where $x = -1$, leaving the answer in the form $ax + by = c$. (3)

[11]

QUESTION 5

(a) Write $f(x) = -x^2 + 4x + 10$ in the form $f(x) = a(x - p)^2 + q$. (3)

(b) If the domain of f is $x \geq k$, determine the minimum value of k such that f^{-1} exists. (1)

(c) Find an expression for $f^{-1}(x)$. (3)

QUESTION 6

- (a) The first three terms of an arithmetic sequence are:

$$2x - 4; \quad x - 3; \quad 8 - 2x$$

Determine the value of x and the sum of the first 15 terms. (7)

- (b) Calculate $\sum_{k=2}^{\infty} 8\left(\frac{1}{2}\right)^{k+2}$ (5)

- (c) For which values of x will the geometric sequence converge:

$$1 + (2 - 3x) + (2 - 3x)^2 + \dots \quad (3)$$

[15]

QUESTION 7

- (a) Sipiwe invests R2 500 each month, starting immediately, in an account yielding an effective interest rate of 6% per annum. Calculate the amount that Sipiwe will have saved after 10 years. (5)

- (b) On 1 January 2023, Mpho takes a loan of R3 000 000. He is offered an interest rate of 7,5% per annum compounded quarterly. He will make quarterly payments of R60 000 at the end of each quarter. Determine the date of the final payment. (4)

[9]**78 marks**

SECTION B**QUESTION 8**

The feasible region in a linear programming problem is described by the following inequalities:

$x \geq 0$

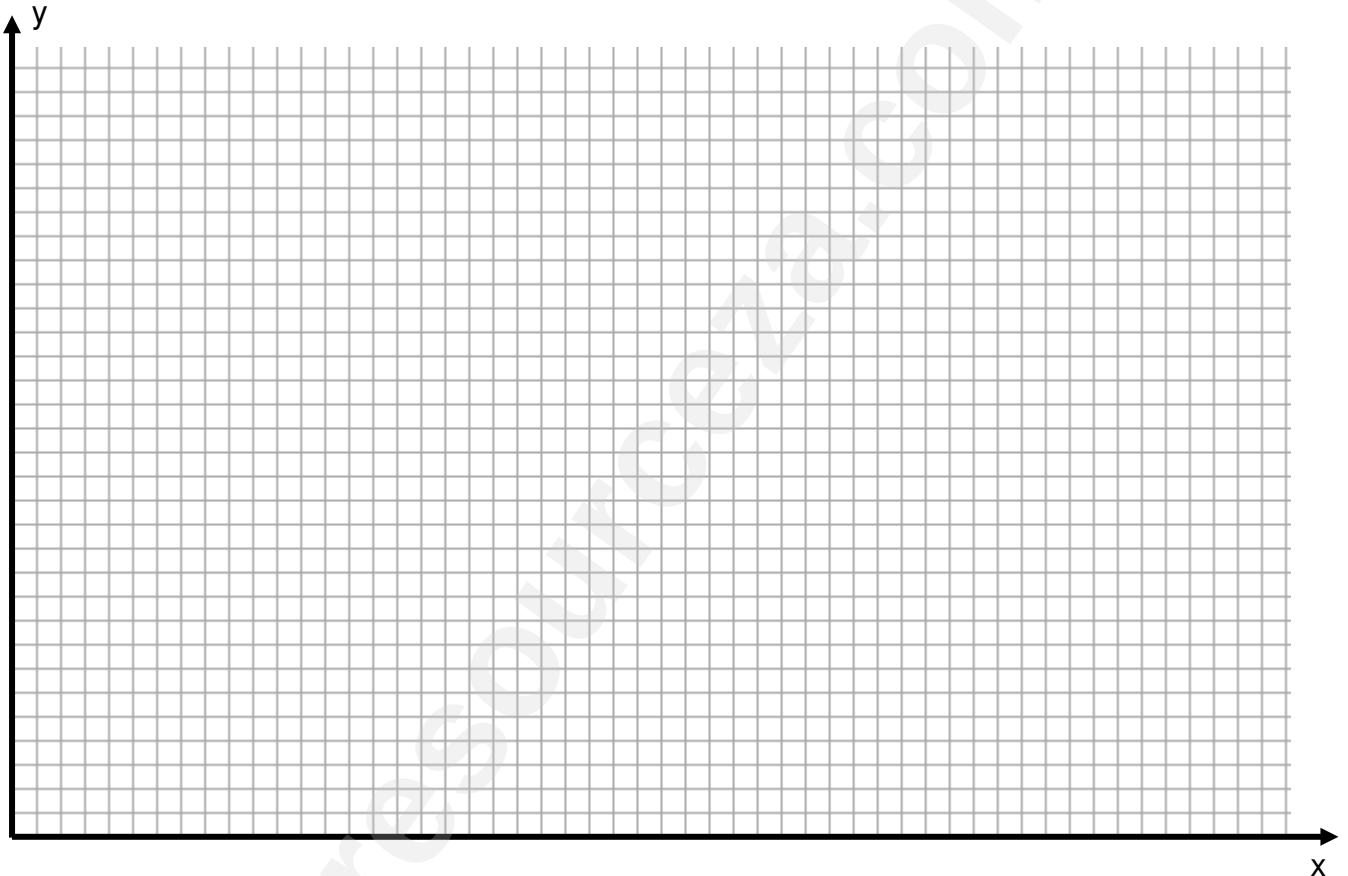
$y \geq 0$

$x + y \geq 2$

$4y + 3x \leq 24$

$1 < x \leq 4$

- (a) On the axes given below, accurately sketch and shade the feasible region. (8)



- (b) Using the graph, determine the maximum value of the objective function.

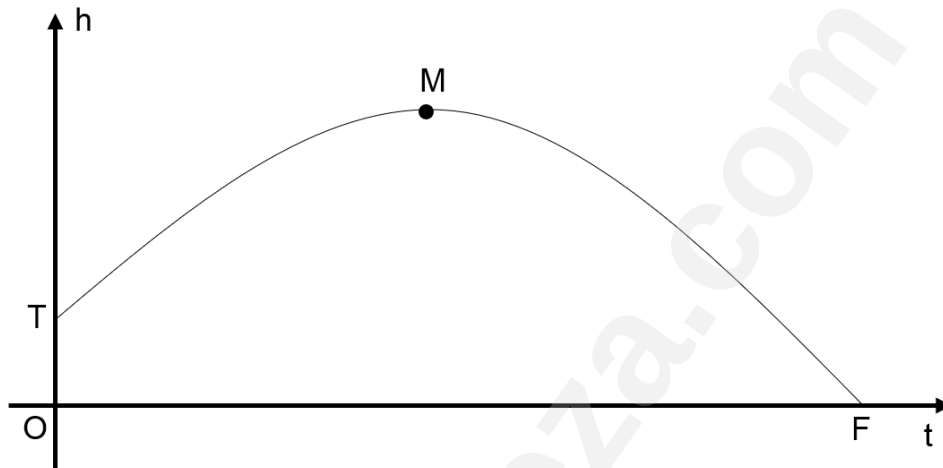
$$P = 3x - 2y \quad (4)$$

[12]

QUESTION 9

The diagram below models the height of a golf ball at time t seconds after it is struck. The point T represents the height at which the ball is struck, M is the maximum height and F is the point at which the ball hits the ground. The equation of the curve is given by:

$$h = -t^2 + 6t + 3$$



(a) From what height is the golf ball struck? (1)

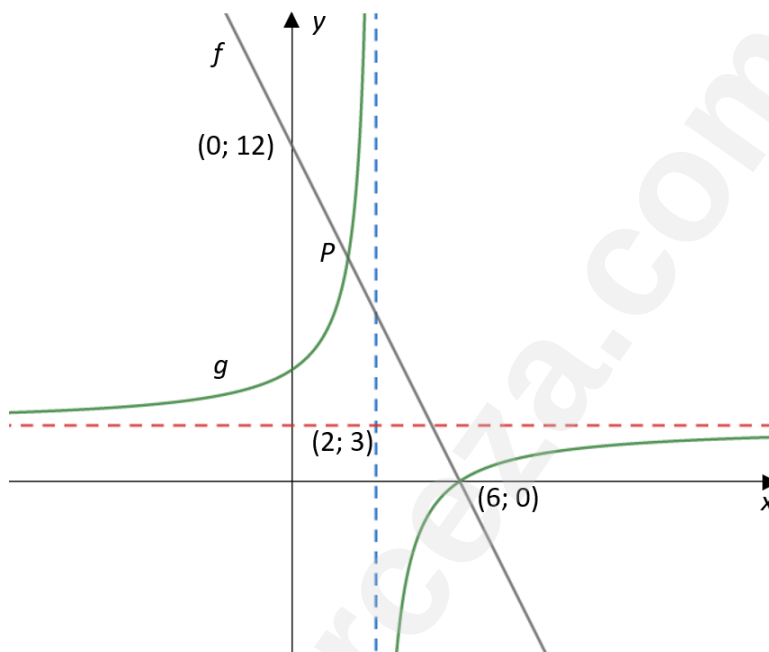
(b) What is the maximum height of the ball? (4)

[5]

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QUESTION 10

Two graphs of the form $f(x) = mx + c$ and $g(x) = \frac{a}{x-p} + q$ are shown below. The asymptotes intersect at $(2; 3)$. The graphs have a common x -intercept at $(6; 0)$ while the y -intercept of f is $(0; 12)$. P is a point of intersection of f and g .



- (a) Determine the parameters m , c , a , p and q .

(5)

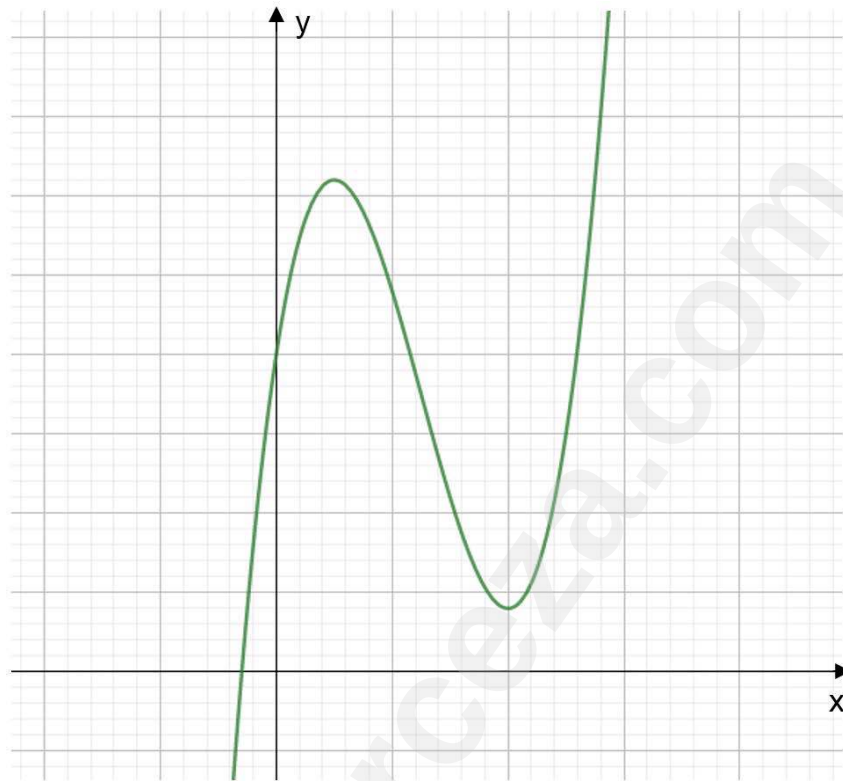
(b) Calculate the coordinates of P . (6)

(c) Use the graph to solve the inequality $f(x) \geq g(x)$. (2)

(d) For which values of k will the equation $f(x - k) = g(x - k)$ have two roots with the same sign? (2)

QUESTION 11

A graph of $y = 2x^3 + ax^2 + bx + 20$ is given, with a stationary point at $(1; 31)$.



- (a) Prove, without substituting the values given, that $a = -15$ and $b = 24$. (7)

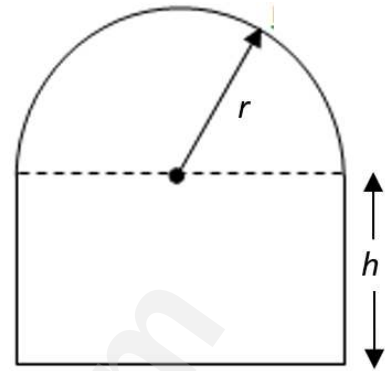
- (b) Hence, determine the other stationary point and the point of inflection. (5)

- (c) Use the graph to solve: $f'(x).f''(x) \geq 0$. (3)

[15]

QUESTION 12

The given shape consists of a semi-circle of radius r cm and a rectangle with a height of h cm.



- (a) Write an expression for the area of the shape, in terms of r and h . (2)

- (b) If the shape is subject to the constraint that the perimeter of the rectangle must be 20 cm, determine the value of r that will yield the maximum possible area of the shape. (5)

- (c) Prove that the value found is a maximum. (2)

QUESTION 13

- (a) Prove, showing all working, that $\sqrt{2,7777\dots}$ is a rational number. (3)

- (b) The first 2022 square numbers are written down:

$$1^2; 2^2; 3^2; 4^2; \dots; 2022^2$$

Find the sum of the remainders of these numbers when divided by 4. (3)

- (c) The equation $x^2 + 4x - 8 = 0$ has roots $x = p$ and $x = q$. Find, without solving for p and q , a quadratic equation with roots $x = p^2$ and $x = q^2$. (4)

- (d) You are given that $1 + 4 + 9 + 16 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Hence calculate: $\sum_{x=1}^{100} (6x^2 - 2x - 1)$ (6)

[16]

72 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

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