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INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
NOVEMBER 2023

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A**VRAAG 1**

1.1 25 minute by A

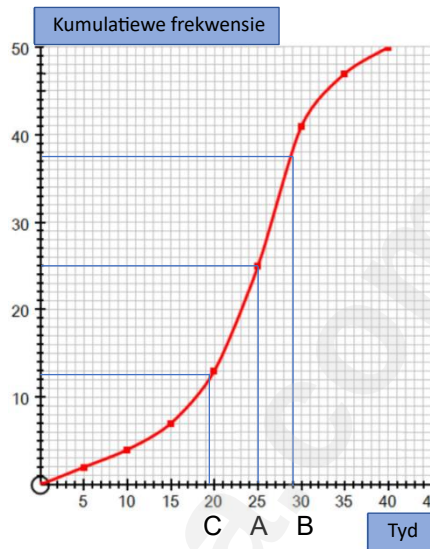
1.2 $0,75 \times 50 = 37,4$

29 minute by B
(28–20)

$0,25 \times 50 = 12,5$

19,5 minute by C

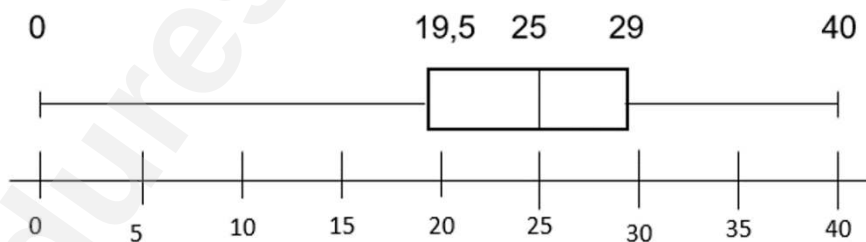
$IKV = 29 - 19,5$
 $= 9,5 \text{ minute}$
(18–20)



1.3 $13 - 7 = 6 \text{ mense (+1 op finale antwoord)}$

1.4 80% van 50 = 40 mense
 $y = 18 \text{ minute (2 punte as slegs 18 min)}$

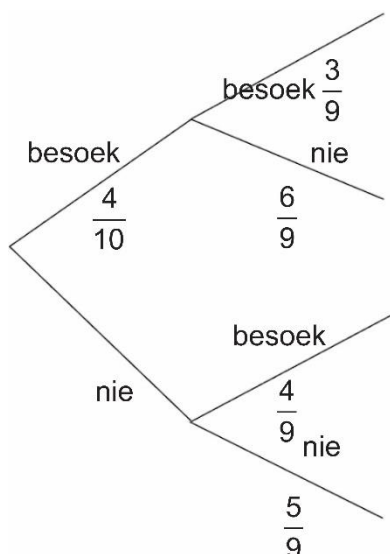
1.5 Min = 0
Maks = 40
 $Q_1 = 19,5$
 $Q_2 = 25$
 $Q_3 = 29$



VRAAG 2

2.1 2.1.1 $\frac{6}{10} = \frac{3}{5}$ of 0,6 of 60%

2.1.2



$$\frac{4}{9} \times \frac{3}{9} + \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{2}{3}$$

OF

$$1 - \frac{6}{10} \times \frac{5}{9} = \frac{2}{3}$$

vir korrekte antwoord of vir korrekte boomdiagram indien antwoord verkeerd is.

2.2 $P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$

$$0,63 = 3P(B) + P(B) - 0 \text{ (onderling uitsluitend)}$$

$$\therefore 0,63 = 4P(B)$$

$$0,1575 = P(B)$$

(0,2 aanvaar indien berekenings dit ondersteun)

VRAAG 3

$$3.1 \quad \text{Middelpunt AB} = \left(\frac{-3+1}{2} ; \frac{-2+4}{2} \right) \\ = (-1 ; 1)$$

Middelpunt van sirkel is $(-1 ; 1)$.

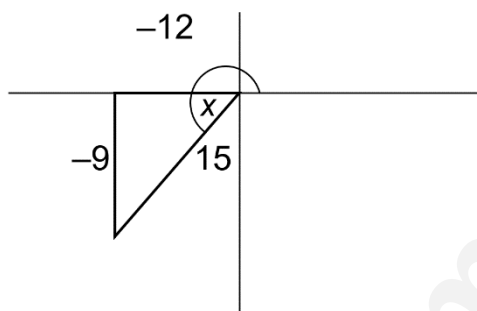
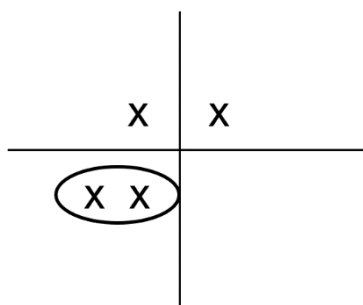
$$AB = \sqrt{(-3-1)^2 + (-2-4)^2} \\ = \sqrt{52} \\ \text{radius} = \sqrt{13}$$

Vergelyking van sirkel is $(x+1)^2 + (y-1)^2 = 13$

$$3.2 \quad m_{OA} = \frac{1-(-2)}{-1-(-3)} \quad O(-1 ; 1) \text{ en } A(-3 ; -2) \\ = \frac{3}{2} \\ M_{\text{raaklyn}} = \frac{-2}{3} \quad (\text{radius} \perp \text{raaklyn}) \\ y - y_1 = m(x - x_1) \\ y - (-2) = -\frac{2}{3}(x - (-3)) \\ y + 2 = -\frac{2}{3}x - 2 \\ y = -\frac{2}{3}x - 4$$

VRAAG 4

4.1



$$\cos x = -\frac{12}{15} = -\frac{4}{5}$$

$$\text{Pythagoras: } 15^2 - (-12)^2 = 81$$

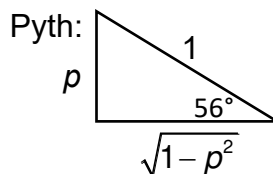
$$\therefore y = -9$$

$$\sin x = -\frac{9}{15} \text{ of } -\frac{3}{5}$$

$$\begin{aligned} 4.2 \quad 4.2.1 \quad \sin(-236)^\circ &= -\sin 236^\circ \\ &= -(-\sin 56^\circ) \\ &= \sin 56^\circ \\ &= p \end{aligned}$$

$$\begin{aligned} \text{OF} \quad \sin(-236)^\circ &= \sin 124^\circ \\ &= \sin 56^\circ \\ &= p \end{aligned}$$

$$\begin{aligned} 4.2.2 \quad \cos 304^\circ &= \cos 56^\circ \\ \cos 56^\circ &= \sqrt{1 - p^2} \end{aligned}$$

**OF**

$$\begin{aligned} \cos^2 56^\circ &= 1 - \sin^2 56^\circ \\ &= 1 - p^2 \\ \therefore \cos 56^\circ &= \sqrt{1 - p^2} \end{aligned}$$

$$\begin{aligned} 4.3 \quad \cos 2x &= \cos[90^\circ - x - 30^\circ] \\ \cos 2x &= \cos(120^\circ - x) \end{aligned}$$

$$\begin{aligned} \text{I: } 2x &= 120^\circ - x + 360^\circ k \\ 3x &= 120^\circ + 360^\circ k \\ x &= 40^\circ + 120^\circ k \quad 120^\circ \end{aligned}$$

$$\begin{aligned} \text{IV: } 2x &= 360^\circ - 120^\circ + x + 360^\circ k \\ x &= 240^\circ + 360^\circ k \\ k &\in \mathbb{Z} \end{aligned}$$

$$\text{In } [-180^\circ; 90^\circ]: x \in \{40^\circ; -80^\circ; -120^\circ\}$$

$$\text{OF } \sin(90^\circ - 2x) = \sin(x - 30^\circ)$$

$$90^\circ - 2x = x - 30^\circ + k \cdot 360^\circ$$

$$-3x = -120^\circ + k \cdot 360^\circ$$

$$x = 40^\circ + k \cdot 120^\circ$$

$$\text{of } 90^\circ - 2x = 180^\circ - x + 30^\circ + k \cdot 360^\circ$$

$$-x = 120^\circ + k \cdot 360^\circ$$

$$x = -120^\circ - k \cdot 360^\circ$$

VRAAG 5

$$\begin{aligned}
 5.1 \quad \text{LK} &= \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)^2 \\
 &= \left(\frac{1}{\sin x} - \frac{1}{\frac{\sin x}{\cos x}} \right)^2 \\
 &= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)^2 \\
 &= \frac{(1 - \cos x)^2}{\sin^2 x} \\
 &= \frac{(1 - \cos x)^2}{1 - \cos^2 x} \\
 &= \frac{(1 - \cos x)^2}{(1 - \cos x)(1 + \cos x)} \\
 &= \frac{1 - \cos x}{1 + \cos x} \\
 &= \text{RK}
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad \frac{1 - \cos x}{1 + \cos x} &= \frac{2}{5} \\
 5 - 5 \cos x &= 2 + 2 \cos x \\
 3 &= 7 \cos x \\
 \frac{3}{7} &= \cos x
 \end{aligned}$$

$$\text{Verwysingshoek} = 64,6^\circ$$

$$\begin{aligned}
 \therefore x &= 64,6^\circ \quad \text{of} \quad x = 360^\circ - 64,6^\circ \\
 &= 295,4^\circ
 \end{aligned}$$

VRAAG 6

$$6.1 \quad p = 2; \quad q = 2; \quad r = 2; \quad t = 1$$

$$6.2 \quad 360^\circ$$

$$6.3 \quad -2$$

$$6.4 \quad -180^\circ \leq x < -90^\circ$$

VRAAG 7

- 7.1 $a = 33^\circ$ $\angle$ e teenoor gelyke sye
 $b = 114^\circ$ binne $\angle$ e van Δ supplementêr
 $c = 57^\circ$ $\angle$ by middelpunt = $2 \times \angle$ by omtrek
 $d = 123^\circ$ teenoorst. $\angle$ e van koordevierhoek supplementêr
 $e = 121^\circ$ teenoorst. $\angle$ e van koordevierhoek supplementêr

- 7.2 7.2.1 $\hat{G} = \hat{C}_1 + \hat{C}_2$ $\angle$ e in dieselfde segment
 maar $\hat{C}_1 = \hat{A}_1$ $\angle$ e in dieselfde segment
 en $\hat{C}_2 = \hat{B}$ $\angle$ e teenoor gelyke sye of gelykbenige driehoek
 $\therefore \hat{G} = \hat{B} + \hat{A}_1$

- 7.2.2 $\hat{D} = \hat{B} + \hat{A}_1$ buite $\angle$ van $\triangle ABD$
 $\therefore \hat{D}_1 = \hat{G} = \hat{B} + \hat{A}_1$
 $\therefore DFGE$ is koordevierh. buite $\angle$ van koordevierh. = teenoorst.
 binne $\angle$ of conv. buite $\angle$ van koordevierh

AFDELING B**VRAAG 8**

$$8.1 \quad \frac{a-1}{2} = 1 \quad \text{en} \quad \frac{b-4}{2} = 0 \text{ of deur inspeksie}$$

$$a-1=2 \quad b-4=0$$

$$a=3 \quad b=4$$

$$8.2 \quad m_{QR} = \frac{4 - (-4)}{3 - (-1)}$$

$$= \frac{8}{4}$$

$$= 2$$

$$\text{Loodregte } m = -\frac{1}{2}$$

$$(1; 0) \quad y - 0 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

$$2y + x - 1 = 0$$

$$8.3 \quad 7x - y + 8 = 0 \quad \text{--- (1)}$$

$$x + 2y - 1 = 0 \quad \text{--- (2)}$$

$$\text{Uit (2): } x = -2y + 1 \quad \text{--- (3)}$$

$$\text{Vervang (3) in (1):} \quad 7(-2y + 1) - y + 8 = 0$$

$$-14y + 7 - y + 8 = 0$$

$$-15y = -15$$

$$y = 1$$

$$x = -2(1) + 1$$

$$x = -1$$

$$\therefore T(-1; 1)$$

Maks 2 punte as geen metode

VRAAG 9

$$9.1 \quad \hat{y} = 4,81 + 1,32x$$

$$9.2 \quad \hat{y} = 4,81 + 1,32(16)$$

R25 930

$$9.3 \quad r = 0,95 \quad \text{Mag nie rond tot } r = 1 \text{ nie}$$

9.4 Wanneer tyd geneem om die app te skryf 0 is

$$\hat{y} = 4,81 + 1,32(0)$$

R4 810

VRAAG 10

$$10.1 \quad \frac{6!}{2!} = 360 \text{ — } =$$

$$10.2 \quad (\text{FBA}) \text{ volg} = \frac{6!}{2!2!} \times 3!$$

$$\begin{aligned} P(\text{FBA}) &= \frac{\frac{6!3!}{2!2!}}{8!} \\ &= \frac{6}{56} \\ &= \frac{3}{28} \end{aligned}$$

VRAAG 11

$$\begin{aligned}
 11.1 \quad & \sin^2 15^\circ + 2\sin 15^\circ \cos 15^\circ + \cos^2 15^\circ \\
 &= 1 + 2\sin 30^\circ \quad 1 \quad 2\sin 30^\circ \\
 &= 1,5 \text{ of } 1\frac{1}{2}
 \end{aligned}$$

$$11.2 \quad \text{In } \triangle ABC: \hat{C} = 180^\circ - (\beta + \alpha) \quad \text{binne } \angle \text{ van } \triangle$$

$$LHS \frac{AC}{\sin \alpha} = \frac{k}{\sin(180^\circ - (\beta + \alpha))}$$

$$AC = \frac{k \sin \alpha}{\sin(\beta + \alpha)}$$

$$\text{In } \triangle ACF: \tan \theta = \frac{FC}{AC}$$

$$\tan \theta \times \frac{k \sin \alpha}{\sin(\beta + \alpha)} = FC$$

$$FC = \frac{k \sin \alpha \tan \theta}{\sin(\beta + \alpha)} \text{ moet volg}$$

VRAAG 12

$$\begin{aligned}
 12.1 \quad & \hat{E}_4 = \hat{E}_1 \quad \text{regoorstaande } \angle \text{ e gelyk} \\
 & \hat{E}_1 = \hat{B}_1 \quad \angle \text{ tussen raaklyn, koord} \\
 & \hat{B}_1 = \hat{D} \quad \text{buite } \angle \text{ van koordevierhoek} \\
 & \therefore \hat{D} = \hat{E}_4 \\
 & \therefore CE = CD
 \end{aligned}$$

sy teenoorst. gelyk $\angle$ s (mag nie gelykbenige $\triangle$ gebruik nie)

$$12.2 \quad \text{In } \triangle CBE \text{ en } \triangle CEA:$$

$$\begin{aligned}
 & \hat{C}_1 = \hat{C}_1 \quad \text{gemeenskaplik} \\
 & \hat{E}_3 = \hat{A} \quad \text{raaklyn-koord-stelling} \\
 & \therefore \hat{B}_2 = \hat{AEC} \quad \text{binne } \angle \text{ e van } \triangle \\
 & \therefore \triangle CEB \sim \triangle CAE \quad \angle \angle \angle
 \end{aligned}$$

$$\begin{aligned}
 12.3 \quad & \frac{CE}{CA} = \frac{CB}{CE} \quad \text{soortgelyke } \triangle \text{'e} \\
 & CE^2 = CA \cdot CB
 \end{aligned}$$

VRAAG 1313.1 13.1.1 $\widehat{D} \equiv 90^\circ$ $\tan \perp$ radius

$$OE = x + 2$$

$$(x+2)^2 = x^2 + 4^2$$

$$x^2 + 4x + 4 = x^2 + 16$$

$$4x = 12 \text{ vereenvoudigde vorm}$$

$$x = 3 \text{ cm}$$

$$13.1.2 \quad V \text{ van keël} - V \text{ van sfeer} = \pi(3)^2 \left(\frac{8}{3} \right) - \text{minus } \frac{4}{3}\pi(6)$$

$$= 50,3 \text{ cm}^3$$

13.2 13.2.1 In $\triangle OXQ$ en $\triangle OYS$:

$$\hat{O} = \hat{O}$$

gemeenskaplik

$$\hat{X} = \hat{Y}$$

ooreenkomstige $\angle$ e $XQ \parallel YS$

$$\hat{Q} = \hat{S}$$

ooreenkomstige $\angle$ e $XQ \parallel YS$ of derde $\angle$ van $\triangle$

$$\therefore \triangle OXQ \parallel \triangle OYS \quad \angle \angle \angle \text{ vir } \parallel$$

$$\frac{OX}{OY} = \frac{XQ}{YS}$$

$$\frac{h}{h+12} = \frac{5}{20}$$

$$20h = 5h + 60$$

$$h = 4$$

13.2.2 Volume van afgeknotte keël
= volume van groter keël – volume van kleiner keël

$$= \frac{1}{3}\pi(20)^2(16) - \frac{1}{3}\pi(5)^2(4)$$

$$= 2100\pi \text{ cm}^3$$

VRAAG 14

14.1 $\frac{CE}{CG} = \frac{CF}{CD}$ lyn $\parallel$ een sy van Δ

$$\therefore \frac{CE}{CG} = \frac{2}{5}$$

Maar $CG = GA$ hoeklyne van parm

$$\therefore \frac{CE}{CA} = \frac{2}{10} = \frac{1}{5}$$

14.2 In $\triangle ECF$ en $\triangle GCD$:

$$\hat{C} = \hat{C}$$

gemeenskaplik

$$\hat{CEF} = \hat{CGD}$$

ooreenkomstige $\angle$ e $EF \parallel GD$

$$\hat{CFE} = \hat{CDG}$$

ooreenkomstige $\angle$ e $EF \parallel GD$

$$\therefore \triangle ECF \sim \triangle GCD \quad \angle \angle \angle$$

$$\text{Sye van } \triangle ECF = \frac{2}{5} \text{ sye van } \triangle GCD$$

$$\therefore \text{opp. van } \triangle ECF = \left(\frac{2}{5}\right)^2 \text{ opp. van } \triangle GCD$$

$$\therefore \text{opp. van } GEFD = \frac{21}{25} \text{ opp. van } \triangle GCD$$

Maar oppervlakte van $\triangle GCD = \triangle GAD = \triangle GAB = \triangle GCB$

(gelyke basisse, gemeenskaplike hoogtes)

$$\therefore \frac{\text{oppervlakte } GEFD}{\text{oppervlakte } ABCD} = \frac{21}{100}$$

OF

$$\triangle CEF \sim \triangle CGD \quad \angle \angle \angle$$

Oppervlak $GEFD = \text{Oppervlak } \triangle GDC - \text{oppervlak } \triangle FCE$

$$= \frac{1}{2} 5 \times 5 \sin \hat{D} - \frac{1}{2} 2 \times 2 \sin \hat{D} \text{ aangesien } \hat{D} = \hat{F}$$

$$= 21 \times \sin \hat{D}$$

Oppervlak van parallelogram = 4x oppervlak van $\triangle GDC$

$$= 4 \times \frac{1}{2} 5 \times 5 \sin \hat{D}$$

$$= 50 \sin \hat{D}$$

$$\frac{21}{50} = \frac{21}{100}$$

Totaal: 150 punte