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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2023

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A**QUESTION 1**

1.1 25 minutes at A

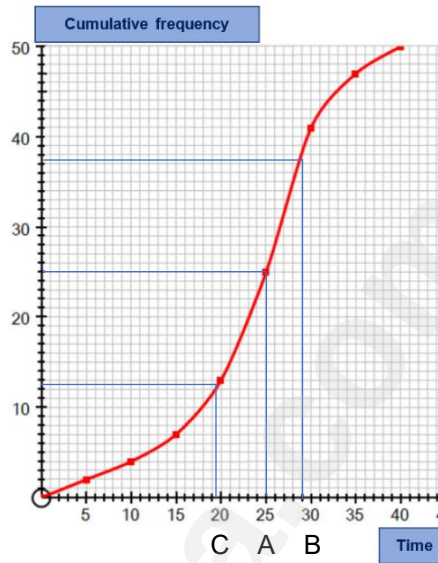
1.2 $0,75 \times 50 = 37,4$

29 min at B
(28-30)

$0,25 \times 50 = 12,5$

19,5 min at C
(18-20)

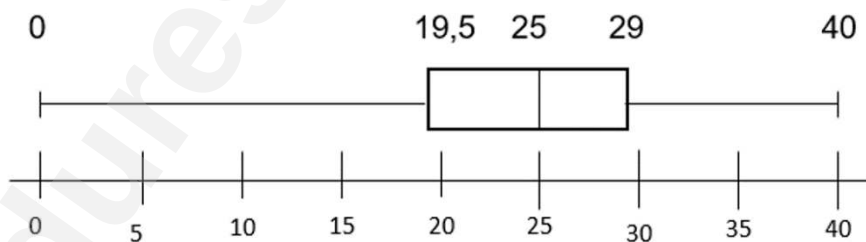
IQR = $29 - 19,5$ or $(30 \text{ to } 28) - (20 \text{ to } 18)$
= 9,5 min



1.3 $13 - 7 = 6$ people (+1 on final answer)

1.4 80% of 50 = 40 people
Y = 18 min (2 marks if only 18 min)

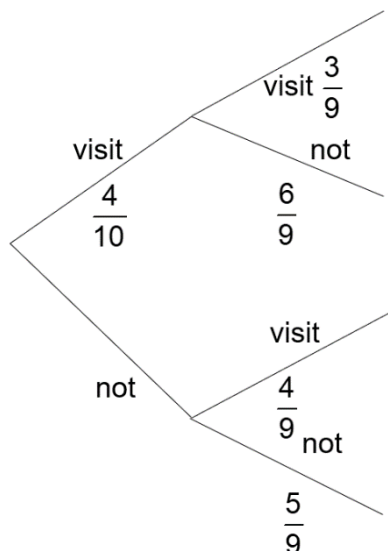
1.5 Min = 0
Max = 40
 $Q_1 = 19,5$
 $Q_2 = 25$
 $Q_3 = 29$



QUESTION 2

2.1 2.1.1 $\frac{6}{10} = \frac{3}{5}$ or 0,6 or 60%

2.1.2



$$\frac{4}{9} \times \frac{3}{9} + \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{2}{3}$$

OR

$$1 - \frac{6}{10} \times \frac{5}{9} = \frac{2}{3}$$

for correct answer or for correct tree diagram if answer is wrong.

2.2 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

0,63 = $3P(B) + P(B) - 0$ (mutually exclusive)

$\therefore 0,63$ = $4P(B)$

0,1575 = $P(B)$

(0,2 accepted if backed up by calculations)

QUESTION 3

$$3.1 \quad \text{Midpt AB} = \left(\frac{-3+1}{2}; \frac{-2+4}{2} \right) \\ = (-1; 1)$$

Centre of circle is $(-1; 1)$

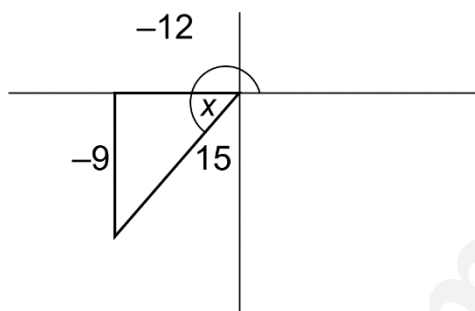
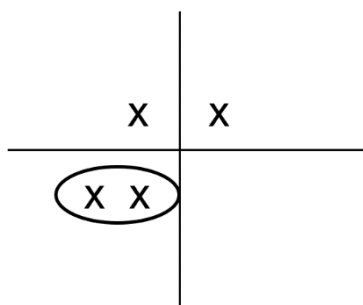
$$AB = \sqrt{(-3-1)^2 + (-2-4)^2} \\ = \sqrt{52} \\ \text{radius} = \sqrt{13}$$

Equation of circle is $(x+1)^2 + (y-1)^2 = 13$

$$3.2 \quad m_{OA} = \frac{1-(-2)}{-1-(-3)} \quad O(-1; 1) \text{ and } A(-3; -2) \\ = \frac{3}{2} \\ M_{\text{tangent}} = \frac{-2}{3} \quad (\text{radius} \perp \text{tangent}) \\ y - y_1 = m(x - x_1) \\ y - (-2) = -\frac{2}{3}(x - (-3)) \\ y + 2 = -\frac{2}{3}x - 2 \\ y = -\frac{2}{3}x - 4$$

QUESTION 4

4.1



$$\cos x = -\frac{12}{15} = -\frac{4}{5}$$

$$\text{Pythagoras: } 15^2 - (-12)^2 = 81$$

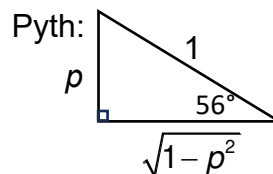
$$\therefore y = -9$$

$$\sin x = -\frac{9}{15} \text{ or } -\frac{3}{5}$$

$$\begin{aligned} 4.2 \quad 4.2.1 \quad \sin(-236)^\circ &= -\sin 236^\circ \\ &= -(-\sin 56^\circ) \\ &= \sin 56^\circ \\ &= p \end{aligned}$$

$$\begin{aligned} \text{OR} \quad \sin(-236)^\circ &= \sin 124^\circ \\ &= \sin 56^\circ \\ &= p \end{aligned}$$

$$\begin{aligned} 4.2.2 \quad \cos 304^\circ &= \cos 56^\circ \\ \cos 56^\circ &= \sqrt{1 - p^2} \end{aligned}$$



OR

$$\begin{aligned} \cos^2 56^\circ &= 1 - \sin^2 56^\circ \\ &= 1 - p^2 \\ \therefore \cos 56^\circ &= \sqrt{1 - p^2} \end{aligned}$$

$$\begin{aligned} 4.3 \quad \cos 2x &= \cos[90^\circ - x - 30^\circ] \\ \cos 2x &= \cos(120^\circ - x) \end{aligned}$$

$$\begin{aligned} \text{I: } 2x &= 120^\circ - x + 360^\circ k \\ 3x &= 120^\circ + 360^\circ k \\ x &= 40^\circ + 120^\circ k \end{aligned}$$

$$\begin{aligned} \text{IV: } 2x &= 360^\circ - 120^\circ + x + 360^\circ k \\ x &= 240^\circ + 360^\circ k \\ k &\in \mathbb{Z} \end{aligned}$$

$$\text{In } [-180^\circ; 90^\circ]: x \in \{40^\circ; -80^\circ; -120^\circ\}$$

$$\text{OR } \sin(90^\circ - 2x) = \sin(x - 30^\circ)$$

$$90^\circ - 2x = x - 30^\circ + k \cdot 360^\circ$$

$$-3x = -120^\circ + k \cdot 360^\circ$$

$$x = 40^\circ + k \cdot 120^\circ$$

$$\text{or } 90^\circ - 2x = 180^\circ - x + 30^\circ + k \cdot 360^\circ$$

$$-x = 120^\circ + k \cdot 360^\circ$$

$$x = -120^\circ - k \cdot 360^\circ$$

QUESTION 5

$$\begin{aligned}
 5.1 \quad \text{LHS} &= \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)^2 \\
 &= \left(\frac{1}{\sin x} - \frac{1}{\frac{\sin x}{\cos x}} \right)^2 \\
 &= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)^2 \\
 &= \frac{(1 - \cos x)^2}{\sin^2 x} \\
 &= \frac{(1 - \cos x)^2}{1 - \cos^2 x} \\
 &= \frac{(1 - \cos x)^2}{(1 - \cos x)(1 + \cos x)} \\
 &= \frac{1 - \cos x}{1 + \cos x} \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad \frac{1 - \cos x}{1 + \cos x} &= \frac{2}{5} \\
 5 - 5 \cos x &= 2 + 2 \cos x \\
 3 &= 7 \cos x \\
 \frac{3}{7} &= \cos x
 \end{aligned}$$

$$\text{Ref angle} = 64,6^\circ$$

$$\begin{aligned}
 \therefore x = 64,6^\circ \quad \text{or} \quad x &= 360^\circ - 64,6^\circ \\
 &= 295,4^\circ
 \end{aligned}$$

QUESTION 6

$$6.1 \quad p = 2; \quad q = 2; \quad r = 2; \quad t = 1$$

$$6.2 \quad 360^\circ$$

$$6.3 \quad -2$$

$$6.4 \quad -180^\circ \leq x < -90^\circ$$

QUESTION 7

- 7.1 $a = 33^\circ$ $\angle$ s opp. equal sides
 $b = 114^\circ$ int. $\angle$ s of Δ supplementary
 $c = 57^\circ$ $\angle$ at centre = $2 \times \angle$ at circumference
 $d = 123^\circ$ opp. $\angle$ s of cyclic quad. supplementary
 $e = 121^\circ$ opp. $\angle$ s of cyclic quad. supplementary

- 7.2 7.2.1 $\hat{G} = \hat{C}_1 + \hat{C}_2$ $\angle$ s in same segment
 but $\hat{C}_1 = \hat{A}_1$ $\angle$ s in same segment
 and $\hat{C}_2 = \hat{B}$ $\angle$ s opp. equal sides or isosceles triangle
 $\therefore \hat{G} = \hat{B} + \hat{A}_1$

- 7.2.2 $\hat{D} = \hat{B} + \hat{A}_1$ ext. $\angle$ of ΔABD
 $\therefore \hat{D}_1 = \hat{G} = \hat{B} + \hat{A}_1$
 $\therefore DFGE$ is a cyclic quad ext $\angle$ of quad = int. opp. $\angle$ or conv. ext
 $\angle$ of cycl. quad

SECTION B**QUESTION 8**

$$8.1 \quad \frac{a-1}{2} = 1 \quad \text{and} \quad \frac{b-4}{2} = 0 \quad \text{or by inspection}$$

$$a-1=2 \quad b-4=0$$

$$a=3 \quad b=4$$

$$8.2 \quad m_{QR} = \frac{4 - (-4)}{3 - (-1)}$$

$$= \frac{8}{4}$$

$$= 2$$

$$\text{Perpendicular } m = -\frac{1}{2}$$

$$(1; 0) \quad y - 0 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$2y + x - 1 = 0$$

$$8.3 \quad 7x - y + 8 = 0 \quad \text{--- (1)}$$

$$x + 2y - 1 = 0 \quad \text{--- (2)}$$

$$\text{From (2): } x = -2y + 1 \quad \text{--- (3)}$$

$$\text{Substitute (3) in (1): } 7(-2y + 1) - y + 8 = 0$$

$$-14y + 7 - y + 8 = 0$$

$$-15y = -15$$

$$y = 1$$

$$x = -2(1) + 1$$

$$x = -1$$

$$\therefore T(-1; 1)$$

Max 2 marks if no method

QUESTION 9

$$9.1 \quad \hat{y} = 4,81 + 1,32x$$

$$9.2 \quad \hat{y} = 4,81 + 1,32(16)$$

R25 930

$$9.3 \quad r = 0,95 \quad \text{May not round to } r = 1$$

9.4 When time taken to write the app is 0

$$\hat{y} = 4,81 + 1,32(0)$$

R4 810

QUESTION 10

$$10.1 \quad \frac{6!}{2!} = 360 \text{ — } =$$

$$10.2 \quad (\text{FBA}) \text{ follow} = \frac{6!}{2!2!} \times 3!$$

$$\begin{aligned} P(\text{FBA}) &= \frac{\frac{6!3!}{2!2!}}{8!} \\ &= \frac{6}{56} \\ &= \frac{3}{28} \end{aligned}$$

QUESTION 11

$$\begin{aligned}
 11.1 \quad & \sin^2 15^\circ + 2\sin 15^\circ \cos 15^\circ + \cos^2 15^\circ \\
 &= 1 + 2\sin 30^\circ \quad 1 \quad 2\sin 30^\circ \\
 &= 1,5 \text{ or } 1\frac{1}{2}
 \end{aligned}$$

$$11.2 \quad \text{In } \triangle ABC: \hat{C} = 180^\circ - (\beta + \alpha) \quad \text{int } \angle \text{ of } \triangle$$

$$LHS \frac{AC}{\sin \alpha} = \frac{k}{\sin(180^\circ - (\beta + \alpha))}$$

$$AC = \frac{k \sin \alpha}{\sin(\beta + \alpha)}$$

$$\text{In } \triangle ACF: \tan \theta = \frac{FC}{AC}$$

$$\tan \theta \times \frac{k \sin \alpha}{\sin(\beta + \alpha)} = FC$$

$$FC = \frac{k \sin \alpha \tan \theta}{\sin(\beta + \alpha)} \text{ must follow}$$

QUESTION 12

$$\begin{aligned}
 12.1 \quad & \hat{E}_4 = \hat{E}_1 \quad \text{vert. opp. } \angle \text{ s equal} \\
 & \hat{E}_1 = \hat{B}_1 \quad \angle \text{ between tan, chord} \\
 & \hat{B}_1 = \hat{D} \quad \text{ext. } \angle \text{ of cyclic quad} \\
 & \therefore \hat{D} = \hat{E}_4 \\
 & \therefore CE = CD \quad \text{sides opp. equal } \angle \text{ s (may not use isosc } \triangle)
 \end{aligned}$$

$$12.2 \quad \text{In } \triangle CBE \text{ and } \triangle CEA:$$

$$\begin{aligned}
 & \hat{C}_1 = \hat{C}_1 \quad \text{common} \\
 & \hat{E}_3 = \hat{A} \quad \text{tan – chord theorem} \\
 & \therefore \hat{B}_2 = \hat{AEC} \quad \text{int } \angle \text{ s of } \triangle \\
 & \therefore \triangle CEB \sim \triangle CAE \quad \angle \angle \angle
 \end{aligned}$$

$$\begin{aligned}
 12.3 \quad & \frac{CE}{CA} = \frac{CB}{CE} \quad \text{Similar } \triangle \text{ s} \\
 & CE^2 = CA \cdot CB
 \end{aligned}$$

QUESTION 1313.1 13.1.1 $\widehat{D} \equiv 90^\circ$ $\tan \perp$ radius

$$OE = x + 2$$

$$(x+2)^2 = x^2 + 4^2$$

$$x^2 + 4x + 4 = x^2 + 16$$

$$4x = 12 \text{ simplified form}$$

$$x = 3 \text{ cm}$$

$$\begin{aligned} 13.1.2 \quad V \text{ of cone} - V \text{ of sphere} &= \pi(3)^2 \left(\frac{8}{3} \right) - \text{minus } \frac{4}{3} \pi(6) \\ &= 50,3 \text{ cm}^3 \end{aligned}$$

13.2 13.2.1 In $\triangle OXQ$ and $\triangle OYS$:

$$\hat{O} = \hat{O}$$

common

$$\hat{X} = \hat{Y}$$

corr. $\angle$ s $XQ \parallel YS$

$$\hat{Q} = \hat{S}$$

corr. $\angle$ s $XQ \parallel YS$ or third $\angle$ of $\triangle$

$$\therefore \triangle OXQ \parallel \triangle OYS$$

 $\angle < \angle$ for $\parallel$

$$\frac{OX}{OY} = \frac{XQ}{YS}$$

$$\frac{h}{h+12} = \frac{5}{20}$$

$$20h = 5h + 60$$

$$h = 4$$

13.2.2 Volume of frustum = volume of bigger cone – volume of smaller cone

$$= \frac{1}{3} \pi (20)^2 (16) - \frac{1}{3} \pi (5)^2 (4)$$

$$= 2100\pi \text{ cm}^3$$

QUESTION 14

14.1 $\frac{CE}{CG} = \frac{CF}{CD}$ line $\parallel$ one side of Δ

$$\therefore \frac{CE}{CG} = \frac{2}{5}$$

But $CG = GA$ diagonals of parm

$$\therefore \frac{CE}{CA} = \frac{2}{10} = \frac{1}{5}$$

14.2 In $\triangle ECF$ and $\triangle GCD$:

$$\hat{C} = \hat{C} \quad \text{common}$$

$$\hat{CEF} = \hat{CGD} \quad \text{corr. } \angle \text{ s } EF \parallel GD$$

$$\hat{CFE} = \hat{CDG} \quad \text{corr. } \angle \text{ s } EF \parallel GD$$

$$\therefore \triangle ECF \parallel \triangle GCD \quad \angle \angle \angle$$

$$\text{Sides of } \triangle ECF = \frac{2}{5} \text{ sides of } \triangle GCD$$

$$\therefore \text{area of } \triangle ECF = \left(\frac{2}{5}\right)^2 \text{ area of } \triangle GCD$$

$$\therefore \text{area of } GEFD = \frac{21}{25} \text{ area of } \triangle GCD$$

But area of $\triangle GCD = \triangle GAD = \triangle GAB = \triangle GCB$

(equal bases, common heights)

$$\therefore \frac{\text{area } GEFD}{\text{area } ABCD} = \frac{21}{100}$$

$$\therefore \frac{\text{oppervlakte } GEFD}{\text{oppervlakte } ABCD} = \frac{21}{100}$$

OR

$$\triangle CEF \parallel \triangle CGD \quad \angle \angle \angle$$

$$\text{Area } GEFD = \text{Area } \triangle GDC - \text{area } \triangle FCE$$

$$= \frac{1}{2} 5 \times 5 \sin \hat{D} - \frac{1}{2} 2 \times 2 \sin \hat{D} \text{ since } \hat{D} = \hat{F}$$

$$= 21 \times \sin \hat{D}$$

$$\text{Area of parallelogram} = 4 \times \text{area of } \triangle GDC$$

$$= 4 \times \frac{1}{2} 5 \times 5 \sin \hat{D}$$

$$= 50 \sin \hat{D}$$

$$\frac{21}{50} = \frac{21}{100}$$

Total: 150 marks